#18: Bayes Rule

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Decomposition into Conditional Probability

Occasionally, we have access to **only** conditional probabilities but need the probability of an event occurring. The probability of an event occurring is _____ **exactly equal to** the sum of two components:

[1]:

[2]:

Mathematically, we express this as the following:

P(**event**) = P(**event** | data)×P(data) + P(**event** | ¬data)×P(¬data)

Example 1: ELISA Test (from Lecture #17)

If a blood sample has the AIDS virus, then there is a 99% probability that ELISA will correctly give a positive result. If the blood sample does not have the AIDS virus then there is still a 6% probability that ELISA will incorrectly give a positive result. About 1% of the blood samples ELISA tests are truly contaminated.

What is the probability of a positive test result, **P(positive result)**?

Bayes Rule

Sometimes, we have the exact opposite conditional data that we need. Bayes Rule provides a way to find a conditional probability:

$$P(h \mid D) = P(D \mid h) \times \frac{P(h)}{P(D)}$$

In English, Bayes Rule tells us we can compute the probability of a hypothesis (**h**) given some data (**D**) given three individual probabilities:

1. The independent probability of the hypothesis being true, P(h),
2. The independent probability of the data being true, P(D), and

3. The conditional probability that the data is true given the hypothesis, P(D | h)

Example 1: Consider the following table of the survival rate and life expectancy in a certain population at birth:

	A=55	A=60	A=65	A=70
Survival, P(A):	0.90	0.85	0.75	0.65

What is the probability of a 60 years old to reach the age of 65?

Example 2: Related to Taylor Swift (from Lecture #17)

A test was developed to see if an individual was related to Taylor Swift. There is a 99% probability that an individual related to Taylor Swift will get a positive result ("true positive"). There is a 6% probability that an individual NOT related to Taylor Swift will get a positive result ("false positive"). About 1% of the world population is related to Taylor Swift.

What is the probability someone is related to Taylor given a positive test result?

Conditionals in Python

Up until today, we have discovered two different control flow techniques in Python:

Control Flow	Python Syntax	Description
	`for i in range( n ):` `  ...`	Repeats indented code n times.
	`def myFunctionName( params ):` `  ...`	Runs indented code when function is called elsewhere in program.

With conditional probability, we need to have a **mechanism to run code conditionally**. In Python, we can do this with an **if-statement**:

```
1 if red == 2:
2   print("The red die rolled a 2.")
```

Four key ideas:

1. [Identical to Pandas]:

2. [Control Flow]:

3. [Syntax]:

4. [Conditional]:

If statements can have an _____ statement for code for cases when the statement is false:

```
1 if red == 2:
2   print("The red die rolled a 2.")
3 else:
4   print("The red die did NOT roll a 2.")
```

Example 4: Write a simulation for the ELISA problem.

If a blood sample has the AIDS virus, then there is a 99% probability that ELISA will correctly give a positive result. If the blood sample does not have the AIDS virus then there is still a 6% probability that ELISA will incorrectly give a positive result. About 1% of the blood samples ELISA tests are truly contaminated.

Practice Problems:

Practice #1: *The following question combines all of the probability rules that we've learned. Suppose you randomly draw from these students.*

	Left-Handed	Ambidextrous	Right-Handed	Totals
Male	**30**	**20**	**266**	**316**
Female	**60**	**27**	**560**	**647**
Totals	**90**	**47**	**826**	**963**

a) What is the chance of getting a female?

b) What is the chance of getting someone who is left-handed?

c) What is the chance you'll get a female if you draw only from the left-handers?

d) What is the chance you'll get a left-hander if you draw only from the females?

e) Draw 3 students *without* replacement. What is the chance that *all* 3 students are left-handed?

f) Draw 3 students *without* replacement. What's the chance that *not all* 3 students are left-handed?

g) Draw 3 students *with* replacement. What is the chance that *at least one* student is right-handed?

Practice #2: What is the probability of getting at least one 5 on six rolls of a die?

Practice #3: What is the probability of rolling a die 3 times and *not* getting all "2"s?