1. Nov 2025 paper 11 question 2

A geometric progression has first term a and common ratio $\cos\theta$, where $0 < \theta < \frac{1}{2}\pi$. It is given that the second term is 8 and the fifth term is $\frac{1}{8}$.

(a) Find the value of θ. Give your answer correct to 3 significant figures. [3]

(b) Find the exact value of the sum to infinity. [2]

2. Nov 2025 paper 11 question 3

In the expansion of

$$(px+3)^5-\left(x^3+\frac{p}{x}\right)^4,$$

the coefficient of x^4 is 216.

Find the value of the positive constant p. [5]

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3. Nov 2025 paper 11 question 9

An arithmetic progression has first term 2 and common difference d. The sum of the first n terms is denoted by S_n.

(a) It is given that $(S_2 - 1)$, S_4, S_9 are the first three terms of a second arithmetic progression.

Find the value of d. [4]

(b) Hence find the difference between the values of the 15th terms of the two arithmetic progressions. [4]

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4. Nov 2025 paper 12 question 2

Find the term independent of x in the expansion of $\left(2x^2 - \frac{3}{x}\right)^6$. [3]

5. Nov 2025 paper 12 question 8

The first three terms of a geometric progression are a, b and c respectively, where a, b and c are positive constants. The first three terms of an arithmetic progression are a, b and $-3c$ respectively.

(a) Show that $a^2 - 10ac + 9c^2 = 0$. [3]

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It is now given that $a = 9$ and c takes the smaller of its two possible values.

(b) **(i)** Find the sum to infinity of the geometric progression. [5]

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(ii) Find the sum of the first 20 terms of the arithmetic progression. [3]

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6. Nov 2025 paper 13 question 1

(a) Expand $\left(2-\frac{1}{2}x\right)^6$ in ascending powers of x up to and including the term in x^3. [3]

(b) Hence find the coefficient of x^3 in the expansion of $\left(3-x+2x^3\right)\left(2-\frac{1}{2}x\right)^6$. [2]

7. Nov 2025 paper 13 question 4

The first, second and third terms of a progression are 20, k and $k-5$ respectively.

(a) Given that the progression is arithmetic, find the 30th term. [2]

(b) Given instead that the progression is geometric, find the sum to infinity. [4]

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8. Nov 2025 paper 15 question 2

A geometric progression has first term $3+4\sqrt{2}$ and second term $5-\sqrt{2}$.

(a) Find the common ratio of the geometric progression. Give your answer in the form $\sqrt{2}+p$, where p is an integer to be found. [3]

(b) Find the sum to infinity of the geometric progression. [2]

9. Nov 2025 paper 15 question 9

In the expansion of $(p+qx)^4$, the coefficient of x is equal to the coefficient of x^2. The constants p and q are both positive.

(a) Find the ratio $p : q$. Give your answer in its simplest form. [3]

(b) It is given that the coefficient of x^3 is 486.

Find the values of p and q. [4]

10. May 2025 paper 11 question 3

The third term of a geometric progression is 18 and the sum of the first three terms is 26. It is given that the common ratio is negative.

(a) Find the tenth term of the progression. Give your answer correct to 3 significant figures. [5]

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(b) Find the exact value of the sum to infinity of the progression. [2]

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11. May 2025 paper 11 question 5

(a) Find the first three terms, in ascending powers of x, in the expansion of each of the following expressions.

(i) $(2-px)^5$ [2]

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(ii) $\left(1-\frac{1}{2}x\right)^4$ [2]

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(b) Given that the coefficient of x^2 in the expansion of $(2-px)^5\left(1-\frac{1}{2}x\right)^4$ is 93, find the possible values of the constant p. [3]

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12. May 2025 paper 12 question 3

The coefficient of x^7 in the expansion of $\left(px^2 + \frac{4}{p}x\right)^5$ is 1280.

Find the value of the constant p. [4]

13. May 2025 paper 12 question 10

(a) The first, second and third terms of an arithmetic progression are $4k$, k^2 and $8k$ respectively, where k is a non-zero constant.

(i) Find the value of k. [2]

(ii) Find the sum of the first 20 terms of the progression. [3]

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(b) The fourth and sixth terms of a geometric progression are 36 and 6 respectively. The common ratio of the progression is positive.

