MAT 2071 - Fall 2019 - CityTech
Prof Reitz

# Exam 1 Review

Sec 1.1-1.8, 2.1-2.11

**NOTE: Problem #16 will NOT be on exam #1**

1. Write each of the sets by listing their elements between curly brackets.

   a. $\{x^2 - 2 : x \in \mathbb{N}\}$ b. $\{x \in \mathbb{Z} : |3x| < 15\}$ c. $\{7x - 1 : x \in \mathbb{Z},\ |7x| \le 21\}$

2. Write each of the sets in set-builder notation.

   a. $\{\ldots, -20, -15, -10, -5, 0, 5, 10, 15, 20, \ldots\}$

   b. $\{1, 4, 9, 16, 25, \ldots\}$

   c. $\{\ldots, \frac{1}{8}, \frac{1}{4}, \frac{1}{2}, 1, 2, 4, 8, \ldots\}$

3. Find the cardinality of each set.

   a. $|\{\emptyset, \{\emptyset\}, \{\{\emptyset\}\}, \{\emptyset, \{\emptyset\}\}\}|$ b. $|\{x \in \mathbb{Z} : |x^3| < 10\}|$

4. Label each statement True or False, and include a brief explanation.

   a. $\{\emptyset\} \subseteq \{1, 2, 3, \emptyset\}$ b. $\{\emptyset\} \in \{1, 2, 3, \emptyset\}$ c. $\{2, 4, 6, 8\} \in P(\mathbb{N})$

   For the remaining parts, let $D = \{7, \pi, \emptyset\}$ and $E = \{e, 3, 5\}$

   d. $\emptyset \in P(D)$ e. $(3, 7) \in D \times E$ f. $\{(7, 5), (\pi, \emptyset)\} \in P(D \times E)$

5. Given sets $A = \{a, b, c\}$, $B = \{c, d, e\}$, $C = \{c, e\}$ and universal set $U = \{a, b, c, d, e, f, g\}$, find each of the following and state the cardinality:

   a. $A \cup C$ b. $A \cap C$ c. $P(A)$ d. $(A \cup B) - C$

   e. $(A \cap B) \cup (B \cap C)$ f. $B \times C$ g. $\overline{A \cup B \cup C}$

   h. $P(C^2)$ i. $(B \times C) - C^2$

6. Given intervals $D = [1, 5]$, $E = (2, 6)$, and $F = [-3, 7]$,

   Write in interval notation: a. $D \cup E$ b. $F - (D \cup E)$ c. $\overline{D} \cap E$

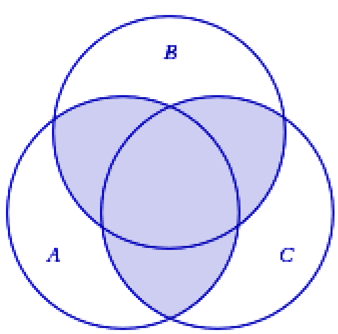
   Sketch in the plane: d. $D \times F$ e. $(F - E) \times \{1, 3, 5\}$

7. Venn diagrams.

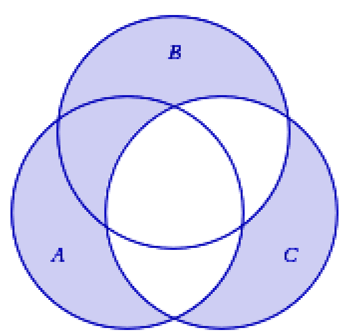
   a. Sketch a Venn diagram for: $A \cap (B \cup C)$

   b. Sketch a Venn diagram for: $\overline{(A \cap B)} \cap C$

   Write an expression for each of the Venn diagrams below.




c.




d.

8. Find the union and intersection of each collection.

   a. Let $A_1 = \{-2, 1\}$, $A_2 = \{-4, 2\}$, $A_3 = \{-6, 3\}$, and in general for each $n \in \mathbb{N}$, $A_n = \{-2n, n\}$.

