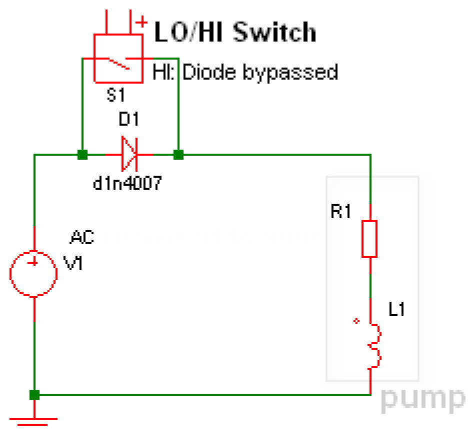
AQUARIUM AIR PUMP

LIMITING OUTPUT POWER MOD FULL COMPLEX ANALYSIS

STAGE 1: Modeling

a. real model



How to model/obtain R and L values of an air-pump?

1. Digital ohm-meter is used to directly measure R
2. "UI" method is used to indirectly calculate L

2.1 Measure AC line RMS voltage **U [V]** with digital multimeter

2.2 Measure AC line pump's RMS current **I [A]** with digital multimeter

2.3 Find effective complex impedance value **Z [Ω]**:
$Z = U/I$

2.4 Find moduo of imaginery impedance part **X_L [Ω]**:

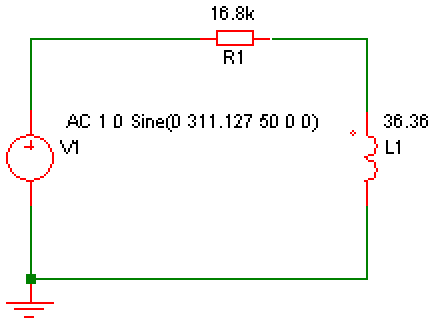
$Z = U/I$
$Z = \sqrt{(R^2 + X_L^2)}$
$X_L = \sqrt{(U^2/I^2 - R^2)}$

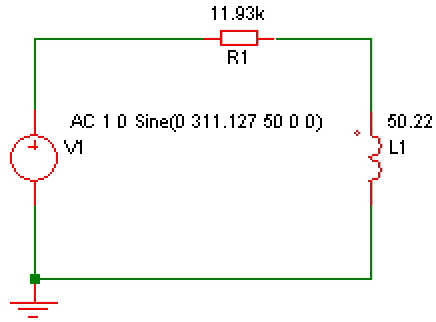
2.5 Find **L [H]**:

$X_L = \omega L = 2*PI*freq*L$
$L = X_L / (2*PI*freq)$

where PI = 3.1415 (constant) and freq = 50 or 60 [Hertz] main power AC line's frequency.

b. simplified models (diode & switch omited = e.g. switch is in HI position)





Two pump models are presented: **ATMAN Champion CX-0078** revision A and revision B (same models, different production batches). The differences are in the electro-magnetic coils: number of windings and used coil wires are different, thus equivalent serial reisistance and inductance are different. Model B has twice the air-pumping power (empirical and experimental conclusion, no actual scientific measurements are taken), it is also much louder (acoustic vibrations are stronger).

Model A:

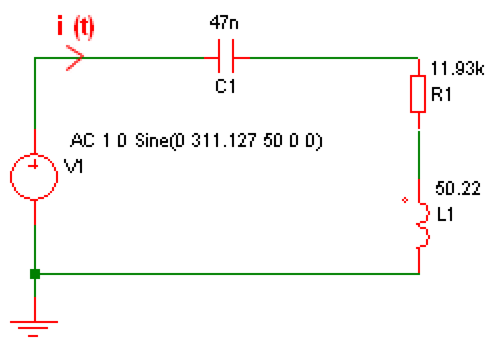
$R_{equivalent}$ = 16.8 [kΩ]
L_{coil} = 36.36 [H]

Model B:

$R_{equivalent}$ = 11.93 [kΩ]
L_{coil} = 50.22 [H]

STAGE 2: Full Complex Analysis with C limit

For simplicity, only second revision (Model B) will be analysed. The procedure is identical for any aquarium pump model with simple coil.



