# Perpendicular Bisectors Investigation

**Instructions and Information:**

- Answer all questions in the answer booklet provided.
- You will have 60 minutes to complete this investigation.
- Use the GeoGebra file provided to help you complete the task.
- You will be assessed against Criteria B and C. Task-specific clarifications for each achievement level can be found in Table 1 and 2 below.

Table 1: Task-specific clarifications for Criterion B Perpendicular Bisectors Investigation

| Achievement level | Descriptor | Clarification |
|---|---|---|
| | The student is able to: | You demonstrated the ability to: |
| 1–2 | i. **apply**, **with teacher support**, mathematical problem-solving techniques to discover **simple patterns**<br>ii. **state predictions** consistent with patterns. | i. **apply**, **with teacher support**, coordinate geometry and straight line equation mathematics to discover a **simple pattern**<br>ii. **state** a **prediction** consistent with the pattern. |
| 3–4 | i. **apply** mathematical problem-solving techniques to discover **simple patterns**<br>ii. **suggest general rules** consistent with **findings** | i. **apply** coordinate geometry and straight line equation mathematics to discover a **simple pattern**<br>ii. **suggest** a **general rule** consistent with the pattern |
| 5–6 | i. **select** and **apply** mathematical problem-solving techniques to discover **complex patterns**<br>ii. **describe patterns** as general rules consistent with **findings**<br>iii. **verify** the validity of these general rules. | i. **select** pattern-finding methods and **apply** coordinate geometry and straight line equation mathematics to discover **complex patterns**<br>ii. **describe** a **pattern** as an algebraic general rule consistent with **findings**<br>iii. **verify** the validity of a general rule by checking it against further examples |
| 7–8 | i. **select** and **apply** mathematical problem-solving techniques to discover **complex patterns**<br>ii. **describe patterns** general rules consistent with **correct findings**<br>iii. **prove**, or **verify and justify,** these general rules. | i. **select** pattern-finding methods and **apply** coordinate geometry and straight line equation mathematics to discover **correct**, **complex patterns**<br>ii. **describe both patterns** as algebraic general rules consistent with **correct findings**<br>iii. **verify** the validity of both general rules by checking them against further examples and **justify or prove** the general rule for the complex pattern by using an algebraic generalisation of your pattern-finding method or a direct derivation using coordinate geometry and straight-line equation mathematics |

Table 2: Task-specific clarifications for Criterion C Perpendicular Bisectors Investigation

| Achievement level | Descriptor | Clarification |
|---|---|---|
| | The student is able to: | You demonstrated the ability to: |
| 1–2 | i. use **limited** mathematical language<br>ii. use **limited forms** of mathematical representation to present information<br>iii. **communicate** through lines of reasoning that are **difficult to interpret.** | i. use **limited** coordinate geometry terminology and notation<br>ii. use **limited** forms of mathematical representation to explain your pattern-finding methods<br>iii. **communicate** your pattern-finding methods through mathematical working steps and word explanations that are **difficult to interpret** |
| 3–4 | i. use **some appropriate** mathematical language<br>ii. use **appropriate forms** of mathematical representation to present information **adequately**<br>iii. **communicate** through lines of reasoning that are **complete**<br>iv. **adequately organize** information using a logical structure. | i. use **some appropriate** coordinate geometry terminology and notation<br>ii. **use appropriate** forms of mathematical representation to explain your pattern-finding and verification methods **adequately**<br>iii. **communicate** your pattern-finding and verification methods through mathematical working steps and word explanations that are **complete**<br>iv. show mathematical working steps that are **adequately** laid out |
| 5–6 | i. **usually** use **appropriate** mathematical language<br>ii. **usually** use **appropriate forms** of mathematical representation to present information **correctly**<br>iii. **usually** move between different forms of mathematical representation<br>iv. communicate through lines of reasoning that are **complete and coherent**<br>v. **present** work that is **usually organized** using a logical structure. | i. usually use **appropriate** coordinate geometry terminology and notation<br>ii. **usually** use **appropriate forms** of mathematical representation to explain your pattern-finding, verification and justification or proof methods **correctly**<br>iii. usually integrate algebra, diagrams/sketches and tables in your discovery, description, verification and justification/proof of the **complex pattern** rule<br>iv. communicate your pattern-finding, verification and justification or proof methods through mathematical working steps and word explanations that are **complete and coherent**<br>v. **present** mathematical working steps that are **usually** well laid out and easy to follow |
| 7–8 | i. **consistently** use **appropriate** mathematical language<br>ii. use **appropriate forms** of mathematical representation to **consistently** present information **correctly**<br>iii. move **effectively** between different forms of mathematical representation<br>iv. communicate through lines of reasoning that are **complete, coherent and concise**<br>v. **present** work that is **consistently organized** using a logical structure. | i. **consistently** use **appropriate** coordinate geometry terminology and notation<br>ii. **consistently** use **appropriate forms** of mathematical representation to explain your pattern-finding, verification and justification or proof methods **correctly**<br>iii. **effectively** integrate algebra, diagrams/sketches and tables in your discovery, description, verification and justification/proof of the **complex pattern** rule<br>iv. communicate your pattern-finding, verification and justification or proof methods through mathematical working steps and word explanations that are **complete, coherent and concise**<br>v. **present** mathematical working steps that are **consistently** well laid out and easy to follow |

