

THE UNITED REPUBLIC OF TANZANIA
PRESIDENT'S OFFICE
REGIONAL ADMINISTRATION AND LOCAL GOVERNMENT



FORM FOUR INTER-REGIONAL EXAMINATION

LINDI, MOROGORO AND MTWARA

CODE: 041

BASIC MATHEMATICS- MARKING SCHEME.

(For Both School and Private Candidates)

SECTION A(60 Marks)

1. (a) Simplify the sum of 85% of 9861 and $\frac{3}{5}$ of 12458. Write your answer correct to two significant figures.

$$\frac{85}{100} \times 9861 + \frac{3}{5} \times 12458$$

1 mark

$$8381.85 + 7474.8 = 15856.65$$

1 mark

$$15856.65 \approx 16000 \text{ in two significant figures}$$

1 mark

- (b) (i) Mr. Tumai Distributed Tshs 1,200,000/= awards to students who passed well in their examinations and their respective teachers as follow: 22% to all students who passed Arts Subjects. 17% to students who passed Mathematics and 26% to students who passed science subjects. The remained amount was distributed to teachers. Required to Find the amount that were awarded to teachers.

$$22\% + 17\% + 26\% + \text{Teachers} = 100\%$$

1 mark

$$65\% + \text{Teachers} = 100\%$$

$$\text{Teachers} = 100\% - 65\%$$

1 mark

$$\text{Teachers} = 35\%$$

1 mark

$$= \frac{35}{100} \times 1,200,000 = 420,000 ; \text{Teachers were awarded Tshs. 420,000/=}$$

2. (a) If $a = \sqrt{5}$, $b = \sqrt{3}$, and $c = \sqrt{2}$. Simplify $\frac{a-b}{b+c}$

$$\frac{a-b}{b+c} = \frac{\sqrt{5}-\sqrt{3}}{\sqrt{3}+\sqrt{2}} \text{ By rationalizing the denominator}$$

$$\frac{\sqrt{5}-\sqrt{3}}{\sqrt{3}+\sqrt{2}} = \frac{\sqrt{5}-\sqrt{3}}{\sqrt{3}+\sqrt{2}} \times \frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}-\sqrt{2}} = \frac{\sqrt{15}-\sqrt{10}-3+\sqrt{6}}{3-\sqrt{6}+\sqrt{6}-2}$$

$$= \sqrt{15} - \sqrt{10} + \sqrt{6} - 3$$

- (b) Describe the applications of Logarithm in real life situations (1mark @, Total = 3marks)

☐ In chemistry:

- ✓ To determine acidity and alkalinity of the pH ($p(t) = - t$)
- ✓ In radioactive decay by geologists (finding Half-life)
- ✓ In carbon dating – Radiometric dating techniques

☐ Measure of population growth

- ✓ Determine increment of certain measurement (Natural and Exponential functions)
- ✓ Predict death rates, population increments (birth rates)

☐ Measure of Earthquakes

- ✓ Measure of magnitude of an Earthquakes
- ✓ Predict the coming of an Earthquake

3. In a certain class MSUVA displayed counting numbers less than 30. From the displayed numbers, OKWI mentioned all numbers completely divisible by 2 and FEITOTO mentioned all numbers that were the multiples of 3.

(a) (i) Outline all numbers that were mentioned by OKWI and FEITOTO in Common

$$\text{OKWI} = \{2, 4, 6, 8, 10, 12, 14, 16, 18, 20, 22, 24, 26, 28\}$$

0.5 marks

$$\text{FEITOTO} = \{3, 6, 9, 12, 15, 18, 21, 24, 27\}$$

0.5 marks

$$\text{Numbers mentioned in common} = \{6, 12, 18, 24\}$$

0.5 marks

(ii) How many numbers were mentioned BY Either OKWI or FEITOTO

$$n(\text{OKWI} \cup \text{FEITOTO}) = n(\text{OKWI}) + n(\text{FEITOTO}) - n(\text{Common numbers})$$

0.5 marks

$$n(\text{OKWI} \cup \text{FEITOTO}) = 14 + 9 - 4 = 19$$

1 mark

(b) A number was selected in random from the number displayed by a teacher, Req. to find the probability that a selected number was mentioned in common.

