BS2250 Intermediate Microeconomics

a) Calculate the optimal consumption bundle and the utility it provides

$U = X^{3/5}Y^{2/5}$

Marginal utility of X, $MU_X = \frac{3}{5} Y^{2/5}X^{-2/5}$

Marginal utility of Y, $MU_Y = \frac{2}{5} Y^{-3/5}X^{3/5}$

MRS = MU_X/MU_Y

MRS = $\frac{3Y}{2X}$

MRS = $\frac{P_X}{P_Y}$

Equating the MRS expressions

$\frac{3Y}{2X} = \frac{25}{20}$

50X = 60Y

X = 6/5Y

Substituting X into the budget constraint; 1,000 = 25X + 20Y

1,000 = 25*6/5Y + 20Y

Y* =20

X* = 24 (6/5*20)

Utility at the optimal consumption bundle:

U = $20^{2/5}24^{3/5}$

U = 22.31

b) Calculate the new optimal consumption bundle and the utility it provides

MRS = $MU_X/MU_Y = \frac{P_X}{P_Y}$

$\frac{3Y}{2X} = \frac{50}{20}$

100X = 60Y

X = 0.6Y

Substituting X into the new budget constraint; 1,000 = 50X + 20Y

1,000 = 30Y + 20Y

Y = 20

X = 12 (0.6 *20)

Utility at the new optimal consumption bundle:

$U = 20^{2/5}12^{3/5}$

U = 14.72

c) Calculate the compensation variation and the equivalent variation of the price increase of good X from £25 to £50

$$U = Y^{\frac{2}{5}} * (\frac{3M}{5Px})^{\frac{3}{5}}$$

$$U = (\frac{3M}{5Px})^{\frac{3}{5}} * (\frac{2M}{5Py})^{\frac{2}{5}}$$

Expenditure function, e (Px, Py, U) = $\frac{3{,}125Px^3Py^2U^5}{108}^{1/5}$

e_0 (Px0, u0) = (3,125*15,625*400*5,529,600/108) ^1/5 = 1,000
e_1 (Px1, u0) = (3,125*125,000*400*5,529,600/108) ^1/5 = 1,515.717
CV = $e_1 - e_0$ = 515.717
e_2 (Px0, u1) = (3,125*15,625*400*691,200/108) ^1/5 = 659.754
EV = $e_2 - e_0$ = -340.246
d) Calculate the substitution and income effects on good X of the increase in PX from £25 to £50 using both compensating and equivalent variation

$$X^* = \overline{U} * \left(\frac{\alpha}{\beta} * \frac{Py}{Px}\right)^{\beta}$$

Substitution effect (SE) using CV:

$$CV = \int_{px0}^{px1} X(U_0, Px, Py)dPx$$

$$CV = \int_{px0}^{px1} U_0(\frac{\alpha}{\beta}\frac{Py}{P_x})^{\beta} dP_x$$

$$CV = \frac{X(U_0,Px_1,Py) - X(U_0,Px_0,P_y)}{1-\beta}(Px_1 - Px_0)$$

$$SE = \frac{CV}{(Px_1 - Px_0)}(1 - \beta)$$

$$SE = \frac{515.717}{25} * 0.6$$

SE = 12.377

Income effect using EV:

$$EV = \int_{px1}^{px0} X(U_1, Px, Py)dPx$$

$$EV = \int_{px1}^{px0} U_1\left(\frac{\alpha}{\beta}\frac{Py}{P_x}\right)^{\beta} dP_x$$