   Find $\bigcup_{i=1}^{\infty} A_i$ and $\bigcap_{i=1}^{\infty} A_i$.

   b. For each $n \in \mathbb{N}$, let $A_n$ be the closed interval of real numbers $A_n = [\frac{1}{n}, n]$. Find $\bigcup_{i=1}^{\infty} A_i$ and $\bigcap_{i=1}^{\infty} A_i$.

c. For each real number $\alpha \in [3, 6]$, let $A_\alpha = \{\alpha, 4\} \times [3, 4]$. Find $\bigcup_{\alpha\in[3,6]} A_\alpha$ and $\bigcap_{\alpha\in[3,6]} A_\alpha$ (give a description in words and a sketch).

9. For each item, determine if it is a statement. If so, determine whether it is True or False. If not, is it an open sentence?
   a. Frogs have yellow blood.
   b. $x^2 - 11 = 7$.
   c. Add 7 to both sides of the equation.
   d. $\mathbb{Z} \subseteq \mathbb{N}$
   e. Is $5 > 3$?
10. Write a truth table for each statement (show your work).
   a. $P \Rightarrow \sim (Q \wedge \sim P)$
   b. $(P \wedge Q) \Leftrightarrow (P \wedge R) \vee (Q \wedge R)$
11. Translate the given statement S into logical form, using statements P, Q and R.
   S: If $x \in \{4, 9, 12, 16\}$ and $x$ is not a perfect square then $x = 12$.
   P: $x = 12$
   Q: $x$ is a perfect square
   R: $x \in \{4, 9, 12, 16\}$
12. Determine whether each pair of statements is logically equivalent by comparing rows in a truth table.
   a. $P \Rightarrow Q$ and $\sim P \vee Q$
   b. $(P \wedge Q) \vee \sim R$ and $(R \Rightarrow P) \wedge (R \Rightarrow Q)$
13. Express (in English) the contrapositive of each statement.
   a. If a frog can jump, then it's alive.
   b. If $x$ is a two-digit even number, then $x$ isn't prime.

You may use the following in problems 14 and 15:
E(n): the number $n$ is even
O(n): the number $n$ is odd
$\mathbb{R}^+$: the set of positive real numbers
$\mathbb{R}^-$: the set of negative real numbers
F: The set of functions
G: The set of polynomial functions

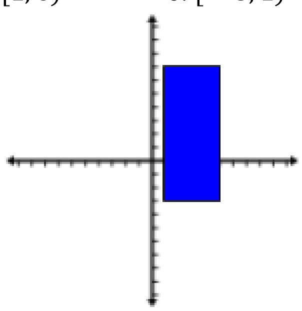
14. Translate each statement to symbols. Is the statement true or false?
   a. For every integer $x$, the number $4x + 7$ is odd.
   b. There exists a real number $x$ such that all real numbers greater than $x$ are positive.
   c. There is no real number $y$ such that all real numbers less than $y$ are negative.
   d. There exists a real number $z$ such that all positive numbers are greater than $z$.
   e. For every function $f$, if $f$ is a polynomial then $f'$ is a polynomial.
   f. If $f$ and $g$ are polynomials then so is the sum $f + g$.
15. Translate to English. Is the statement true or false?
   a. $\forall x \in \mathbb{R}, (x \in \mathbb{N} \Rightarrow x \in \mathbb{Z})$
   b. $\forall m, n \in \mathbb{Z}\,(E(m) \wedge O(n)) \Rightarrow O(m \cdot n)$
   c. $\forall f \in F, \sim (f \in G) \Rightarrow \sim (f = 2x^2 + 6x + 1)$
16. Find the negation of each sentence. For a) and b), give your answer in words. For c) and d), give your answer first in symbols, then in words.
   a. $a$ and $b$ are both positive

b. every integer is either prime or composite
c. $\forall x \in \mathbb{R}\ \exists y \in \mathbb{R}\ y^2 = x$
d. $(p\ is\ even) \Rightarrow (q\ is\ odd)$

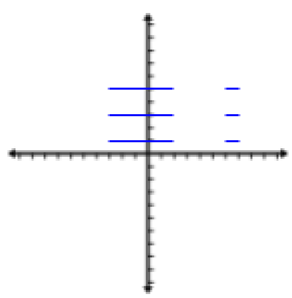
# Exam 1 Review ANSWER KEY

*If you discover an error please let me know, either in class, on the OpenLab, or by email to jreitz@citytech.cuny.edu. Corrections will be posted on the "Exam Reviews" page.*