$$P(\text{common number}) = \frac{n(\text{Common number})}{n(\text{Counting number less than 50})}$$

1.5 marks

$$P(\text{common number}) = \frac{4}{39}$$

1.5 marks

4. (a) Given that $\underline{a} = 4i + 3j$ and $\underline{b} = 6i - 3j$, Required to find the value of “h” and “k”

$$\text{if } h\underline{a} + k\underline{b} = 10i + j$$

$$\text{given that } \underline{a} = 3i + 2j \text{ and } \underline{b} = 4i - 3j$$

$$h\underline{a} + k\underline{b} = h(3i + 2j) + k(4i - 3j)$$

$$= 3hi + 2hj + 4ki - 3kj \quad (1\text{mark})$$

$$\text{Comparing } h\underline{a} + k\underline{b} = 10i + j$$

$$3hi + 2hj + 4ki - 3kj = 10i + j$$

$$3hi + 4ki = 10i$$

$$3h + 4k = 10 \dots\dots\dots (i) \quad (0.5 \text{ marks})$$

$$2hj - 3kj = j$$

$$2h - 3k = 1 \dots\dots\dots (ii) \quad (0.5 \text{ marks})$$

Solving equation (i) and (ii) Simultaneously, $k = 1$ and $h = 2$ (1 mark)

(b) A perpendicular line from the point $P(2,-4)$ to the line meets the line at point $Z(-1,3)$.

Required to find:

i. Distance $\overline{PZ}$

$$\text{Distance } \overline{PZ} = \sqrt{(2 + 1)^2 + (-4 - 3)^2} = \sqrt{58} = 7.615 \text{ unit of length} \quad (1.5 \text{ marks})$$

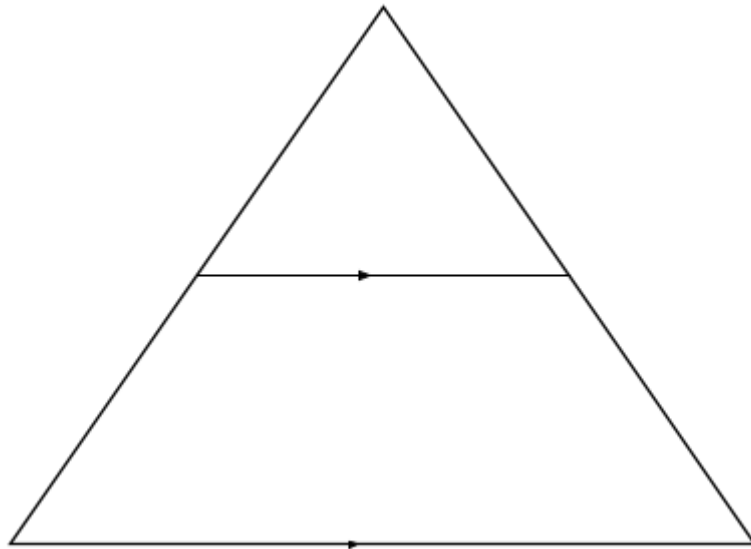
ii. If the point $Z(-1,3)$ is a mid-point of the line $\overline{PQ}$, find the coordinates of point Q

$$\text{Midpoint} = \left(\frac{x+2}{2}, \frac{y-4}{2} \right)$$

$$(-1, 3) = \left(\frac{x+2}{2}, \frac{y-4}{2} \right) \text{ by comparing, we get; } (x, y) = (-4, 10)$$

$$\text{Hence } Q(x, y) = (-4, 10) \quad (1.5 \text{ marks})$$

5. (a) In the following figure; $\overline{ED} \parallel \overline{BC}$ and $\overline{AB} = \overline{AC}$; If $\overline{AB} = 13\text{cm}$ and $\overline{BC} = 10\text{cm}$, find the Area of BCDE



E

A

D

1 mark

B

C

0.5 marks

6.5cm

6.5cm

0.5 marks

10cm

$$\hat{BAC} = \hat{EAD}$$

(common Angle)

0.5 marks

$$\hat{ABC} = \hat{AED}$$

(alternate angle $\overline{ED} \parallel \overline{BC}$)

