Using the Gallagher Index, which voting and distribution method proportions seats in legislatures most representative of the electorate?

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Introduction

I was first introduced to different voting methods in my Comparative Government class. I became fascinated by comparing different voting methods from different countries. I saw how First Past The Post voting in countries like Canada and the United Kingdom can often over represent the winning party, and sometimes even make the popularly elected party lose. In First Past The Post, the electorate have one vote to choose their preferred candidate for public office. Whichever candidate receives the most votes, wins — regardless of whether or not they won the majority of the votes. I was astounded to find that parties ruled big majorities without receiving over 50% of the votes, as in the Conservative Party win in the 2015 United Kingdom election.[1] And sometimes I found that parties that did not win the most votes still received the most seats, as in the Liberal Party win in the 2019 Canadian Election.[2]

I investigated further and found that some countries like Australia and New Zealand rank their candidates using the Single Transferable Voting system. The electorate rank their candidates from most favorite to least favorite. If no candidate gets 50% number one votes, then the candidate with the least number one votes is out, their second choice votes going to other candidates. This keeps happening until one candidate gets 50%.

When finding the most accurate voting system, I came across Score Voting. Many electoral reformers touted it as the future of democracy, where there was no trade off with your vote.[3] This means that if you score candidate A a 5/5, you can still score candidate B a 5/5, unlike in Single Transferable Vote where you can only put one candidate as your first choice. In

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[1]Morey, Stephen. “Voting System Gives Tories a Result Most UK Voters Didn't Want.” The Conversation, 11 May 2015, theconversation.com/voting-system-gives-tories-a-result-most-uk-voters-didnt-want-41595.
[2]Smith, Kyle. “Justin Trudeau Lost the Popular Vote. Will Outrage Ensue?” National Review, National Review, 22 Oct. 2019, www.nationalreview.com/corner/justin-trudeau-lost-the-popular-vote-will-outrage-ensue/.
[3]“Score Voting.” The Center for Election Science, www.electionscience.org/library/score-voting/.

score voting, the electorate scores all the candidates. Whichever candidate gets the highest average score wins.

Investigation

To see which voting method was the most representative of the electorate, I conducted a poll of 151 people. I first asked what their favorite color was between Blue, Green, Purple, Red, and Orange, using First Past the Post (choosing one color), Single Transferable Vote (ranking the colors), and Score Vote (scoring each color). I split the 151 responses into 15 electoral constituencies/districts, each with nearly the same number of people to mimic constituencies/districts in federal elections (One constituency/district had eleven people, while the other fourteen had ten people). The responses were split into 15 constituencies based on the time of their response to leave out any substantial confounding variable. For example, the first ten people to complete the poll were put in one constituency/district while the last ten people to complete the poll were put in another constituency/district etc. The poll was written in a way so that people were blind. They did not know it was about electoral reform and assumed it was just about their favorite colors. When they continued to the next page, then the poll explained they had just voted for their favorite color using 3 different voting methods. Next, they were asked questions about whether the United States should have electoral reform, and which voting method they prefered.

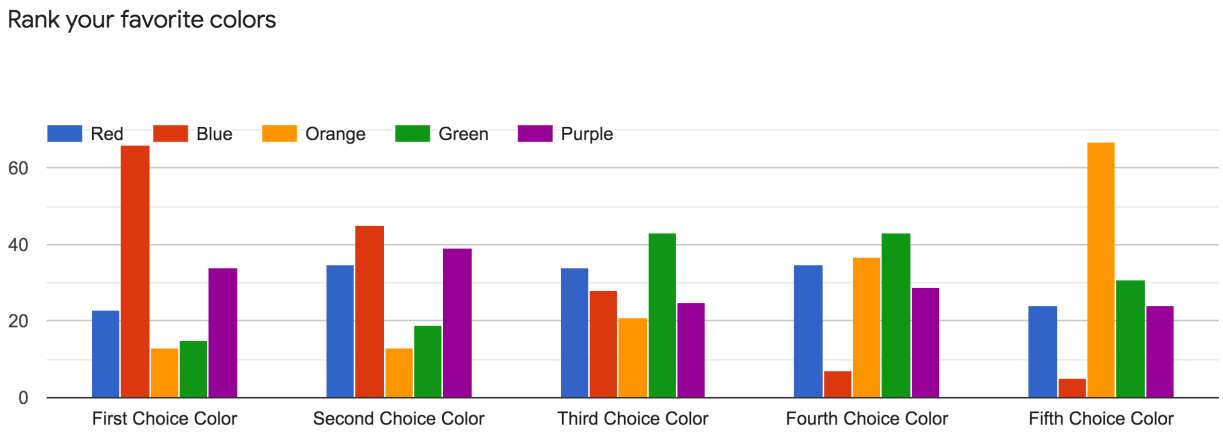
Results from the Poll by Constituency and Voting System

