

FUNCTION & NOTATION NOTES

- A **function** is a mathematical rule that assigns exactly one output to each input.
 - The set of all possible input values is referred to as the **domain** of the function.
 - ✓ The input values are assigned as x -values.
 - The set of all possible output values is referred to as the **range** of the function.
 - ✓ The input values are assigned as y -values, or often notated as $f(x)$.

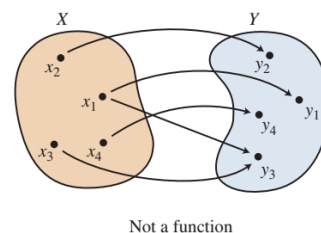
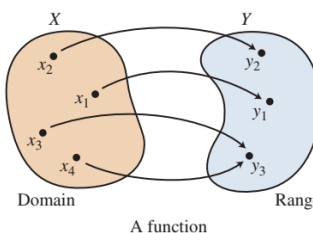
DEFINITION Function, Domain, and Range

A function from a set D to a set R is a rule that assigns to every element in D a unique element in R . The set D of all input values is the **domain** of the function, and the set R of all output values is the **range** of the function.

- To indicate that y comes from the function acting on x , we use Euler's elegant function notation
 - Read as "y equals f of x"
 - or "the value of f at x."

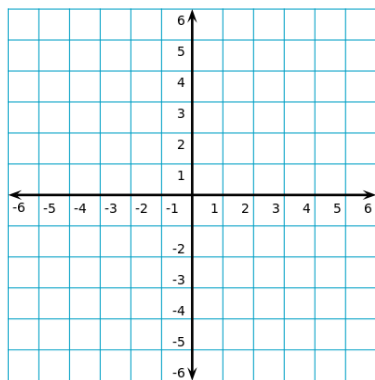
$$y = f(x)$$

- A function can also be viewed as a **mapping** of the elements of the domain onto the elements of the range.
- A function may map the same output value from different input values.
 - ✓ However, a single input value cannot map to multiple output values!
- The **domain** of a function may be individual data points $\{x_1, x_2, x_3, x_4\}$, or can be continuous set of points indicated by interval notation.
 - The same can be said for the **range** of the function. (Ex. $\{y_1, y_2, y_3\}$ or an interval of points)

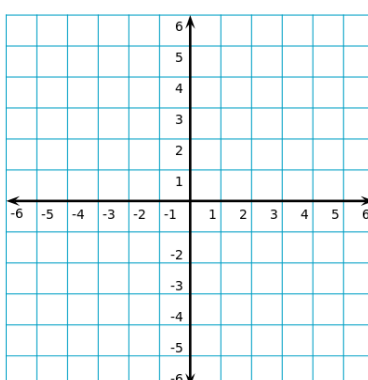


EXAMPLE 1 – Find the domain and range of these functions algebraically. Use graphing software to verify.

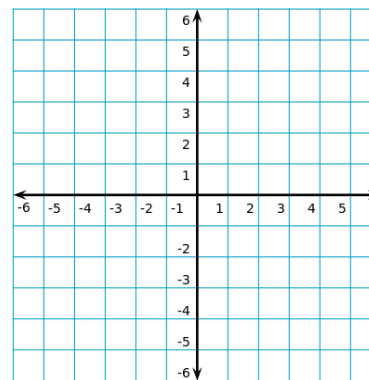
1. $f(x) = \frac{1}{2}x^4 - 2x^2 + 1$



2. $f(x) = \sqrt{x+3}$



3. $g(x) = \frac{x}{x+4}$



VERTICAL LINE TEST

- The **graph of the function $y = f(x)$** is the set of all points $(x, f(x))$, in the domain of f .
 - This means we are matching domain values along the x -axis with their range values along the y -axis to get the ordered pairs that yield the graph of $y = f(x)$.

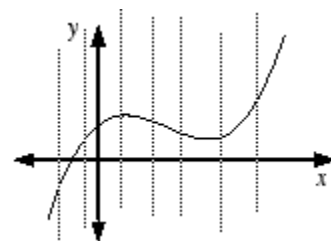
- ★ To assure the graph represents a function, we use the **Vertical Line Test**.

- ✓ The *vertical line test* uses a vertical line, or series of vertical lines, to verify that each input value only intersects the graph at one output value.