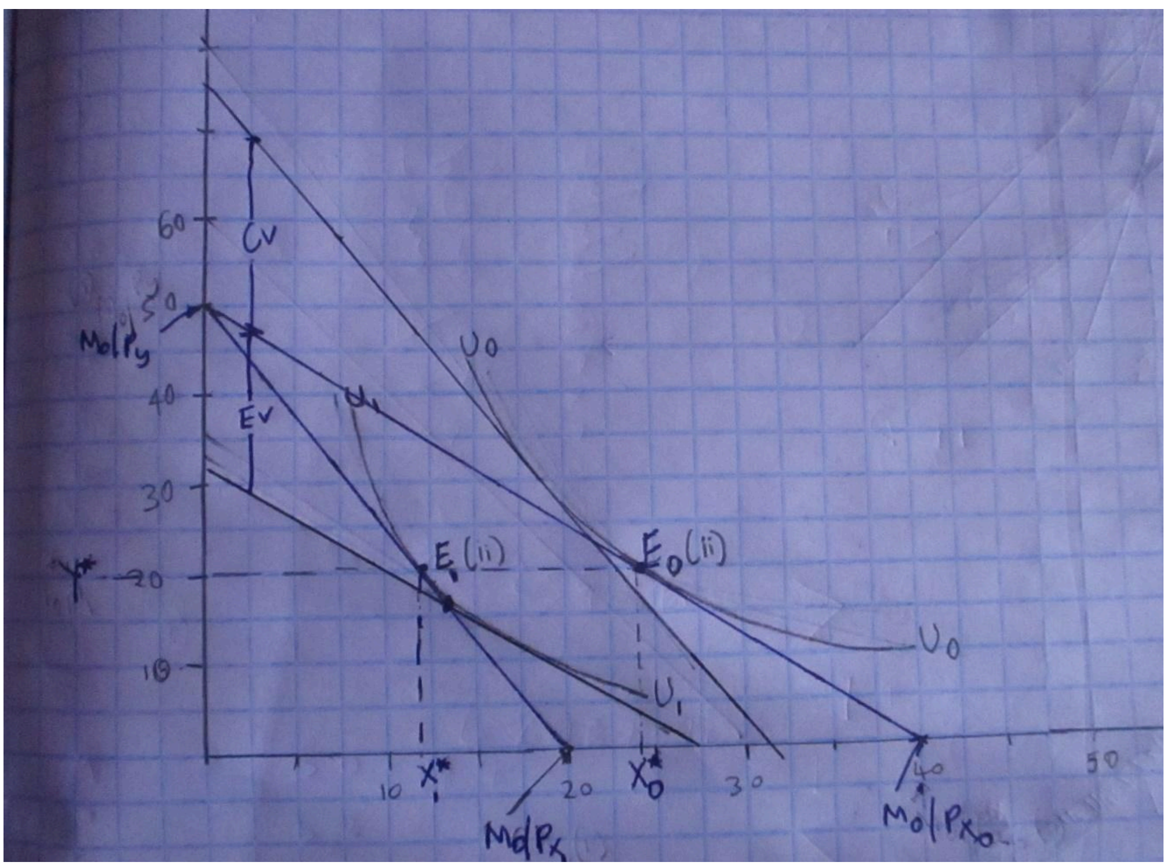
$$X(U_0, Px_0, Py) - X(U_0, Px_1, P_y) = \frac{EV}{(Px_0 - Px_1)}(1 - \beta)$$

$$IE = \frac{EV}{(Px_0 - Px_1)}(1 - \beta)$$

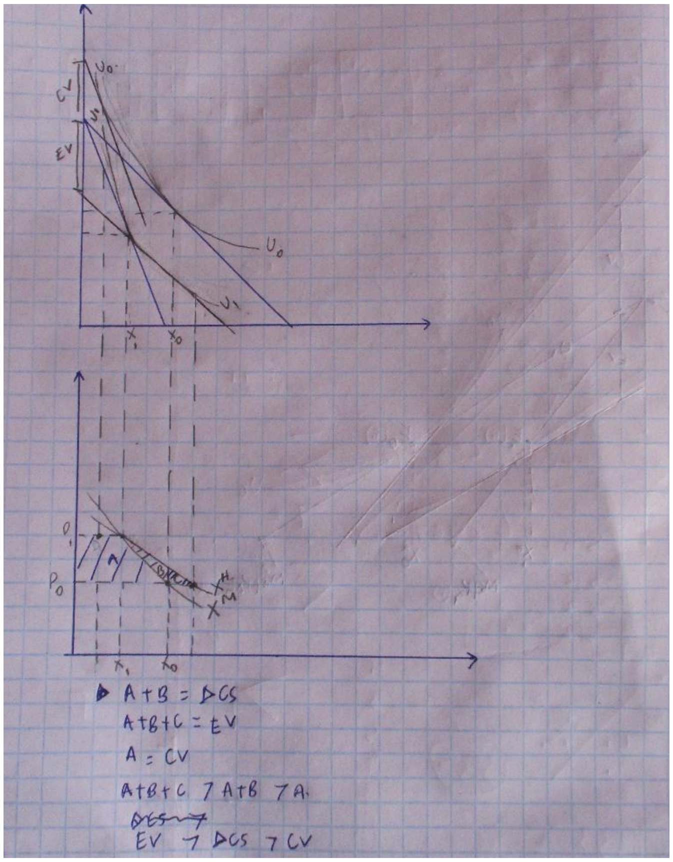
$$IE = \frac{-340.246}{(-25)}(0.6)$$

IE = 1.866

d) Graph of price increase of good X from 25 to 50



2. Using an indifference curve/budget constraint diagram, illustrate the compensating and equivalent variation of a price increase of an inferior good.



3. Under what circumstances would the three different measures of welfare (CV, EV and the ΔCS) provide similar monetary values?

The Slutsky equation is given as:

$$\frac{\partial X^M}{\partial Px} = \frac{\partial X^H}{\partial Px} - X\frac{\partial X}{\partial M}$$

The equation decomposes the effect of price on demand into two: first is the compensation to the consumer to keep their utility constant and the second is the income effect which has the equivalent effect of reducing the consumer's income to the level of utility that would exist from the price increase. The compensation variation is given by the Hicksian derivative while the change in consumer surplus is given by the Marshallian derivative. The equivalent variation is given by the product of quantity demanded and the income effect. As the income effect approaches zero, change in consumer surplus, compensating variation, and equivalent variation all approach equality. In the absence of an income effect (dX/dM = 0), the three measures will be equal.