Constituency	First Past the Post	Single Transferable Vote	Score Vote
1	Green	Green	Blue
2	Blue	Blue	Blue
3	Blue	Blue	Blue
4	Blue	Blue	Blue
5	Blue	Blue	Blue
6	Purple	Purple	Blue
7	Blue	Blue	Blue
8	Red	Red	Blue
9	Blue	Blue	Blue
10	Blue	Purple	Purple
11	Blue	Blue	Blue
12	Blue	Blue	Purple
13	Blue	Blue	Blue
14	Blue	Red	Blue
15	Blue	Blue	Blue

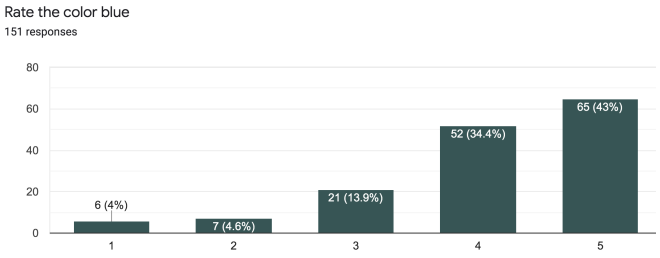
This table depicts all the constituencies/districts and their results based on the different voting methods. The winner is sometimes the same using all three voting methods, like in Constituency 13 where Blue won in all voting methods. In other cases, the winner changes based on the voting method. In Constituency 1, Green wins using First Past the Post and Single Transferable Vote, but Blue wins using Score Vote.

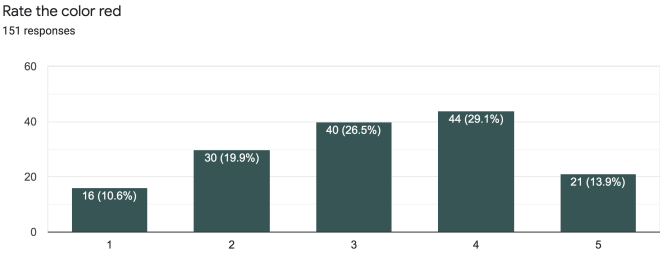
Party	First Past the Post	Single Transferable Vote	Score Vote
Red Total	1	2	0
Blue Total	12	10	13
Orange Total	0	0	0
Green Total	1	1	0
Purple Total	1	2	2

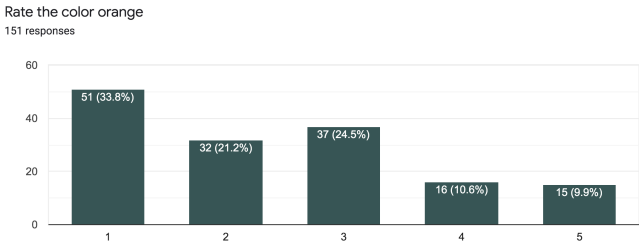
The total seats won per color are depicted in this table. Score vote seems to give Blue a bigger majority than with First Past the Post voting. Single Transferable Vote seems to represent more parties compared to the other two voting systems.

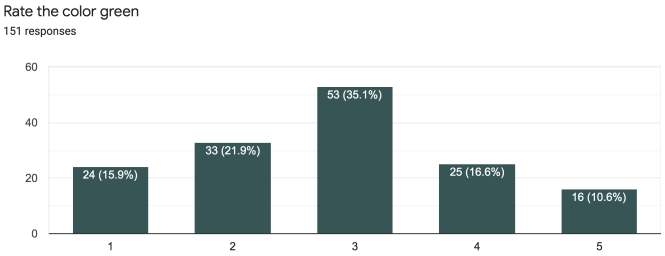


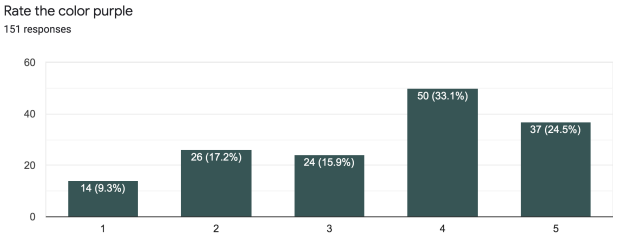
This bar graph above depicts the results of the poll with Single Transferable Vote. It seems that Blue was a very popular first choice color, Purple as the second most popular first choice color. Orange overwhelmingly seems to be the least favorite color. (Note: Red and Blue are not color coordinated).