0.5 marks

$$\begin{array}{l} \hat{B}\hat{C}A = \hat{E}\hat{D}A \quad \text{(alternate angle } \overline{ED} \parallel \overline{BC}) \\ \triangle ABC \sim \triangle AED \quad \text{(AAA)} \end{array}$$

$$\text{Hence } \frac{13}{6.5} = \frac{6.5}{\overline{ED}} \quad \text{and } \overline{ED} = 5\text{cm}$$

$$\text{Area of } \triangle ABC = \frac{1}{2} \times 12 \times 5 = 30$$

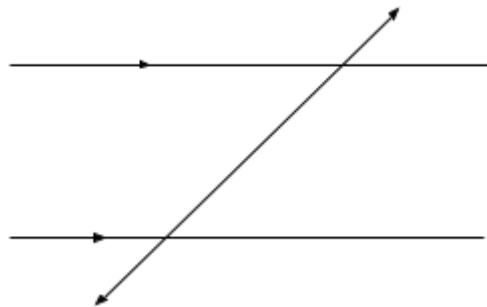
$$\text{SINCE } \overline{AB} = \overline{BC} = 2 \times \overline{AE} = 2 \times \overline{AD} \text{ and } \overline{BC} = 2 \times \overline{ED}$$

$$\text{Therefore Area of } \triangle ABC = 2 \times \text{Area of } \triangle AED$$

$$\text{Hence Area of } BCDE = \text{Area of } \triangle ABC - \triangle AED = \frac{\text{Area of } \triangle ABC}{2} = \frac{30}{2}$$

$$\text{Area of } BCDE = 15\text{cm}^2$$

(b) By considering the Alternating opposite angles theorem, required to draw the diagram hence identify the corresponding angles



1 mark

1 mark

a

b

x

y

1 mark

$$b = x$$

$$a = y$$

6. (a) The time “t” taken to buy fuel at LAKE OIL PETROL STATION in Morogoro varies directly as the number of vehicle “V” in the queue and inversely as the number of pumps “P” available in the station. In petrol station with 10 pumps it takes 20 minutes to fuel 40 vehicles.

i. Required to write vehicle “v” in terms of pumps “P” and time “t”

$$t \propto \frac{v}{p} \text{ which implies that } t = k \frac{v}{p} \quad (0.5 \text{ marks})$$

$$v = \frac{pt}{k} \quad (0.5 \text{ marks})$$

ii. Req. to find the time it will take to fuel 60 vehicles in a station with 4 pumps.

$$\text{From, } t = \frac{kv}{p} \text{ this implies } k = \frac{pt}{v} \quad (1 \text{ mark})$$

$$\text{when } p = 10, t = 20 \text{ and } v = 40 \text{ therefore } k = \frac{10 \times 20}{40} = 5 \quad (1 \text{ mark})$$

when $v = 60$, $p = 4$ and $k = 5$, therefore $t = \frac{5 \times 60}{4} = 75$ minutes

(b) Mayele bought 3 bottles of juice of capacity 350 ml and Dialo bought 1 bottle of juice of capacity 1 litre.

i. required to find Who had more juice to drink.

$$\text{Mayele juice} = 3 \times 350 \text{ ml} = 1050 \text{ ml} = 1.05 \text{ litres}$$

$$\text{Dialo juice} = 1 \text{ litre} \quad \textbf{(1 mark)}$$

Mayele had more Juice to drink

ii. required to find How much more

$$\text{amount} = 1.05 \text{ litres} - 1 \text{ litre} = 0.05 \text{ litre} = 50 \text{ ml} \quad \textbf{(1 mark)}$$

7. (a) Require to give the meaning of the following terms as used in Accounts

- (i) **Cash book:** Is the book/account which records cash transactions only.
- (ii) **Assets:** Are properties or possessions of the business; Forexample buildings, motor vans, etc **(1mark @)**
- (iii) **Credit transaction:** Are transactions where by payments are done later.