Vertical Line Test

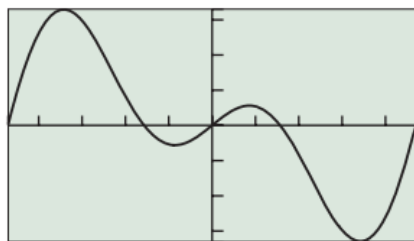
A graph (set of points (x, y)) in the xy -plane defines y as a function of x if and only if no vertical line intersects the graph in more than one point.

Vertical Line Test
The graph is a function since there are no vertical lines that hit the graph more than once.

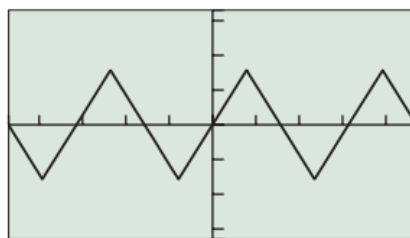


EXAMPLE 1 – Use the Vertical Line Test to determine if the following represent a function.

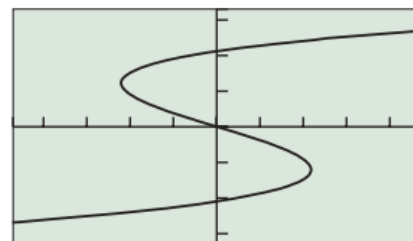
1.



2.

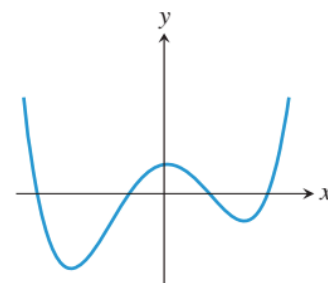


3.



CONTINUITY NOTES

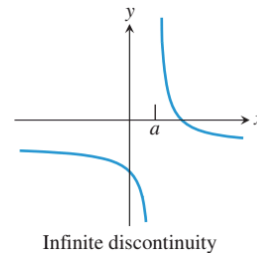
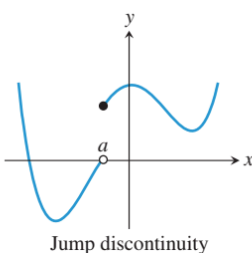
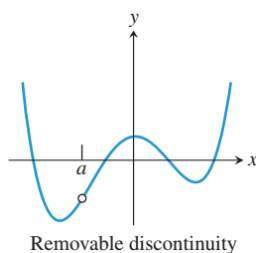
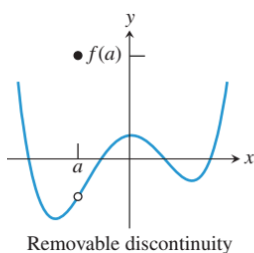
- A function is **continuous** at a point if you can draw its graph without lifting your pencil at that point.
 - A function may be *discontinuous* due to:
 - Asymptotes
 - Undefined sections of the function
 - ✓ This will be indicated by an empty circle.
 - Jumps in piecewise type functions.



Continuous at all x

TYPES OF DISCONTINUITY

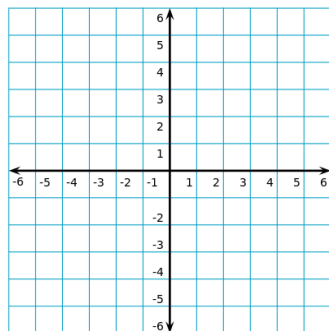
- Removable Discontinuity** - Functions with an undefined input value that may, or may not, be able to be filled by redefining the function at that point to plug the hole.
- Jump Discontinuity** - Often piecewise functions that cannot be redefined at a single point
- Infinite Discontinuity** - Typically graphs that contain a vertical asymptote.



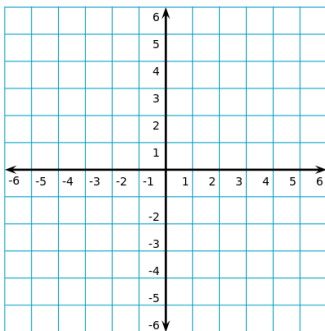
EXAMPLE 3 – Determine if the functions are continuous or discontinuous. Use graphing software to verify.