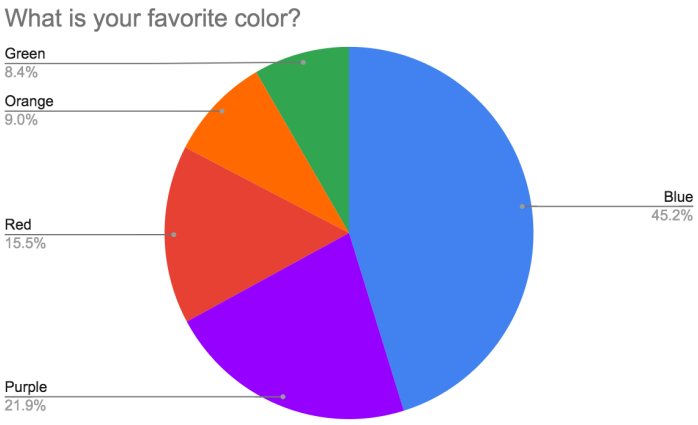


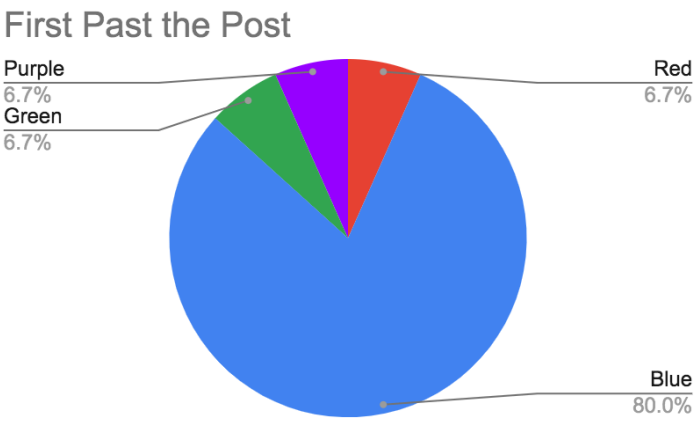


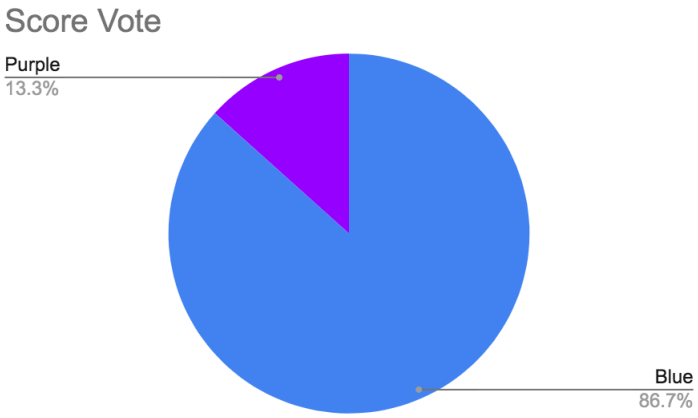


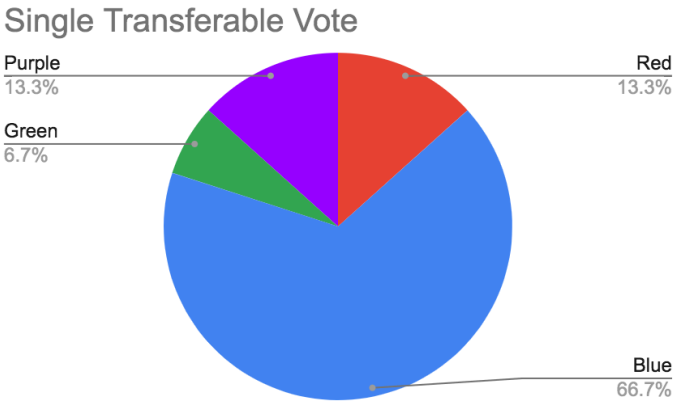


The bar graphs above depict the results of the poll using Score Vote. Respondents were asked to rate these colors, 5 being most favorite and 1 being least favorite. Blue seems to be the more favorite, with majority of those polled rating it either a 4 or 5. Orange seems to be the least favorite, with majority of those polled rating it either a 1 or 2.