For a series RLC circuit, total complex impedance $\underline{\mathbf{Z}}$ is equal to:

$\underline{Z} = \underline{Z}_R + \underline{Z}_L + \underline{Z}_C$

$\underline{Z} = R + j\omega L + 1/j\omega C$

$\underline{Z} = 11930 + j314.159*50.22 + 1/(j314.159*47*10^{-9})$

$\underline{Z} = 11930 + j15708 + 67725/j$

the last part 67725/j can be multiplied and divided by term j/j without changing the overall value, thus the equation becomes:

$\underline{Z} = 11930 + j15708 + (67725*j)/(j*j)$

since imaginary unit is defined as $j=\sqrt{(-1)}$, thus $j*j = j^2 = -1$

$\underline{Z} = 11930 + j15708 – j67725$

$\underline{Z} = 11930 + j\ (15708 – 67725)$

$\underline{Z} = 11930 – j52017$, from here we see that the total impedance $\underline{Z}$ is now **capacitive load** in nature.

Z (Real Part) = 11930 [Ω]
Z (Imaginery Part) = 52017 [Ω]

Euler's tranformation:

$\underline{Z} = |Z| * e^{j\Theta}$

where:

$|Z| = \sqrt{}$ (Real Part^2 + Imaginary Part^2)
$|Z|$ = moduo $\underline{Z}$ (effective (RMS) value of total impedance $\underline{Z}$)

Θ = arc tan (Imaginery Part / Real Part)
phase shift of total impedance $\underline{Z}$

From Euler's transformation we will gather complex argument Θ which will explain phase shifts between main current and voltage:

$|Z| = \sqrt{}(11930^2 + (-52017)^2)$

$|Z| = 53367.5$ [Ω]

Θ = arc tan (-52017 / 11930)

Θ= arc tan (-4.36)

Θ = -77.08 [degrees] (note: switch Calculator's mode to degrees!)

$\underline{Z} = 53367.5 * e^{j(-77.08)}$ [Ω] written in Euler's form

Now we can calculate AC current in the series RLC circuit:

$\underline{I} = \underline{U} / \underline{Z}$

$\underline{I} = (220 * e^{j0}) / (53367.5 * e^{j(-77.08)}$ [A], note: we assume 0 phase shift of AC Main's power voltage e^{j0}

$\underline{I} = 0.00412 * e^{j77.08}$ [A]

From this value we can clearly see that the load is trully capacitive, the total current is LEADING AC Main's voltage for +77.08 degrees.

Let's find out U_{pump} and U_C:

$\underline{Z}_{pump} = R + j\omega L$

$\underline{Z}_{pump} = 11930 + j15708$

$\underline{Z}_{pump} = |Z_{pump}| * e^{j\Theta pump}$

$|Z_{pump}| = \sqrt{}(11930^2 + 15708^2) = 19725$ [Ω]

Θ_{pump} = arc tan (15708 / 11930) = 52.78 [degrees]

$\underline{Z}_{pump} = 19725 * e^{j+52.78}$

$\underline{U}_{pump} = \underline{Z}_{pump} * \underline{I}$

$\underline{U}_{pump} = 19725 * e^{j+52.78} * 0.00412 * e^{j77.08} = 81.3 * e^{j129.86}$ [V]

U_{pump} (RMS) = 81.3 [V]

$\underline{U}_C = \underline{Z}_C * \underline{I}$

$\underline{U}_C = 1/j\omega C * \underline{I}$

$\underline{Z}_C = -j67725 = |Z_C| * e^{j\Theta C}$

Θ_C = arc tan (-67725 / 0) = arc tan (- oo) = -90 [degrees]