If discontinuous, identify the type of discontinuity and at what input does it occur.

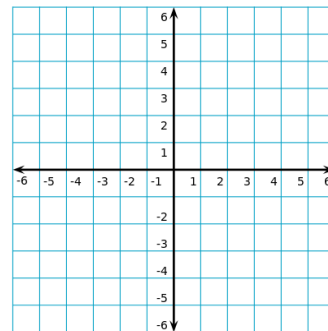
1. $f(x) = \frac{x+3}{x-2}$



2. $g(x) = (x+3)(x-2)$

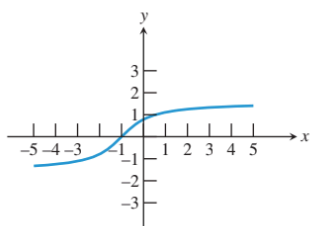


3. $h(x) = \frac{x^2-4}{x-2}$

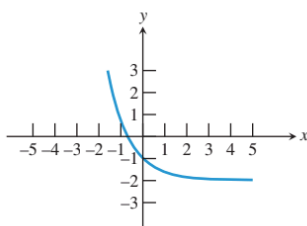


INCREASING AND DECREASING FUNCTIONS

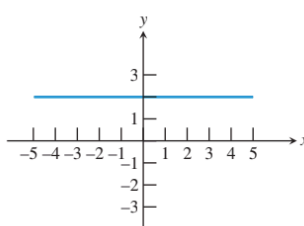
- A function is **increasing** on an interval if, as the input x gets *larger*, the output $f(x)$ gets *larger*.
- A function is **decreasing** on an interval if, as the input x gets *larger*, the output $f(x)$ gets *smaller*.
- A function is **constant** on an interval if, as the input x gets *larger*, the output $f(x)$ remains **constant**.



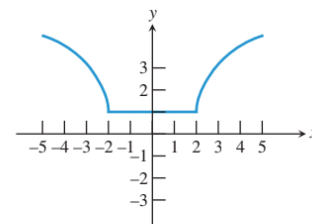
Increasing



Decreasing



Constant



Decreasing on $(-\infty, -2]$
Constant on $[-2, 2]$
Increasing on $[2, \infty)$

★ Think about the overall slope, or average slope, of the function during the defined interval.

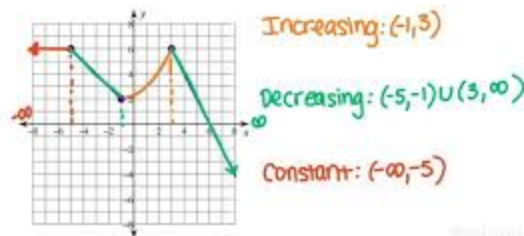
☆ Look at the overall change in y -values (Δy) of the interval defined by the change in x -values (Δx).

DEFINITION Increasing, Decreasing, and Constant Function on an Interval

A function f is **increasing** on an interval if, for any two points in the interval, a positive change in x results in a positive change in $f(x)$.

A function f is **decreasing** on an interval if, for any two points in the interval, a positive change in x results in a negative change in $f(x)$.

A function f is **constant** on an interval if, for any two points in the interval, a positive change in x results in a zero change in $f(x)$.



Increasing: $(-5, 3)$

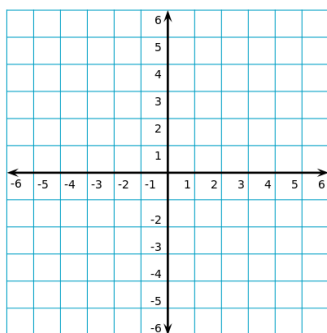
Decreasing: $(-1, 3) \cup (3, \infty)$

Constant: $(-\infty, -5)$

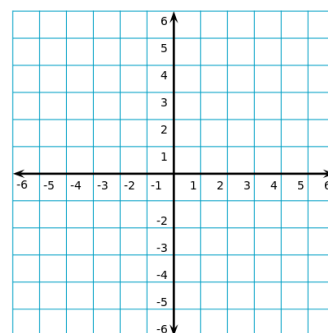
➤ We identify **increasing and decreasing intervals** by the respective input values, or x -values.

EXAMPLE 4 – For each function, tell the intervals on which it is increasing and the intervals on which it is decreasing. Use graphing software to verify.

