ISRAEL INSTITUTE FOR ADVANCED STUDIES

ISRAEL SCIENCE FOUNDATION

Einstein Institute of Mathematics
האוניברסיטה העברית בירושלים
THE HEBREW UNIVERSITY OF JERUSALEM

The 24th Midrasha Mathematicae

Random Schrödinger Operators and Random Matrices

14-18 May 2023

Abstracts

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Sunday, 14 May

Sasha Sodin: Quasi-one-dimensional random operators

We shall discuss the spectral properties of quasi-one-dimensional random operators (such as the Anderson model on a strip), including some results from the 1980-s, some more recent ones, and a few open questions.

Ofer Zeitouni: The extremal landscape for the characteristic polynomial of the CβE

(Joint work with Elliot Paquette)

I will describe the behavior of the characteristic polynomial of a CβE near high values of its modulus. The approach is based on the recursions satisfied

by the associated orthogonal polynomials.

Mylene Maida: Number-rigidity for point process and related properties

For Poisson point process, the number of points inside a given (bounded) domain is independent of the configuration outside this domain. On the contrary, several interesting point processes have the intriguing property that the number of points inside a domain is almost surely a deterministic function of the configuration outside; it is called *number rigidity.* We will present several examples of processes having this property, in particular spectra of some random operators. If time allows, we will discuss the dual property of *tolerance,* through the prism of DLR equations.

Benjamin Gess: Fluctuations in conservative systems and SPDEs

Fluctuations are ubiquitous in non-equilibrium conservative systems. The analysis of their large deviations lead to macroscopic fluctuation theory (MFT), a general framework for non-equilibrium statistical mechanics. MFT is based on a constitutive formula for large fluctuations around thermodynamic variables, and can be justified from fluctuating hydrodynamics. The latter postulates conservative, singular SPDEs to describe fluctuations in systems out of equilibrium. Both theories are informally linked via zero noise large deviations principles for SPDEs. In this talk, we demonstrate how the analysis of large deviations of interacting particle systems and conservative SPDEs lead to intricate and open problems for PDEs with irregular coefficients. In the last part of the talk, we present a positive result in this direction, by proving the well-posedness of parabolic-hyperbolic PDEs with irregular coefficients.

Monday, 15 May

Yan Fyodorov: Resonances in wave reflection from a disordered medium: nonlinear σ-model approach

Using the framework of supersymmetric non-linear $\sigma-$ model we develop general non-perturbative characterisation of universal features of the density of S-matrix poles (resonances) in the complex energy plane for the model of waves incident on and reflected from a disordered medium via a single M-channel waveguide/lead. Explicit expressions for the pole density are derived, in particular, in weakly localized/diffusive regime as well as in the regime of strong localization in (quasi-)1D geometry. The latter geometry can be modelled by a finite-rank non-Hermitian deformation of random banded matrices.
We identify several salient features of the pole density, with statistics of poles with small imaginary part reflecting exponential localization and the rest reflecting decaying states located in the vicinity of the attached waveguide.
For multimode waveguides an intermediate powerlaw asymptotics is shown to emerge, reflecting diffusive nature of semiclassical decay.
The presentation will be mainly based on arXiv:2211.03376, which is the joint work with M. Skvortsov and K. Tikhonov.

Mira Shamis: On the abominable properties of the Almost Mathieu operator with Liouville frequencies

(Based on joint work with A. Avila, Y. Last, and Q. Zhou)

We show that, for sufficiently well approximable frequencies, several spectral characteristics of the Almost Mathieu operator can be as poor as at all possible in the class of all discrete Schroedinger operators. For example, the modulus of continuity of the integrated density of states may be no better than logarithmic. Other characteristics to be discussed are homogeneity, the Parreau-Widom property, and (for the critical AMO) the Hausdorff content of the spectrum.

Gady Kozma: Linearly reinforced random walk

Linearly reinforced random walk is a self-interacting process. Given any graph and initial weights on the edges, the walker, at each step, moves according to the weights at this step and then updates the weights by adding 1 to the edge it just crossed. We will survey what is known for this model, and its conjectured connection to the Anderson model.

Tuesday, 16 May

Alan Edelman: Symmetric Spaces and Random Matrix Theory

(Joint work with Sungwoo Jeong)

Inspired by a lecture of Martin Zirnbauer's from many years ago (and help through email correspondence) we take a close look at the interaction of symmetric spaces, random matrix theory, the Cartan decomposition, and numerical linear algebra. No knowledge of symmetric spaces is assumed.

Benedek Valko: Scaling limits of the truncated circular beta ensemble

(Joint with Yun Li , building on joint work with Balint Virag)

If one removes the first row and column of a unitary matrix then the resulting matrix has eigenvalues that lie inside the unit disk. Zyczkowski and Sommers (2000) derived the joint probability density function of the eigenvalues of a truncated uniformly chosen unitary matrix, and showed that they have determinantal structure. Peres and Virag (2003) showed that the point process limit of these eigenvalues can be described as the zero set of a random (Gaussian) analytic function.

The circular beta ensemble is a one parameter family of distributions generalizing the joint eigenvalue distribution of a uniformly chosen unitary matrix. Killip and Kozhan (2017) provided a random matrix model that can be considered the truncated circular beta ensemble, and described the spectrum via a random recursion. We derive and describe the point process limit of the truncated circular beta ensemble together with the scaling limit of the normalized characteristic polynomials. The limiting objects are closely connected to the random analytic function appearing as the limit of the normalized characteristic polynomials of the (full) circular beta ensemble.