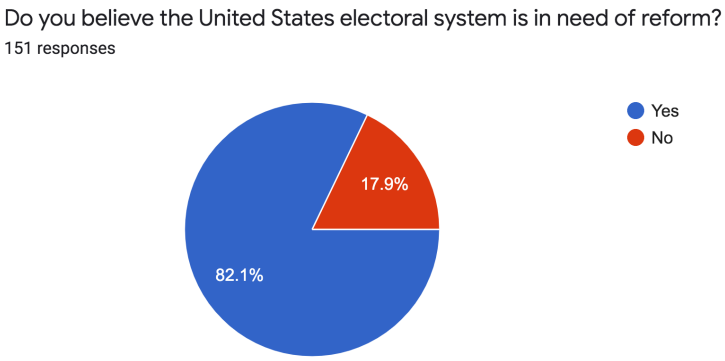


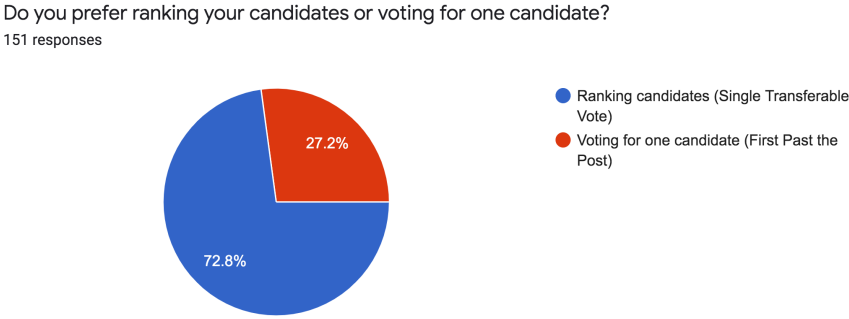


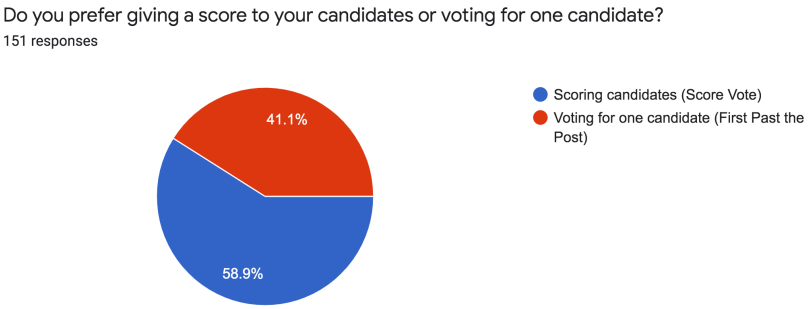


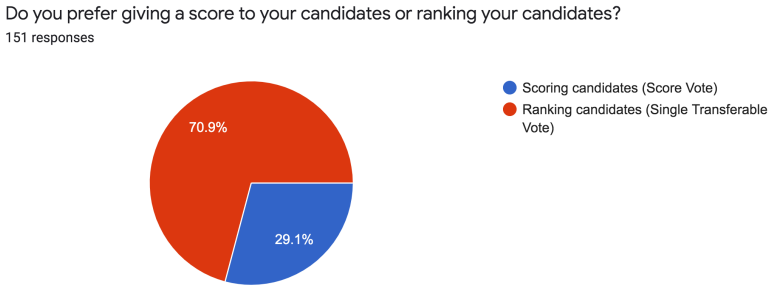


These pie charts visually depict the representation each color gets. The more a pie chart looks like the “What is your favorite color?” pie, the more representative it is of the electorate. Again, Single Transferable Vote seems to be more representative than the other voting systems.









These are all results from the poll conducted to gauge respondent's opinions on general electoral reform in the United States and their opinions about the different voting methods. Majority of respondents believe there is a need for electoral reform within the United States. And the preferred voting system seems to be Single Transferable Vote, with First Past the Post being the least liked out of the three. This was surprising to me because my personal opinion was to score candidates, largely because there is no trade off between how you vote for different candidates.