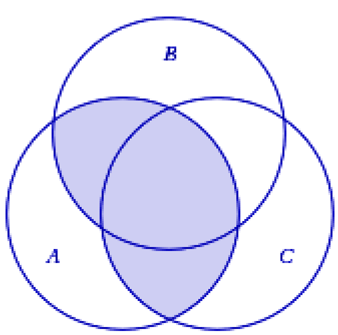
1. a. { -1, 2, 7, 14, 23, … }  b. { -4, -3, -2, -1, 0, 1, 2, 3, 4 }  c. { -22, -15, -8, -1, 6, 13, 20 }
2. a. $\{ 5x : x \in \mathbb{Z} \}$  b. $\{ x^2 : x \in \mathbb{N} \}$  c. $\{ 2^x : x \in \mathbb{Z} \}$  d. $\{ 3 \cdot 2^x : x \in \mathbb{Z} \}$
3. a. 4  b. 5
4. a. T - the only element of the set on the left, $\emptyset$, is also an element of the set on the right.
   b. F - the set "$\{\emptyset\}$" does not appear inside the set on the right.
   c. T - the set is a subset of $\mathbb{N}$, so it is an element of $P(\mathbb{N})$
   d. T - $\emptyset$ is a subset of D, so it is an element of $P(D)$
   e. F - $(3, 7)$is an element of $E \times D$, NOT of $D \times E$
   f. F - We check to see if the set on the left is a subset of $D \times E$. For this to be true, each member should be an ordered pair with the first element from D and second element from E. $(7, 5)$ fits this description, but $(\pi, \emptyset)$ does not.
5. a. $\{a, b, c, e\}$, the cardinality is 4  b. $\{c\}$, 1
   c. $\{\{a, b, c\}, \{a, b\}, \{b, c\}, \{a, c\}, \{a\}, \{b\}, \{c\}, \emptyset\}$, 8 d. $\{a, b, d\}$, 3
   e. $\{c, e\}$, 2  f. $\{(c, c), (c, e), (d, c), (d, e), (e, c), (e, e)\}$, 6  g. $\{f, g\}$, 2
   h. $\{ \{(c, c), (c, e,), (e, c), (e, e,) \}, \{(c, c), (c, e,), (e, c) \}, \{(c, c), (c, e,), (e, e,)\}, \{(c, c), (e, c), (e, e,) \}$
   $\{ (c, e,), (e, c), (e, e,) \}, \{ (c, c), (c, e,)\}, \{ (c, c), (e, c)\}, \{ (c, e,), (e, c)\}, \{ (c, c), (e, e,) \},$
   $\{(c, e,), (e, e,) \}, \{(e, c), (e, e,) \}, \{(c, c)\}, \{(c, e,)\}, \{(e, c)\}, \{(e, e,) \}, \emptyset \}$, 16
   i. $\{(d, c), (d, e)\}$, 2
6. a. $[1, 6)$  b. $[-3, 1) \cup [6, 7]$  c. $(5, 6)$



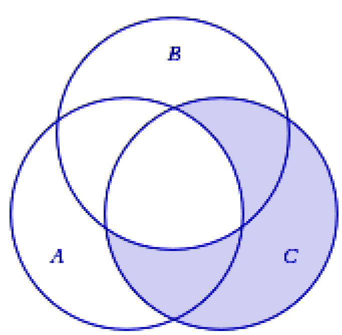
d.



e.




7. a.




b.

   c. There are many possible solutions, including$(A \cap B) \cup (A \cap C) \cup (B \cap C)$
   d. There are many possible solutions,
   including $((A \cup B) - C)) \cup (C - (A \cup B))$, and $(A \cup B \cup C) - ((B \cap C) \cup (A \cap C))$

8. a. $\bigcup_{i=1}^{\infty} A_i = \{..., -8, -6, -4, -2, 1, 2, 3, 4, ...\}$ and $\bigcap_{i=1}^{\infty} A_i = \emptyset$

b. $\bigcup_{i=1}^{\infty} A_i = (0, \infty)$ and $\bigcap_{i=1}^{\infty} A_i = \{1\}$

c. $\bigcup_{\alpha\in[3,6]} A_\alpha = [3, 6] \times [3, 4]$ and $\bigcap_{\alpha\in[3,6]} A_i = \{4\} \times [3, 4]$