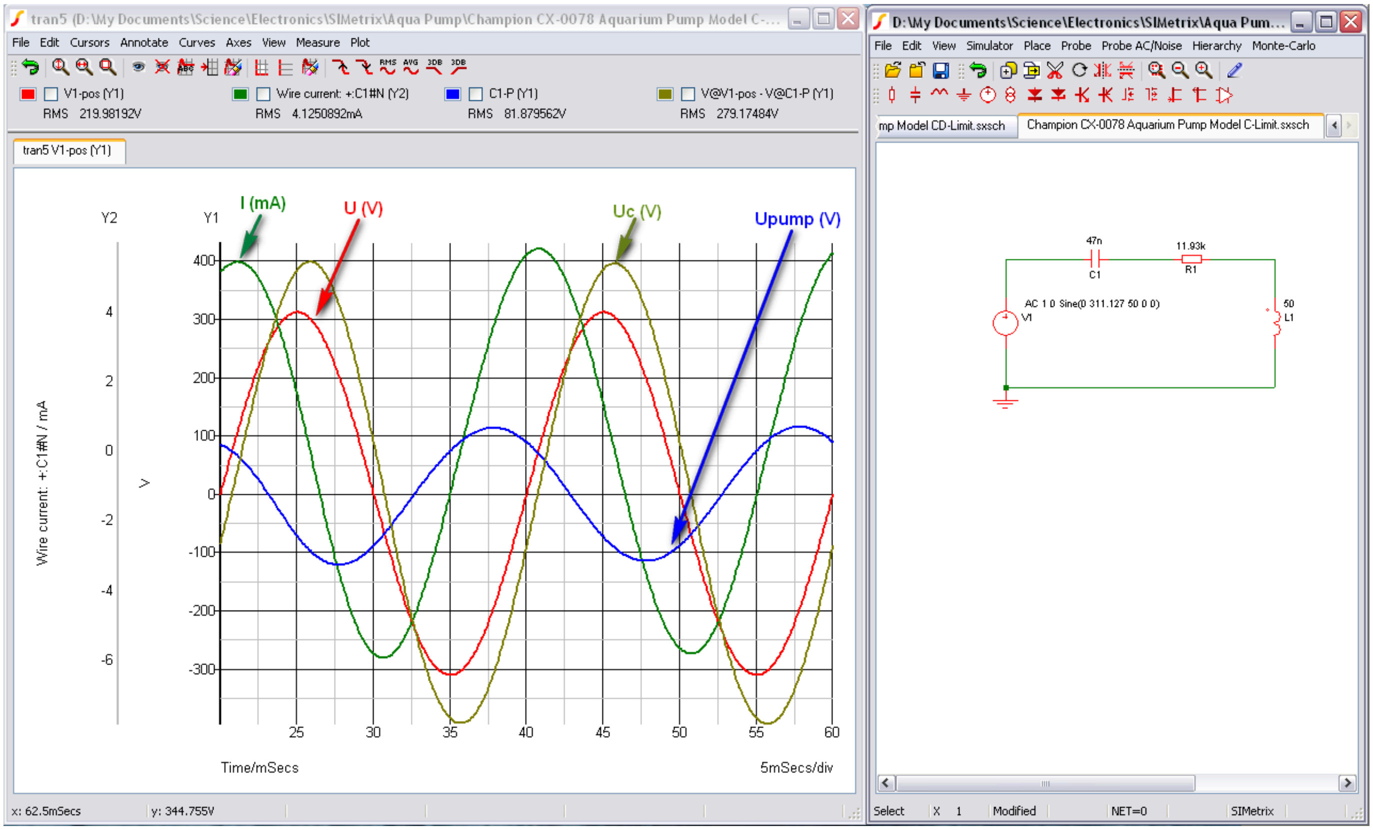
$\underline{Z}_C = 67725 * e^{j(-90)}$

$\underline{U}_C = 67725 * e^{j(-90)} * 0.00412 * e^{j77.08} = 279.027 * e^{j(-13)}$ [V]

U_C (RMS) = 279.027 [V]

From the result we clearly see that the Capacitor's voltage is behind current exactly 90 degrees. In real life, ideal capacitors do not exist, thus complex capacitor modeling is achived with parallel resistor (~10 ... 1000 MΩ, equivalent serial resistance ESR (~ 0.1 Ω) and parasitic serial inductance Lp (~ nH).

SIMetrix simulation agrees with above calculation:



Appendix: Complex circle and mathematical positive/negative phase convention