The Gallagher Index

While visually Single Transferable Vote is the most representative voting method, I wanted to mathematically find the most representative electoral system. The Gallagher Index does just that by calculating the disproportionality of elections and election systems.[4] The Gallagher Index takes the difference between the popular vote (V_i) and the percentage of seats won (S_i). With that difference, the index squares it, finds the total for all parties, divides it by 2,

[4]Becker, Byron Weber. "Modelling Elections ." Submission to ERRE: Special Committee on Electoral Reform, House of Commons - Canada, www.ourcommons.ca/Content/Committee/421/ERRE/Brief/BR8454480/br-external/BeckerByronWeber-e.pdf.

and takes the square root of that quotient. A Gallagher Index of less than 5 is considered proportional.[5] Shown below are the calculations for the Gallagher Index for Blue.

$$\sqrt{\frac{1}{2}\sum_{i=1}^{n}(v_i - s_i)^2} =$$

$$\sqrt{\frac{1}{2}\sum_{i=1}^{n}(45.20 - 80.00)^2 + \ldots.} =$$

$$\sqrt{\frac{1}{2}\sum_{i=1}^{n}(-34.80)^2 + \ldots.} =$$

[5]Scarpaleggia, Francis. "STRENGTHENING DEMOCRACY IN CANADA: PRINCIPLES, PROCESS AND PUBLIC ENGAGEMENT FOR ELECTORAL REFORM - Report of the Special Committee on Electoral Reform." 42nd PARLIAMENT, 1st SESSION, House of Commons - Canada, Dec. 2016, www.ourcommons.ca/Content/Committee/421/ERRE/Reports/RP8655791/errerp03/errerp03-e.pdf.

$$\sqrt{\frac{1}{2}\sum_{i=1}^{n}(1211.04) + \ldots.} =$$

$$\sqrt{\frac{1}{2}(1604.9547)} = \sqrt{(802.47735)} \approx 28.33$$

The same calculations were applied to the other voting systems. Here are the results:

First Past The Post Gallagher Index					
	% of Votes	Seats Won	% of Seats	Difference	Difference Squared
Red	15.50%	1	6.67%	8.83	77.9689
Blue	45.20%	12	80.00%	-34.8	1211.04
Orange	9.00%	0	0.00%	9	81
Green	8.40%	1	6.67%	1.73	2.9929
Purple	21.90%	1	6.67%	15.23	231.9529
Total	100.00%	15	100.00%		1604.9547
				Halved	802.47735
				Gallagher Index	28.33

Single Transferable Vote Gallagher Index					
	% of Votes	Seats Won	% of Seats	Difference	Difference Squared

Red	15.50%	2	13.33%	2.17	4.7089
Blue	45.20%	10	66.67%	-21.47	460.9609
Orange	9.00%	0	0.00%	9	81
Green	8.40%	1	6.67%	1.73	2.9929
Purple	21.90%	2	13.33%	8.57	73.4449
Total	100.00%	15	100.00%		623.1076
				Halved	311.5538
				Gallagher Index	17.65

Score Vote Gallagher Index					
	% of Votes	Seats Won	% of Seats	Difference	Difference Squared
Red	15.50%	0	0.00%	15.5	240.25
Blue	45.20%	13	86.67%	-41.47	1719.7609
Orange	9.00%	0	0.00%	9	81
Green	8.40%	0	0.00%	8.4	70.56
Purple	21.90%	2	13.33%	8.57	73.4449
Total	100.00%	15	100.00%		2185.0158
				Halved	1092.5079
				Gallagher Index	33.05

For every single electoral system, the Gallagher Index was higher than 5. Single Transferable Vote was closest to 5, but still at 17. None of the electoral systems I had polled actually made the legislature mathematically representative of the electorate. Not satisfied with the results, I calculated my own system of distributing seats using Single Transferable Vote, the closest voting system to a Gallagher Index of 5.

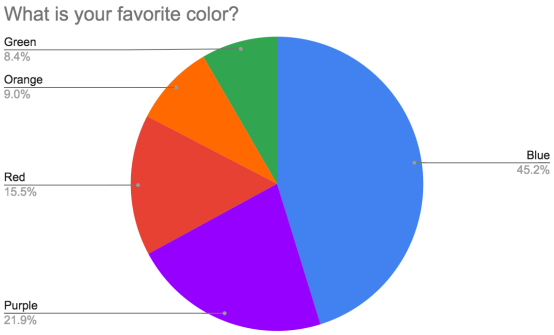
A New Electoral System

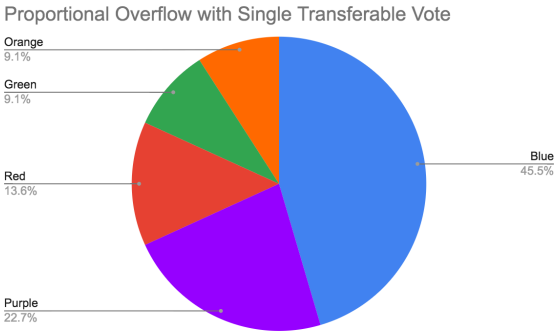
Using the Single Transferable Vote results from the poll, I wanted to make the seat distributions as proportional as possible. I did this by finding the total number of seats necessary for the percentage of seats won by the biggest party to be equal to percentage of votes won by that party.