Find the sum to infinity of the progression. Give your answer in the form $\frac{a}{\sqrt{b}-c}$, where a, b and c are integers. [5]

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14. May 2025 paper 13 question 2

The first two terms of a geometric progression are

$$4\sin^2\theta, \quad 8\sin^3\theta,$$

where θ is an angle such that $0 < \theta < \frac{1}{6}\pi$.

Given that the sum to infinity of the progression is $\frac{1}{2}$, find the value of θ. Give your answer in the form $\sin^{-1} k$, where k is a rational number. [4]

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15. May 2025 paper 13 question 4

(a) Find the first three terms in the expansion of $\left(2-\frac{3}{2}x\right)^5$ in ascending powers of x. [3]

(b) Use your answer to part **(a)**, with a suitable value of x, to find an approximation to 1.985^5. [3]

16. May 2025 paper 13 question 6

An arithmetic progression has first term a and common difference 2. The Nth term is 55 and the sum of the first $3N$ terms is 5760.

Find the values of N and a. [6]

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17. May 2025 paper 15 question 2

In the expansion of $(3+ax)^5+(6-x)^4$, the coefficient of x^2 is six times the coefficient of x.

Find the possible values of the constant a. [5]

18. May 2025 paper 15 question 6

Each year, on her birthday, Ananya receives some money from each of her parents.

On Ananya's first birthday, her father gives her \$10. Every subsequent year, her father gives her \$5 more than he gave her the previous year.

On Ananya's first birthday, her mother also gives her \$10. Every subsequent year, her mother gives her 20% more than she gave her the previous year.

(a) Show that on Ananya's eleventh birthday she receives more from her mother than from her father. [3]

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(b) Find the total amount of money Ananya receives up to and including her eighteenth birthday. [5]

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19. March 2025 paper 12 question 3

(a) Find the complete expansion of $\left(2x-\frac{3}{x}\right)^4$. [4]

(b) Hence determine the coefficient of x^2 in the expansion of $\left(x^2+5\right)\left(2x-\frac{3}{x}\right)^4$. [2]

20. March 2025 paper 12 question 5

An arithmetic progression has first term 5 and common difference 6.

For this progression, find the sum of all the terms that lie between 150 and 400. [6]

21. March 2025 paper 12 question 8

A geometric progression is such that its second term is -120 and its sum to infinity is 160.

(a) Find the common ratio. [4]

(b) The first nine terms of the progression are now removed.

Find the sum to infinity of the remaining terms of the progression. [3]

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22. Nov 2024 paper 11 question 1

In the expansion of $\left(kx+\frac{2}{x}\right)^4$, where k is a positive constant, the term independent of x is equal to 150.

Find the value of k and hence determine the coefficient of x^2 in the expansion. [4]

23. Nov 2024 paper 11 question 10

An arithmetic progression has first term 5 and common difference d, where $d > 0$. The second, fifth and eleventh terms of the arithmetic progression, in that order, are the first three terms of a geometric progression.

(a) Find the value of d. [3]

(b) The sum of the first 77 terms of the arithmetic progression is denoted by S_{77}. The sum of the first 10 terms of the geometric progression is denoted by G_{10}.

Find the value of $S_{77} - G_{10}$. [5]

24. Nov 2024 paper 12 question 2

The first term of an arithmetic progression is -20 and the common difference is 5.

(a) Find the sum of the first 20 terms of the progression. [2]

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It is given that the sum of the first $2k$ terms is 10 times the sum of the first k terms.

(b) Find the value of k. [3]

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25. Nov 2024 paper 12 question 4

Find the term independent of x in the expansion of each of the following:

(a) $\left(x+\frac{3}{x^2}\right)^6$ [2]

(b) $(4x^3-5)\left(x+\frac{3}{x^2}\right)^6$. [4]

26. Nov 2024 paper 13 question 1

An arithmetic progression has fourth term 15 and eighth term 25.

Find the 30th term of the progression. [3]

27. Nov 2024 paper 13 question 3

(a) Find the coefficients of x^3 and x^4 in the expansion of $(3-ax)^5$, where a is a constant. Give your answers in terms of a. [3]

(b) Given that the coefficient of x^4 in the expansion of $(ax+7)(3-ax)^5$ is 240, find the positive value of a. [3]

28. Nov 2024 paper 13 question 6

The first term of a convergent geometric progression is 10. The sum of the first 4 terms of the progression is p and the sum of the first 8 terms of the progression is q. It is given that $\frac{q}{p} = \frac{17}{16}$.