(b) A company bought two cars for Tshs 25,000,000/= each. If one car was sold at a profit of 18% and another was sold at a loss of 6%. In the whole transactions there were no loss. What was the profit made by a company?

$$\text{Car 1: Profit} = \frac{18}{100} \times 25,000,000 = 4,500,000 \quad \text{1 mark}$$

$$\text{Car 2: Loss} = \frac{6}{100} \times 25,000,000 = 1,500,000 \quad \text{1 mark}$$

$$\begin{aligned} \text{Profit made} &= \text{Profit of Car 1} - \text{Loss of car 2} \\ &= 4,500,000 - 1,500,000 = 3,000,000 \quad \text{1 mark} \end{aligned}$$

8. (a) A BODA BODA driver rates after each kilometre. The fare is Tshs. 1000/= for the first kilometre and raise by Tshs. 500/= for each additional kilometre. If MROPE want to travel 10 kilometres by BODA BODA. What will he be charged by BODA BODA driver?

$$\text{First kilometre } (A_1) = 1,000$$

$$\text{Amount added to preceeding } (d) = 500$$

Required to find charge (A_{50}) after travelling 50km ($n = 10$)

$$A_n = A_1 + (n - 1)d \quad \text{1 mark}$$

$$A_{10} = 1,000 + (10 - 1)500 \quad \text{1 mark}$$

$$\text{1 mark}$$

$$A_{10} = 5,500 \text{ Tshs.}$$

(b) The number 19683 is in which term in the following Geometric sequence 3, 9, 27, ...?

$$G_n = 19683, G_1 = 3, r = 3 \text{ required to find number of terms "n"}$$

$$\text{0.5 mark}$$

$$G_n = G_1 r^{n-1} \quad \text{0.5 mark}$$

$$19683 = 3 \times 3^{n-1} \quad \text{1 mark}$$

$$6561 = 3^{n-1} \quad \text{1 mark}$$

$$3^8 = 3^{n-1}$$

$$n - 1 = 8, n = 9$$

32768 is the Nineth term of the sequence

9. (a) If the square of the hypotenuse of an isosceles right-angled triangle is 128 cm^2 find the length of each other sides

let a side be x

$$x^2 + x^2 = 128$$

1 mark

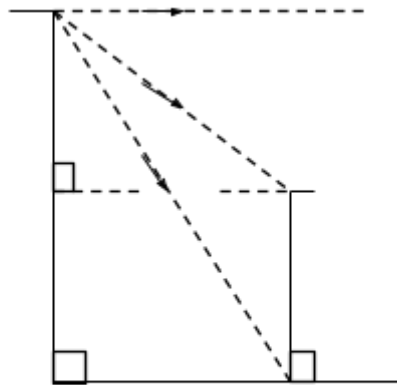
$$2x^2 = 128$$

1 mark

$$x^2 = 64, \quad x = 8 \text{ cm}$$

1 mark

- (b) From the top of a tower of height 60m the angles of depression of the top and the bottom of a building are observed to be 30° and 60° respectively. Find the height of the building.



0.5 marks

0.5 marks

0.5 marks

0.5 marks

1 mark

30°

30°

60°

30°

A

B

C

D

E

$$\overline{AD} = 60 \text{ m}$$

$$\tan 60^\circ = \frac{\overline{AD}}{\overline{DC}} = \frac{60}{\overline{DC}}$$

$$\overline{DC} = \frac{60}{\sqrt{3}}$$

$$\text{Also, } \tan 30^\circ = \frac{\overline{AE}}{\overline{BE}} = \frac{60 - \overline{DE}}{\overline{BE}}$$

$$\overline{BE} = \sqrt{3}(60 - \overline{DE})$$

But $\overline{DC} = \overline{BE}$

$$\frac{60}{\sqrt{3}} = \sqrt{3}(60 - \overline{DE})$$

$\overline{DE} = \overline{BC} = 40\text{cm}$ **height**

10. (a) If the length of each side of a square is increased by 6cm, the area become increased by 144cm^2 of the area of the small square. Required to find the length of one side of the original square.