**Scenario:**

After learning about coordinate geometry, you've had an idea for an app that helps people to choose optimal meeting points by automatically creating a line that represents an equal distance between any two chosen points. To do this, you'll need to know how to code the equation of that line based on the coordinates of any chosen points.

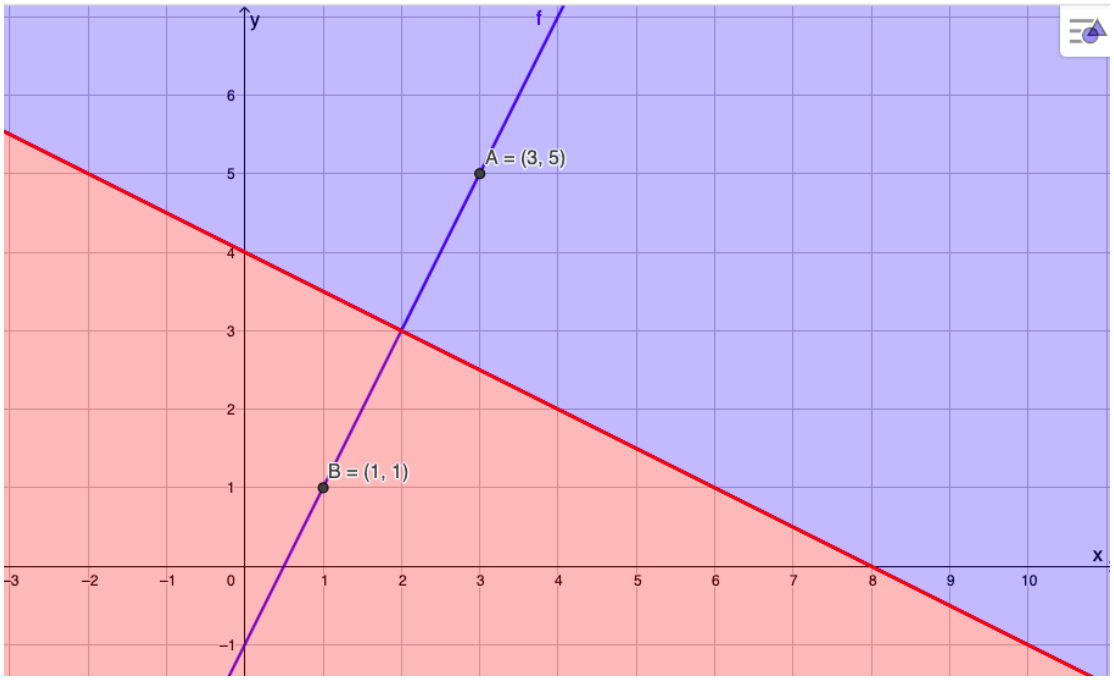


Figure 1: all points in the red region are closer to point B than point A, and all points in the blue region are closer to point A than point B. The red line is called a **perpendicular bisector** and represents all locations that are the same distance from both points. It is perpendicular to line AB and intersects it exactly halfway between A and B.

You decide to start by investigating a rule for calculating the gradient of the perpendicular bisector in terms of the coordinates of points A and B.

**Instructions:**

**Part 1**

1. **Choose** any two points A and B in a Cartesian coordinate system with integer coordinates (see example below). Use this Geogebra file to help you find:

   a) The equation of the line that passes through points A and B

   b) The equation of the perpendicular bisector of line AB.

   DO NOT use the same points as those shown in Figure 1 and Figure 2.

**Notes on using the Geogebra app:**

Drag points A and B by first clicking on the white arrow in the top left corner and then dragging the points around, or by dragging the sliders in the left sidebar. The equations of line AB and its perpendicular bisector are displayed (in general form) in the bottom left corner. **You will need to rearrange them into gradient-intercept form yourself.**

**DO NOT construct a new line for every pair of points – move points A and B to new locations and the perpendicular bisector will move with it.**