Find the two possible values of the sum to infinity. [5]

29. March 2024 paper 12 question 6

It is given that the coefficient of x^3 in the expansion of

$$(2+ax)^4(5-ax)$$

is 432.

Find the value of the constant a. [5]

30. March 2024 paper 12 question 8

(a) An arithmetic progression is such that its first term is 6 and its tenth term is 19.5 .

Find the sum of the first 100 terms of this arithmetic progression. [4]

(b) A geometric progression $a_1, a_2, a_3, \ldots$ is such that $a_1 = 24$ and the common ratio is $\frac{1}{2}$.

The sum to infinity of this geometric progression is denoted by S. The sum to infinity of the even-numbered terms (i.e. $a_2, a_4, a_6, \ldots$) is denoted by S_E.

Find the values of S and S_E. [4]

31. May 2024 paper 11 question 3

The coefficient of x^3 in the expansion of $(3+ax)^6$ is 160.

(a) Find the value of the constant a. [2]

(b) Hence find the coefficient of x^3 in the expansion of $(3+ax)^6(1-2x)$. [3]

32. May 2024 paper 11 question 8

(a) The first three terms of an arithmetic progression are 25, $4p-1$ and $13-p$, where p is a constant.

Find the value of the tenth term of the progression. [4]

(b) The first three terms of a geometric progression are $25, 4q-1$ and $13-q$, where q is a positive constant.

Find the sum to infinity of the progression. [4]

33. May 2024 paper 12 question 1

The coefficient of x^2 in the expansion of $(1-4x)^6$ is 12 times the coefficient of x^2 in the expansion of $(2+ax)^5$.

Find the value of the positive constant a. [3]

34. May 2024 paper 12 question 6

The first and second terms of an arithmetic progression are $\tan\theta$ and $\sin\theta$ respectively, where $0 < \theta < \frac{1}{2}\pi$.

(a) Given that $\theta = \frac{1}{4}\pi$, find the exact sum of the first 40 terms of the progression. [4]

The first and second terms of a geometric progression are $\tan\theta$ and $\sin\theta$ respectively, where $0 < \theta < \frac{1}{2}\pi$.

(b) (i) Find the sum to infinity of the progression in terms of θ. [2]

(ii) Given that $\theta = \frac{1}{3}\pi$, find the sum of the first 10 terms of the progression. Give your answer correct to 3 significant figures. [3]

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35. May 2024 paper 13 question 1

Find the coefficient of x^2 in the expansion of

$$(2-5x)(1+3x)^{10}.$$ [4]

36. May 2024 paper 13 question 6

The first term of an arithmetic progression is 1.5 and the sum of the first ten terms is 127.5 .

(a) Find the common difference. [2]

(b) Find the sum of all the terms of the arithmetic progression whose values are between 25 and 100. [5]

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37. May 2024 paper 13 question 10

The geometric progression $a_1, a_2, a_3, \ldots$ has first term 2 and common ratio r where $r > 0$.
It is given that $\frac{9}{2}a_5 + 7a_3 = 8$.

(a) Find the value of r. [3]

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(b) Find the sum of the first 20 terms of the geometric progression. Give your answer correct to 4 significant figures. [2]

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(c) Find the sum to infinity of the progression $a_2, a_5, a_8, \ldots$. [3]

38. Nov 2023 paper 11 question 1

(a) Expand $(1 + 3x)^6$ in ascending powers of x up to, and including, the term in x^2. [2]

(b) Hence find the coefficient of x^2 in the expansion of $(1 - 7x + x^2)(1 + 3x)^6$. [2]

39. Nov 2023 paper 11 question 7

The sum of the first two terms of a geometric progression is 15 and the sum to infinity is $\frac{125}{7}$. The common ratio of the progression is negative.

Find the third term of the progression. [7]

40. Nov 2023 paper 12 question 1

The coefficient of x^3 in the expansion of $(3 + 2ax)^5$ is six times the coefficient of x^2 in the expansion of $(2 + ax)^6$.

Find the value of the constant a. [4]

41. Nov 2023 paper 12 question 5

The first, second and third terms of a geometric progression are $\sin\theta$, $\cos\theta$ and $2 - \sin\theta$ respectively, where θ radians is an acute angle.