$$\text{Area of small square} = \text{Length} \times \text{Length} = (\text{Length})^2$$

$$\text{Area of Large square} = (6 + \text{Length})(6 + \text{length})$$

0.5 marks

$$\text{But, Area of large square} = 144 + \text{Area of small square}$$

0.5 marks

$$36 + 12\text{Length} + (\text{Length})^2 = 144 + (\text{Length})^2$$

0.5 marks

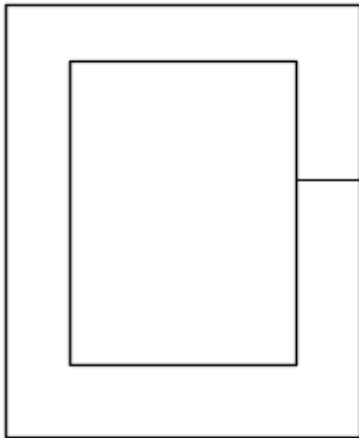
$$36 + 12\text{Length} = 144$$

$$12\text{Length} = 108$$

1 mark

$$\text{Length} = 9\text{cm}$$

- (b) A large rectangular garden in a park is 120m wide and 150m long. A contractor is called in to add a brick walkway to surround this garden by the same width. If the area of the walkway is 2800m², how wide is the walkway?



0.5 marks

0.5 marks

0.5 marks

1 mark

0.5 marks

120m

150m

x

$$\text{Area of small rectangle} = 120 \times 150 = 18000 \text{ m}^2$$

$$\text{Area of large rectangle} = (120 + 2x)(150 + 2x)$$

$$\text{But, Area of large} = \text{Area of small} + \text{Area walkway}$$

$$(120 + 2x)(150 + 2x) = 18000 + 2800$$

$$18000 + 540x + 4x^2 = 18000 + 2800$$

$$4x^2 + 540x - 2800 = 0$$

$$x^2 + 135x - 700 = 0$$

$$x = 5 \text{ or } x = -140$$

$$\text{Since, Length} \neq \text{negative value, } x = 5\text{m}$$

The walkway is 5m wide

SECTION B(40 Marks)

11. (a) In certain research the data were summarized as shown in a table below:

Class Mark	10	15	20	25	30	35
Frequency	3	2	10	5	4	1

Required to reconstruct a frequency distribution table including class interval and frequency.

Let $a - b$, be the first class interval. Then $\frac{a+b}{2} = 10$

hence $a + b = 20$ (i) **(0.5mark)**

class Size = $15 - 10 = 5$ (Consecutive Classmarks)

But Also, Class Size = Upper boundary – Lower boundary

Class Size = $b + 0.5 - a + 0.5$

$5 = b - a + 1$

$b - a = 4$(ii) **(1mark)**

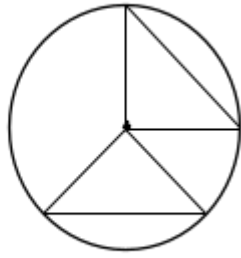
Solving (i) and (ii) simultaneously we get $a = 8, b = 12$ **(1.5 mark)**

The first interval is $8 - 12$

interval	8 – 12	13 – 17	18 – 22	23 – 27	28 – 32	33 – 37
Frequency	3	2	10	5	4	1

(2 marks)

(b) Required to prove that equal chords of a circle subtend equal angles at a centre.



A

B

C

D

O

$\text{chord } \overline{AB} = \text{chord } \overline{DC}$ Given (1 mark)

$\overline{AO} = \overline{DO} = \overline{BO} = \overline{CO}$ radii (1 mark)

$\triangle AOB \cong \triangle COD$ SSS (1 mark)

Hence $\hat{AOB} = \hat{COD}$ (1 mark)

(1 mark)

12. (a) Two towns P and Q on the Latitude 48° are 370 Nautical miles apart. Find the difference in their longitude.

$Nm \text{ Distance} = (60\theta \cos \alpha) Nm$ (1 mark)

where $\theta = \text{Difference in longitude (to be found)}$

and $\alpha = \text{Latitude} = 48^\circ N$ (Given)

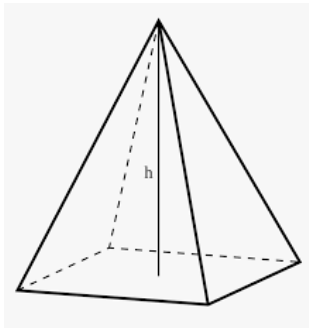
$\text{Distance} = 370 Nm$ (Given)

Therefore, $370 Nm = (60\theta \cos 48^\circ) Nm$ (1 mark)

$\theta = 9.2^\circ$ (1 mark)

(b) A pyramid with vertex V and edges VA, VB, VC, VD each 15cm long has a rectangular base ABCD where $AB = CD = 10\text{cm}$ and $AD = BC = 8\text{cm}$.

i. Sketch the pyramid using the above information.