$$\frac{10}{45.20\%} \approx 22$$

Then, I multiplied the percentage of votes won for every other party by 22. Next, I added x number of seats to the number of seats that party already won, making the percentage of seats won equal to the percentage of votes won.

Calculations to Redistribute Seats - Proportional Overflow with Single Transferable Vote					
Color	Seats Won 2020	Vote Percentage	Seat Percentage	Proportional	Seat Change
Blue	10	45.20%	66.67	10	0
Purple	2	21.90%	13.33	5	3
Red	2	15.50%	13.33	3	1
Orange	0	9.00%	0	2	2
Green	1	8.40%	6.67	2	1
Total	15	100.00%	100	22	





This new system is called Proportional Overflow with Single Transferable Vote. Proportional Overflow because of the additional seats that don't represent a geographic constituency but rather a political party (i.e. the 3 additional seats for Purple). This is how it's different from straight Proportional Representation, where candidates don't win a constituency but rather a national vote, which suffers from the fact that politicians don't represent a subset of a population but rather the country as a whole. Notice how visually the Proportional Overflow with Single Transferable Vote is nearly identical to the electorate's opinion. Here is the Gallagher Index.

Proportional Overflow with Single Transferable Vote Gallagher Index					
Party	% of Votes	Seats Won	% of Seats	Difference	Difference Squared
Red	15.50%	3	13.64%	1.86	3.4596
Blue	45.20%	10	45.45%	-0.25	0.0625
Orange	9.00%	2	9.09%	-0.09	0.0081
Green	8.40%	2	9.09%	-0.69	0.4761
Purple	21.90%	5	22.73%	-0.83	0.6889
Total	100.00%	22	100.00%		4.6952
				Halved	2.3476
				Gallagher Index	1.53

The Gallagher Index of 1.53 shows that this system is very proportional to the will of the people. This system accurately distributes votes to seats in legislatures, accurately putting power with the consent of the governed.

Limitations

Like any research paper, this has its own limitations. In the poll, there was a Voluntary Response Bias because when I posted it online, people who wanted to complete the survey did it. There was also Nonresponse Bias due to the fact that when I asked some people to complete it, they refused to take it. There was also Undercoverage Bias because the poll was online. People who do not have internet access could not have conducted the survey. There was also Sampling Bias because most of the people who completed the survey were my peers and attended my school. This sample is not representative of the entire population.

Additionally, I later realized it was very difficult to actually apply the Gallagher Index to Score Voting. This is because voters do not have a trade off when it comes to voting. If a voter wanted to, they could give 5/5 to all candidates and all parties. So, the Score Vote results could actually be more representative of the electorate's will than the first poll conducted asking "What is your favorite color?"

Another limitation is that having a system that is the most representative does not necessarily mean it is the best. One big benefit of traditional First Past the Post electoral systems is that it gives stability to the government. The biggest party wins a big enough majority not to form a risky coalition that could fall apart before the next election. Proportional Overflow with Single Transferable Vote relies on the fact that parties can work together to form coalitions to reach a majority in the legislature. However, if no coalition can get a majority, it leads to a chaotic government that some could argue is worse than having a non representative government.

Conclusion

This started off with me being surprised that democracy was not always upheld by the popularly elected and ended with me creating a new way to distribute seats in legislatures. I was surprised by the poll I conducted, thinking that Score Vote would be the most representative of the electorate and most people like it. But the opposite happened, where Single Transferable Vote had the most proportionate results and most people liked this system over Score Voting. If I were to do this again, I would conduct the poll with more people using Simple Random Sample. I would also apply the Gallagher Index to the real world in elections across the world. I would continue to find other mathematical equations that calculate proportionality and see which one is better. In the end, there is still a sizable population that believe electoral systems need to change, especially in the United States. While not all problems will be solved with a more proportional and representative government, and could bring more risk, it is a step in the right direction.

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