(a) Find the value of θ. [3]

(b) Using this value of θ, find the sum of the first 10 terms of the progression. Give the answer in the form $\frac{b}{\sqrt{c}-1}$, where b and c are integers to be found. [3]

42. Nov 2023 paper 13 question 4

(a) Expand the following in ascending powers of x up to and including the term in x^2.

(i) $(1 + 2x)^5$. [1]

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(ii) $(1 - ax)^6$, where a is a constant. [2]

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In the expansion of $(1 + 2x)^5(1 - ax)^6$, the coefficient of x^2 is -5.

(b) Find the possible values of a. [4]

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43. Nov 2023 paper 13 question 5

The first, second and third terms of a geometric progression are $2p + 6$, $5p$ and $8p + 2$ respectively.

(a) Find the possible values of the constant p. [3]

(b) One of the values of p found in **(a)** is a negative fraction.

Use this value of p to find the sum to infinity of this progression. [4]

44. March 2023 paper 12 question 4

The circumference round the trunk of a large tree is measured and found to be 5.00 m. After one year the circumference is measured again and found to be 5.02 m.

(a) Given that the circumferences at yearly intervals form an arithmetic progression, find the circumference 20 years after the first measurement. [2]

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(b) Given instead that the circumferences at yearly intervals form a geometric progression, find the circumference 20 years after the first measurement. [3]

45. March 2023 paper 12 question 6

In the expansion of $\left(\frac{x}{a}+\frac{a}{x^2}\right)^7$, it is given that

$$\frac{\text{the coefficient of } x^4}{\text{the coefficient of } x} = 3.$$

Find the possible values of the constant a. [6]

46. May 2023 paper 11 question 2

(a) Find the first three terms in the expansion, in ascending powers of x, of $(2 + 3x)^4$. [2]

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(b) Find the first three terms in the expansion, in ascending powers of x, of $(1 - 2x)^5$. [2]

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(c) Hence find the coefficient of x^2 in the expansion of $(2 + 3x)^4(1 - 2x)^5$. [2]

47. May 2023 paper 11 question 6

The first three terms of an arithmetic progression are $\frac{p^2}{6}$, $2p - 6$ and p.

(a) Given that the common difference of the progression is not zero, find the value of p. [3]

(b) Using this value, find the sum to infinity of the geometric progression with first two terms $\frac{p^2}{6}$ and $2p - 6$. [2]

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48. May 2023 paper 12 question 2

The coefficient of x^4 in the expansion of $(x + a)^6$ is p and the coefficient of x^2 in the expansion of $(ax + 3)^4$ is q. It is given that $p + q = 276$.

Find the possible values of the constant a. [4]

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49. May 2023 paper 12 question 9

The second term of a geometric progression is 16 and the sum to infinity is 100.

(a) Find the two possible values of the first term. [4]

(b) Show that the nth term of one of the two possible geometric progressions is equal to 4^{n-2} multiplied by the nth term of the other geometric progression. [4]

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50. May 2023 paper 12 question 3

(a) Give the complete expansion of $\left(x+\frac{2}{x}\right)^5$. [2]

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(b) In the expansion of $(a+bx^2)\left(x+\frac{2}{x}\right)^5$, the coefficient of x is zero and the coefficient of $\frac{1}{x}$ is 80.

Find the values of the constants a and b. [4]

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51. May 2023 paper 13 question 8

A progression has first term a and second term $\frac{a^2}{a+2}$, where a is a positive constant.

(a) For the case where the progression is geometric and the sum to infinity is 264, find the value of a. [5]

(b) For the case where the progression is arithmetic and $a = 6$, determine the least value of n required for the sum of the first n terms to be less than -480. [5]

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52. Nov 2022 paper 11 question 4

The coefficient of x^2 in the expansion of $\left(1+\frac{2}{p}x\right)^5+(1+px)^6$ is 70.

Find the possible values of the constant p. [6]

53. Nov 2022 paper 11 question 7

A tool for putting fence posts into the ground is called a 'post-rammer'. The distances in millimetres that the post sinks into the ground on each impact of the post-rammer follow a geometric progression. The first three impacts cause the post to sink into the ground by 50 mm, 40 mm and 32 mm respectively.

(a) Verify that the 9th impact is the first in which the post sinks less than 10 mm into the ground. [3]

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(b) Find, to the nearest millimetre, the total depth of the post in the ground after 20 impacts. [2]

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(c) Find the greatest total depth in the ground which could theoretically be achieved. [2]

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