V

B

C

A

D

O

15cm

8cm

10 cm

1.5 marks

- ii. Calculate the height “VO” of the pyramid where “O” is the centre of the rectangle.

Consider $\triangle AOV$,

$$\overline{VO}^2 + \overline{AO}^2 = \overline{AV}^2 \text{ but } \overline{AO} = \frac{1}{2}\overline{AC}$$

$$\overline{VO}^2 = 15^2 - \frac{1}{4}\overline{AC}^2 \dots\dots\dots (i) \quad \textbf{(0.5 marks)}$$

consider $\triangle ABC$,

From Pythagoras theorem

$$\overline{AC}^2 = \overline{AB}^2 + \overline{BC}^2$$

$$\overline{AC}^2 = 10^2 + 8^2 \text{ hence } \overline{AC}^2 = 164 \dots\dots\dots (ii) \quad \textbf{(0.5 marks)}$$

By Subst equation (ii) into (i)

$$\overline{VO} = h = 13.6\text{cm} \quad \textbf{(1mark)}$$

- iii. Calculate the angle between the base and edge

From $\triangle AOV$, let θ represent the angle between $\overline{VA}$ and $\overline{AO}$

$$\text{Then, } \cos \theta = \frac{\overline{AO}}{\overline{VA}} = \frac{\frac{\sqrt{164}}{2}}{15} \quad \textbf{(1mark)}$$

Hence $\theta = 64.7^\circ$ (0.5 marks)

13. (a). (i). Find the possible values of x if the matrix $\begin{pmatrix} x+1 & 9 & 3 \\ x & 5 \end{pmatrix}$ has no inverse.

(ii). By using matrix method, solve the equations $\begin{cases} 4x - 9 = -3y \\ 5x - 17 - 2y = 0 \end{cases}$

(i). Let $A = \begin{pmatrix} x+1 & 9 & 3 \\ x & 5 \end{pmatrix}$

Since matrix A has no inverse

Then

$$|A| = 0 \quad (0.5 \text{ marks})$$

So

$$(x+1)(x-5) - (3 \times 9) = 0 \text{ expanding } x^2 - 5x + x - 5 - 27 = 0 \text{ hence } x^2 - 4x - 32 = 0$$

Solving quadratically we get (1mark)

$$\underline{x = 8 \text{ or } -4}$$

(ii). Given that $\begin{cases} 4x - 9 = -3y \\ 5x - 17 - 2y = 0 \end{cases}$

$$\text{Rearranging } \begin{cases} 4x + 3y = 9 \\ 5x - 2y = 17 \end{cases}$$

$$\text{Let } B = \begin{pmatrix} 4 & 3 & -9 \\ 5 & -2 & -17 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 9 \\ 17 \end{pmatrix}$$

$$|B| = -8 - 15 \text{ hence } |B| = -23 \quad (0.5 \text{ marks})$$

$$B^{-1} = \frac{1}{-23} \begin{pmatrix} -2 & -3 & -5 & 4 \end{pmatrix}$$

$$B^{-1} = \begin{pmatrix} \frac{2}{23} & \frac{3}{23} & \frac{5}{23} & \frac{-4}{23} \end{pmatrix}$$

(1mark)

Then

$$B^{-1}B = B^{-1} \begin{pmatrix} 9 \\ 17 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} \frac{2}{23} & \frac{5}{23} & \frac{3}{23} & \frac{-4}{23} \end{pmatrix} \begin{pmatrix} 9 \\ 17 \end{pmatrix}$$

(1mark)

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} \frac{18}{23} & \frac{51}{23} & \frac{45}{23} & \frac{-68}{23} \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} \frac{69}{23} & \frac{-23}{23} \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 3 \\ -1 \end{pmatrix} \text{ hence } \underline{x = 3 \text{ and } y = -1} \quad (1 \text{ mark})$$

(b). A transformation is given $x' = 2x + 5y$ and $y' = -x$, Required to find the image of the equation $2x - 3y = 6$ under the transformation matrix which performs the above transformations.