9. a. A statement, False
b. Not a statement, an open sentence
c. Not a statement, not an open sentence
d. A statement, False
e. Not a statement, not an open sentence

10. a.

| P | Q | $Q \wedge\sim P$ | $\sim(Q \wedge\sim P)$ | $P \Rightarrow \sim(Q \wedge\sim P)$ |
|---|---|---|---|---|
| T | T | F | T | T |
| T | F | F | T | T |
| F | T | T | F | T |
| F | F | F | T | T |

b.

| P | Q | R | $P \wedge Q$ | $P \wedge R$ | $Q \wedge R$ | $(P \wedge R) \vee (Q \wedge R)$ | $(P \wedge Q) \Leftrightarrow (P \wedge R) \vee (Q \wedge R)$ |
|---|---|---|---|---|---|---|---|
| T | T | T | T | T | T | T | T |
| T | T | F | T | F | F | F | F |
| T | F | T | F | T | F | T | F |
| T | F | F | F | F | F | F | T |
| F | T | T | F | F | T | T | F |
| F | T | F | F | F | F | F | T |
| F | F | T | F | F | F | F | T |
| F | F | F | F | F | F | F | T |

11. a. $(R \wedge\sim Q) \Rightarrow P$ b.

| P | Q | R | $R \wedge\sim Q$ | $(R \wedge\sim Q) \Rightarrow P$ |
|---|---|---|---|---|
| T | T | T | F | T |
| T | T | F | F | T |
| T | F | T | T | T |
| T | F | F | F | T |
| F | T | T | F | T |
| F | T | F | F | T |
| F | F | T | T | F |

| | | | | |
|---|---|---|---|---|
| F | F | F | F | T |

12. a. Yes, they are logically equivalent.

| P | Q | $P \Rightarrow Q$ | $\sim P \vee Q$ |
|---|---|---|---|
| T | T | T | T |
| T | F | F | F |
| F | T | T | T |
| F | F | T | T |

b. Yes, they are logically equivalent

| P | Q | R | $P \wedge Q$ | $(P \wedge Q) \vee \sim R$ | $(R \Rightarrow P)$ | $(R \Rightarrow Q)$ | $(R \Rightarrow P) \wedge (R \Rightarrow Q)$ |
|---|---|---|---|---|---|---|---|
| T | T | T | T | T | T | T | T |
| T | T | F | T | T | T | T | T |
| T | F | T | F | F | T | F | F |
| T | F | F | F | T | T | T | T |
| F | T | T | F | F | F | T | F |
| F | T | F | F | T | T | T | T |
| F | F | T | F | F | F | F | F |
| F | F | F | F | T | T | T | T |

13. a. If a frog is not alive, then it can't jump.
    b. If x is prime, then it is not a two-digit even number.
14. a. $\forall x \in \mathbb{Z},\ O(4x + 7)$ b. $\exists x \in \mathbb{R}\, \forall y \in \mathbb{R}\, (y > x \Rightarrow y \in \mathbb{R}^{+})$
    c. $\sim \exists y \in \mathbb{R}\, \forall x \in \mathbb{R}\, (x < y \Rightarrow x \in \mathbb{R}^{-})$ d. $\exists z \in \mathbb{R}\, \forall y \in \mathbb{R}^{+}\ y > z$
    e. $\forall f \in F,\ (f \in G \Rightarrow f\,' \in G)$ f. $f, g \in G \Rightarrow (f + g) \in G$
15. a. For every real number *x*, if *x* is a natural number then x is an integer. TRUE.
    b. For all integers *m* and *n*, if *m* is even and *n* is odd, then *mn* is odd. FALSE.
    c. For every function *f*, if *f* is not a polynomial then *f* is not equal to $2x^2 + 6x + 1$. TRUE.
16. a. *a* is negative or *b* is negative.
    b. There is an integer that is neither prime nor composite.
    c. $\exists x \in \mathbb{R}\, \forall y \in \mathbb{R},\ y^2 \neq x$. There is a real number *x* such that for all real numbers *y*, we have $y^2 \neq x$.
    d. $(p\ is\ even) \wedge (q\ is\ even)$. Both p and q are even.