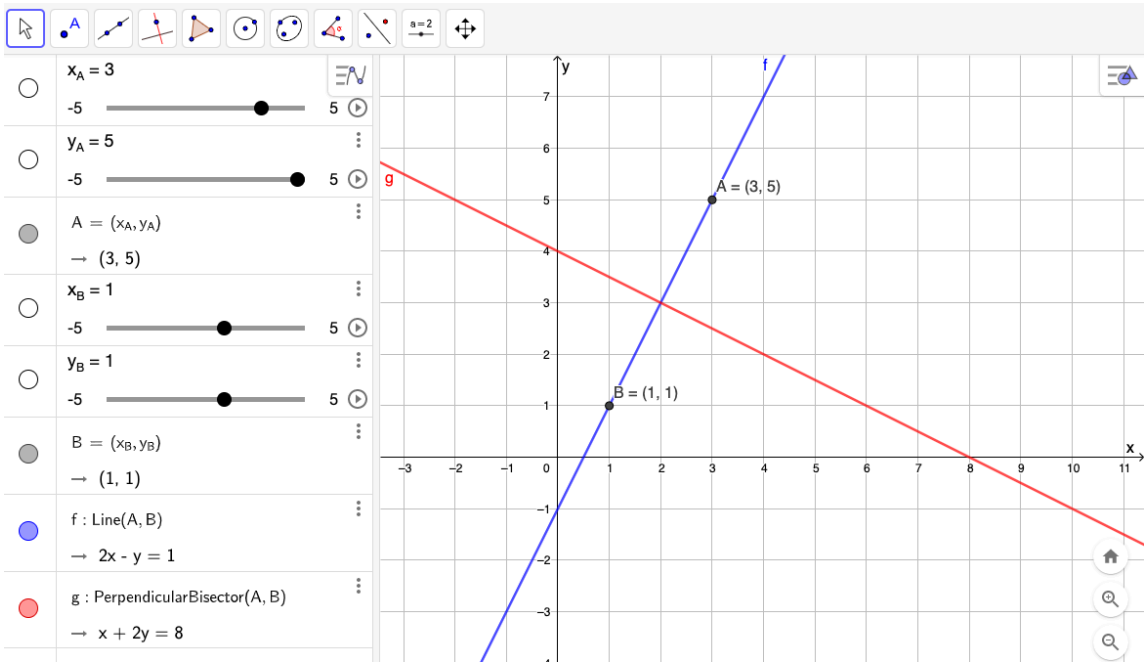


Figure 2: Finding the perpendicular bisector (red) of line AB (blue) in the Geogebra app.

**Reminder**: the gradient-intercept form of the equation of a straight line is

$$y = mx + c$$

2. Repeat this for multiple pairs of points A and B (at least 4), and display your results in a table like this:

| Point A $(x_A, y_A)$ | Point B $(x_B, y_B)$ | Equation of line through points A and B in gradient-intercept form | Equation of perpendicular bisector of AB in gradient-intercept form |
|---|---|---|---|
| | | | |
| | | | |
| | | | |
| | | | |

Make sure you choose different combinations of points A and B that make the gradient of the line through points A and B both positive and negative, and write the coefficients of your equations as fractions, not decimals. You should not choose the point (0,0).

3. **Predict** how the gradient of the line through points A and B and gradient of its perpendicular bisector are related. You may wish to further process your results from Question 2 by adding the following columns:

| Gradient of line through points A and B | Gradient of perpendicular bisector of AB |
|---|---|
| | |
| | |
| | |
| | |

4. Using the results of your investigation from Question 2 and 3, or otherwise, **suggest** a general rule for the gradient of the perpendicular bisector of AB in terms of the $x$ and $y$ coordinates of points A and B, $(x_A, y_A)$ and $(x_B, y_B)$.

5. **Verify** that your rule works by adding rows to your tables from Questions 2 and 3 and then comparing the values obtained with the results obtained from your general rule.

**Part 2**

Next, as an intermediate step towards our fully generalised app code, we'll investigate a general rule for the equation of the perpendicular bisector between two points when one of the points is at the origin.

1. Let point B be fixed at (0, 0). Choose any point A $(x_A, y_A)$ not on the $x$ or $y$ axis and then find the equation of the perpendicular bisector of AB in ***general form***.

   **Reminder**: we can write the general form of the equation of a straight line as

   $$ax + by = c$$

   where $a$ and $b$ are integers.

   **Important Note**: $a$ and $b$ represent the coefficients of the equation of straight line, whereas A and B represent points in 2D space with coordinates $(x_A, y_A)$ and $(x_B, y_B)$.

2. Let $x_A = a$ and $y_A = b$. Investigate how the coefficient $c$ of the ***general form*** (that is, $ax + by = c$) of the equation of the perpendicular bisector of AB varies as point A $(x_A, y_A)$ is changed. Display your results in another table.

3. **Describe** a general rule for the coefficient $c$ of the ***general form*** of the equation of the perpendicular bisector of AB in terms of $x_A$ and $y_A$.

4. **Verify** that the general rule you described in Question 3 works.

5. **Justify** or **prove** that your general rule from Question 3 is correct.