Given

$$x' = 2x + 5y$$

$$y' = -x$$

Then

$$\text{The transformation matrix} = \begin{pmatrix} 2 & 5 & -1 & 0 \end{pmatrix} \quad (1 \text{ mark})$$

Required to find the image of $2x - 3y =$.

Its x and y intercept points are:

A(0,-2) and B(3,0).

Their images are obtain as

$$A' = (2 \quad 5 \quad -1 \quad 0) (0 \quad -2) \quad (1\text{mark})$$

$$A' = (-10, 0)$$

Also

$$B' = (2 \quad 5 \quad -1 \quad 0) (3 \quad 0)$$

$$B' = (6, -3) \quad (1\text{mark})$$

$$M = \frac{y_2 - y_1}{x_2 - x_1} \quad M = \frac{-3 - 0}{6 - (-10)} \quad M = \frac{-3}{16}$$

Also to find equation

$$M = \frac{y - y_0}{x - x_0} \quad M = \frac{y - 0}{x - (-10)} \quad \text{such that } \frac{-3}{16} = \frac{y}{x+10} \quad (1\text{mark})$$

$$-3x - 30 = 16y \quad \text{rearranging, } -3x - 16y = 30$$

$$\text{Its image is } \underline{3x + 16y = -30} \quad (1\text{mark})$$

14. (a) A function f is defined as;

$$f(x) = \begin{cases} x - 4, & \text{if } x < 2 \\ 15, & \text{if } -2 < x < 2 \\ x^2, & \text{if } x > 2 \end{cases}$$

Req. to Find

- | | | |
|-------|---|---------|
| (i) | $f(1.95) = 15$ | 1 mark |
| (ii) | $f(15) = 15^2 = 225$ | 1 mark |
| (iii) | Domain of $f = \{x \in \mathbb{R}, x \neq -2 \text{ and } x \neq 2\}$ | 2 marks |
| (iv) | The Type of a function f is STEP FUNCTION | 1 mark |

(b) A school is preparing a trip for 400 students. The company who is providing the transportation has 10 buses of 50 seats each and 8 buses of 40 seats, but only 9 drivers available. The rental cost for a large bus is 800,000/= and 600,000/= for the small bus. How many buses of each type should be used for the trip for the least possible cost?

CONSTRAINTS

$$Eq1: 40x + 50y \geq 400$$

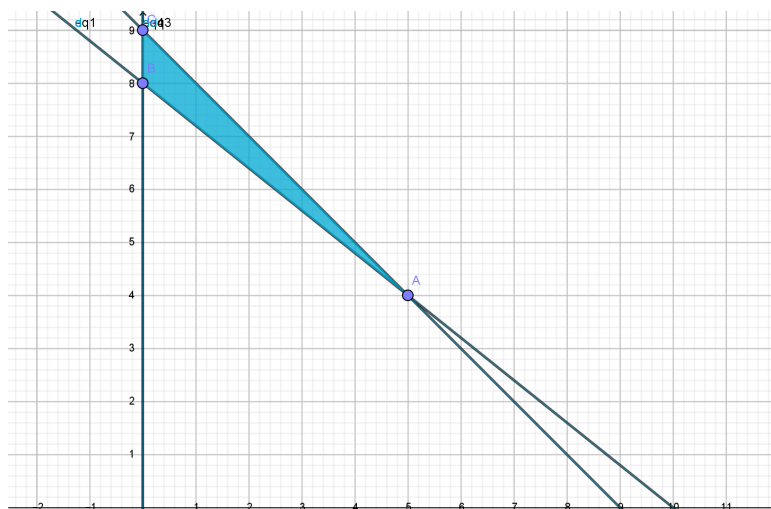
0.5 marks

$$Eq2: x + y \leq 9$$

0.5 marks

$$Eq3: x \geq 0$$

$$Eq4: y \geq 0$$



2 marks

$$A = (6, 4)$$

$$B = (0, 8)$$

$$C = (0, 9)$$

$$\text{Objective Function; } F(x, y) = 600,000x + 800,000y$$

$$F(5, 4) = 6,200,000$$

1 mark

$$F(0, 8) = 6,400,000$$

$$F(0, 9) = 7,200,000$$

1 mark

Hence the minimum cost is 6,200,00, hence 5 small buses and 4 large buses should be used