1. $f(x) = (x + 2)^2$

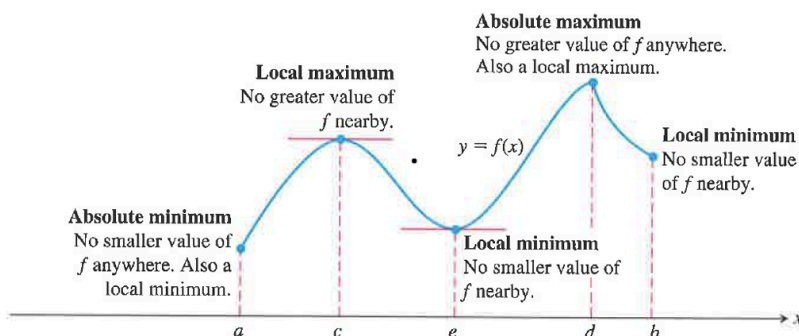


2. $g(x) = \frac{x^2}{x^2 - 1}$



RELATIVE EXTREMES NOTES

- The extreme values of the function (or local extrema) can be characterized as either **local maxima** or **local minima**.
- ✓ If a *local maximum (or minimum)* is also the largest (or smallest) value of $f(x)$ over the entire function, then it is called an **absolute maximum (or minimum)**.
- 👁 The distinction can be easily seen graphically.
- ★ We identify the **extreme values** by their output value, or y-value.



DEFINITION Local and Absolute Extrema

A **local maximum** of a function f is a value $f(c)$ that is greater than or equal to all range values of f on some open interval containing c . If $f(c)$ is greater than or equal to all range values of f , then $f(c)$ is the **maximum** (or **absolute maximum**) value of f .

A **local minimum** of a function f is a value $f(c)$ that is less than or equal to all range values of f on some open interval containing c . If $f(c)$ is less than or equal to all range values of f , then $f(c)$ is the **minimum** (or **absolute minimum**) value of f .

Local extrema are also called **relative extrema**.

EXAMPLE 5 – For each function, identify any local maxima or minima, as well as absolute maxima or minima. Use graphing software to verify.

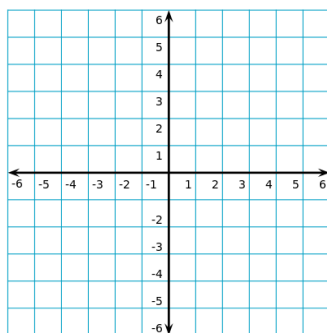
1. $f(x) = x^3 - 4x$

Local Minimum:

Local Maximum:

Absolute Minimum:

Absolute Maximum:



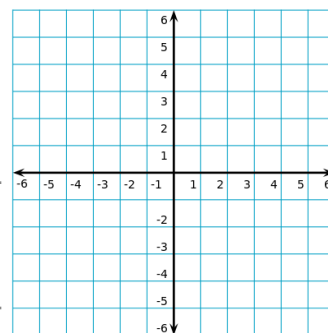
2. $g(x) = x^4 - \frac{1}{2}x^2$

Local Minimum:

Local Maximum:

Absolute Minimum:

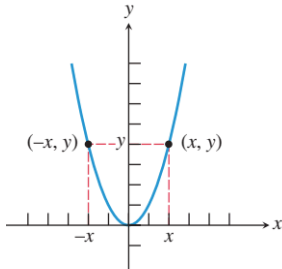
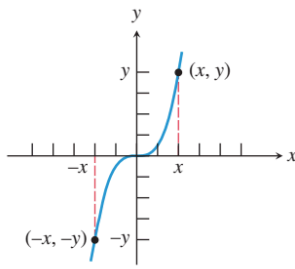
Absolute Maximum:



FUNCTION SYMMETRY NOTES

★ We will study two basic types of function symmetry

- I. y -axis symmetry
- II. Origin symmetry

	y-axis Symmetry	Origin Symmetry																												
Definition	The graph looks the same on both sides of the y-axis.	The graph has 180° rotational symmetry about the origin.																												
How to Recognize (Graphically)																														
How to Recognize (Algebraically)	Plugging in x and $-x$ will result in the same y. $f(x) = f(-x)$	If (x, y) is on the graph, then so is $(-x, -y)$. $f(-x) = -f(x)$																												
How to Recognize (Numerically)	<table><tr><th>x</th><th>$f(x)$</th></tr><tr><td>-3</td><td>9</td></tr><tr><td>-2</td><td>4</td></tr><tr><td>-1</td><td>1</td></tr><tr><td>1</td><td>1</td></tr><tr><td>2</td><td>4</td></tr><tr><td>3</td><td>9</td></tr></table>	x	$f(x)$	-3	9	-2	4	-1	1	1	1	2	4	3	9	<table><tr><th>x</th><th>y</th></tr><tr><td>-3</td><td>-27</td></tr><tr><td>-2</td><td>-8</td></tr><tr><td>-1</td><td>-1</td></tr><tr><td>1</td><td>1</td></tr><tr><td>2</td><td>8</td></tr><tr><td>3</td><td>27</td></tr></table>	x	y	-3	-27	-2	-8	-1	-1	1	1	2	8	3	27
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Highest Degree (or Exponent)	Functions will typically have an <u>even number</u> for the <i>largest</i> exponent. ★ This is why we refer to these as even functions.	Functions will typically have an <u>odd number</u> for the <i>largest</i> exponent. ★ This is why we refer to these as odd functions.																												
How to Check Numerically	- Pick an x-value in the domain to substitute into the function and simplify. - Pick the opposite of the x-value chosen before and substitute into the function and simplify. The opposite x-value must produce the same y-value.	- Pick an x-value in the domain to substitute into the function and simplify. - Pick the opposite of the x-value chosen before and substitute into the function and simplify. The opposite x-value should produce the opposite y-value.																												

! MANY FUNCTIONS ARE NEITHER EVEN NOR ODD!

😎 Most real-world models fall into this category.

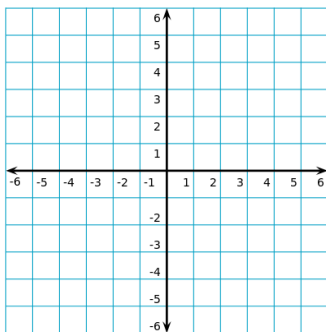
! DO NOT ALWAYS TRUST THE EXPONENTS TO DETERMINE IF THE FUNCTION IS EVEN OR ODD!

☀️ Verify with either a graph, or by algebraic or numerical means.

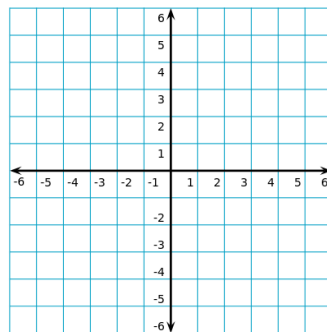
EXAMPLE 6 – For each function, determine whether the function is **odd**, **even**, or **neither**.

Verify with either graphing software or by algebraic/numeric means.

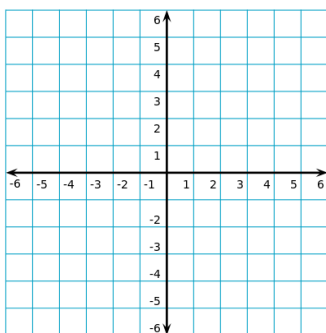
1. $f(x) = x^2 - 3$



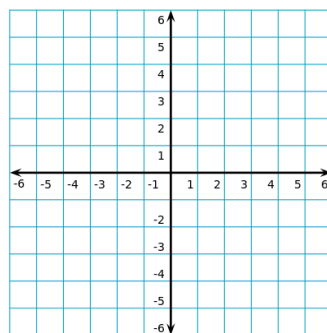
2. $f(x) = \sqrt{x - 2}$



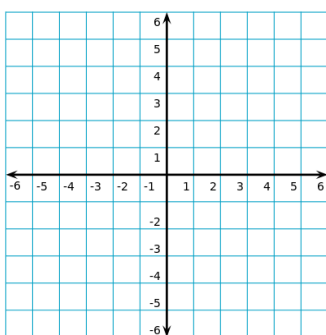
3. $f(x) = x^5 + x$



4. $f(x) = |x| + 2$



5. $f(x) = x^2 + x$



6. $f(x) = \frac{2}{x^2}$