Elliot Paquette: The scaling limits of the characteristic polynomial of the Gaussian-beta-ensemble

The Gaussian beta-ensemble is a 1-parameter generalization of the Gaussian orthogonal/unitary/symplectic ensembles which retains some integrable structure, through an explicit tridiagonal matrix representation. Its eigenvalues have been shown to converge in law to the Sine-beta and Airy-beta point processes, depending on whether one looks in the spectral bulk or the spectral edge. These point processes, in general beta, have been characterized as the eigenvalues of stochastic operators, among other ways.

In this talk, we present joint work with Gaultier Lambert on the scaling limits of the characteristic polynomials themselves, as random real analytics functions. At the spectral edge, this results in a new limiting object, the stochastic Airy function. In the bulk, we show this is the stochastic zeta function of Valko-Virag. We'll show how this can be anticipated from the tridiagonal representation of the Gaussian-beta ensemble and then give some descriptions of the limit objects.

Wednesday, 17 May

Laszlo Erdos: Condition numbers and eigenvector overlaps for random matrices

It is well known that eigenvalues of general non-Hermitian matrices can be very unstable under tiny perturbations but adding a small noise regularises this instability. The quantity governing this effect, called the eigenvalue condition number in numerical linear algebra, is also well known in random matrix theory as the eigenvector overlap. We present several recent results on almost optimal lower and upper bounds on this key quantity. For the lower bound we need to prove the strong form of quantum unique ergodicity (QUE) for the singular vectors of non-Hermitian random matrices. The upper bound requires very different tools: here we prove a Wegner type estimate for non-Hermitian matrices. The talk is based upon joint works with G. Cipolloni, J. Henheik, H.-C. Ji, O. Kolupaiev and D. Schroder.

Martin Zirnbauer: Field Theory of Random Schrödinger Operators

The spectrum of a self-adjoint operator is known to decompose into three parts, which are called pure point (pp), absolutely continuous (ac), and singular continuous (sc). In the traditional physics approach to Anderson (de-) localization for random Schrödinger operators, only the first two types of spectrum are featured: ac spectrum comes with spatially extended stationary states (a.k.a. metallic regime), while the energy eigenfunctions for eigenvalues in the pp spectrum are localized (a.k.a. insulating regime). Now, over the last few years there have been various predictions of a possible third regime, called NEE (for non-ergodic extended), where the eigenstates are fractal, matching the

phenomenology expected for the case of sc spectrum. There exists, however, an ongoing debate as to whether NEE/sc can be a true thermodynamic phase (instead of just a finite-size effect or an exotic feature that needs fine-tuning to a critical point).
In this talk, I will first review the standard field-theory approach, developed by Wegner, Efetov and others, for random Schrödinger operators in the metallic and insulating regimes. Motivated by a recent proposal for the conformal field theory of the integer quantum Hall transition, I will then describe a field-theoretical scenario for the elusive case of random Schrödinger operators with sc spectrum (NEE phase). Distinct from the usual sigma model, the proposed scenario is supported by an exact solution of Wegner´s (N=1)-orbital model on a Bethe lattice. It is expected to be generic for moderately strong disorder in high space dimension.

Cyril Labbe: On the 1d Anderson Hamiltonian with white noise potential - first part

This talk will focus on the random Schrodinger operator on R obtained by perturbing the Laplacian with a white noise. First, I will present an Anderson localization result for this operator: the spectrum is pure point and the eigenfunctions exponentially localized. Second, I will present various results on the eigenvalues and eigenfunctions of the finite-volume approximation of this operator in the infinite volume limit: it turns out that many different behaviors can be observed according to the energy regime one focuses on. This is based on a series of joint works with Laure Dumaz.

Laure Dumaz: On the 1d Anderson Hamiltonian with white noise potential - second part

In the second part of the talk, I will present in more details the delocalized part of the spectrum of the random Schrodinger operator on finite volume. For energies of the order of the size of the box, the operator converges to a non-trivial operator that we call Critical Schrodinger operator. Moreover, its eigenfunctions follow a universal shape given by the exponential of a Brownian motion plus a drift, a behavior already observed by Rifkind and Virag in tridiagonal matrix models. This is based on a series of joint works with Cyril Labbé.

Thursday, **18 May**

Balint Virag: Eigenvectors of the square grid plus GUE

Eigenvectors of the GUE-perturbed discrete torus with uniform boundary conditions retain some product structure for small perturbations but converge to discrete Gaussian waves for large perturbations. In joint work with Andras Meszaros, we determine where the phase transition happens.

Tatyana Shcherbina: Universality of the second correlation function of the

In this talk we consider non-Hermitian random $n\times n$ matrices of the form $H=A+H_0$, where H_0 is a standard Ginibre matrix with iid Gaussian entries, and A is a rather general matrix. Such matrices are important in communication theory, where A is considered as a ``signal", and $H_0$ as a ``noise" matrix. Using the Girko approach and supersymmetry techniques to obtain the suitable integral representation, we show the limiting behavior of the second correlation function of such matrices in the bulk of the spectrum coincides with that for the pure Ginibre matrices. The talk is based on a joint work with Ievgenii Afanasiev and Mariya Shcherbina.

Frederic Klopp: The entanglement entropy for the ground state of a system of interacting fermions submitted to a random field

* The entanglement entropy for the ground state of a system of interacting fermions submitted to a random field.

* In condensed matter physics, the behavior of the entanglement entropy a state is argued to be a marker of its many body localization/thermalization properties. In the talk, we shall present the computation of the entanglement entropy of the ground state of a system of interacting fermions submitted to a random field in the thermodynamic limit. The talk is based on the PhD thesis of and joint work with V. Ognov.