Roll No…………………….. **Total No. of Printed Pages: [2]**

Total No. of Questions: [9]

B.Tech. (CSE, IT, Textile, AI& ML, CCE, IoT and Cyber Security including Block Chain Technology)

(Semester-1st)

MATHEMATICS – I (CALCULUS, LINEAR ALGEBRA)

Subject Code: BMATH1101

Paper ID: [19110008]

Time: 03 Hours **Maximum Marks: 60**

Instruction for candidates:

1. Section A is compulsory. It consists of 10 parts of two marks each.
2. Section B consist of 5 questions of 5 marks each. The student has to attempt any 4 questions out of it.
3. Section C consist of 3 questions of 10 marks each. The student has to attempt any 2 questions.

Section – A **(2 marks each)**

Q1. (a). State the Rabbe's Test.

(b). Write the relation between Beta and Gamma function.

(c). Check the validity of the Rolle's theorem for function $f(x) = |x|$ on the interval $[-1, 1]$.

(d). Define Rank of Matrix.

(e). State the Rank – Nullity theorem.

(f). Discuss the convergence of series $\sum(\sqrt[3]{(n^3+1} - n)$

(g). State and prove the Cauchy's second theorem on limits for sequence.

(h). If $v = \cos^{-1}\frac{x+y}{\sqrt{x}+\sqrt{y}}$, Show that $x\frac{\partial v}{\partial x} + y\frac{\partial v}{\partial y} + \frac{1}{2}\cot v = 0.$

(i). State the Taylor's theorem (with Lagrange's form of reminder).

(j). Find the Rank of the matrix A = [1 2 3 4 5 6 2 1 2].

Section – B **(5 marks each)**

Q2. Prove that $\nabla.\left\{r\nabla\left(\frac{1}{r^3}\right)\right\} = \frac{3}{r^4}$

Q3. Expand tan x in the powers of $\left(x - \frac{\pi}{4}\right)$ upto first four terms.

Q4. If z = tan $^{-1}\left(\frac{x^3+y^3}{x-y}\right)$. Prove that

$$x^2\frac{\partial^2 z}{\partial x^2} + y^2\frac{\partial^2 z}{\partial y^2} + 2xy\frac{\partial^2 z}{\partial x \partial y} = (1 - 4Sin^2 z)Sin2z.$$

Q5. State and prove the Cayley-Hamilton Theorem.

Q6. Using the Cauchy's Integral test, discuss the convergence or divergence of the series $\sum_{n=1}^{\infty} \frac{1}{n^p}$, $p > 0$.

Section – C **(10 marks each)**

Q7. Find the volume of the solid formed by the revolution of the curve $y^2(2a - x) = x^3$ about its asymptotes.

Q8. Find the Characteristic equation of the matrix $A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$ and verify that it is satisfied by A and hence A^{-1}.

Q9. If s_1 and s_2 are positive and $s_{n+1} = \sqrt{s_n s_{n-1}}$,prove that the sequences s_1 , s_3 , s_5 ,…..; s_2 , s_4 , s_6 …..are the one increasing and the other decreasing and show that their common limit is $(s_1 s_2^2)^{\frac{1}{3}}$.