School of Interwoven Arts and Scieces (SIAS)
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Krea University Mathematics PhD Entrance Exam and Interview Syllabus

0. Foundations in Mathematics

Syllabus:

1. **Vocabulary and grammar of mathematics and basic properties of sets and numbers**: Mathematical statements, AND, OR, negation, contrapositive, converse, quantifiers. Unions, intersections, complements, products. Ordered pairs, cartesian products.
2. **Functions**: Functions: definitions, properties (injective/surjective/bijective etc).
3. **Equivalence relations:** Relations, equivalence relations (definitions and properties), equivalence classes, partitions, and transversals. Introduction to cardinality. Examples including congruence mod n.
4. **Cardinality:** Definition of countable and uncountable sets. Countability of Q, uncountability of power sets of infinite sets. Counting Injections/surjections/bijections between finite sets.
5. **Partial orders:** Partial orders: definition and examples (including orders on number systems, divisibility and subset containment). Well-ordering principle in N.
6. **Order in the Reals:** Upper and lower bounds, least upper and greatest lower bounds, LUB axiom, Archimedean principle, Density of Rationals.
7. **Topics in cardinality:** Schroeder-Bernstein theorem, Zorn's lemma, comparability of cardinals.
8. **Basic Methods in Combinatorics:** The Pigeon-Hole Principle and its Generalization, The Method of Mathematical Induction, Weak Induction, Strong Induction, Induction on multiple arguments

9. **Enumerative Combinatorics:** Counting problems, Permutations, Combinations, Strings over Finite Alphabet, Choice Problems, Binomial Theorem, Multinomial Theorem, Theory of Partitions, Compositions, Set Partitions, Integer Partitions, Cycles in Permutations, Permutations with Restricted Cycle Structure, The Principle of Inclusion and Exclusion and its Applications, Generating Functions, Ordinary and Exponential Generating Functions, Recurrence Relations, Pattern avoidance in Permutations.
10. **Graph Theory:** The notion of graphs, Eulerian and Hamiltonian paths and Cycles, Directed Graphs, Isomorphisms, Multiply Connected Graphs, Spanning Trees, Graphs and adjacency matrices, Bipartite Graphs, Matching Theory, Planar Graphs, Euler's theorem, Graph Colourings, Chromatic Polynomial.
11. **Partially Ordered Sets:** Posets, Lattices, Mobius Function of a Poset, Mobius Inversion Formula.
12. **Divisibility Theory in the Integers:** Divisibility, The Division Algorithm, The Greatest Common Divisor, The Euclidean Algorithm, The Fundamental Theorem of Arithmetic, Linear Diophantine Equations
13. **Primes and Their Distribution:** Prime Numbers, The Infinitude of Primes, Primes of Special Form, Perfect Numbers and Mersenne Primes, The Distribution of Primes, Factorization Methods
14. **Congruences:** Congruence Modulo n, Properties of Congruences, Linear Congruences, The Chinese Remainder Theorem, Systems of Linear Congruences
15. **Fermat's and Euler's Theorems:** Fermat's Little Theorem, Pseudoprimes and Carmichael Numbers, Euler's Theorem, Applications of Fermat–Euler Results
16. **Number-Theoretic Functions:** Arithmetic Functions, The Divisor Counting Tau Function and The Sum of Divisors Sigma Function, Multiplicative Functions, The Möbius Function, The Euler Phi Function, Properties of $\varphi(n)$
17. **Primitive Roots:** The Order of an Integer Modulo n, Primitive Roots, Existence of Primitive Roots, Indices and Discrete Logarithms
18. **Quadratic Residues:** Quadratic Residues and Nonresidues, The Legendre Symbol, Euler's Criterion, Properties of the Legendre Symbol, Quadratic Reciprocity, Supplementary Laws
19. **Quadratic Forms:** Binary Quadratic Forms, Representation of Integers, Reduction of Quadratic Forms
20. **Diophantine Equations:** Linear Diophantine Equations, Pell's Equation, Methods of Solution, Applications
21. **Continued Fractions:** Simple Continued Fractions, Convergents, Best Approximations, Continued Fractions and Pell's Equation

Recommended books:

1. **A Foundation Course in Mathematics by A. Kumar, B.K. Sharma, and S. Kumaresan**
2. **Naive Set Theory by P.R. Halmos**
3. **Proofs and Ideas: A Prelude to Advanced Mathematics by B.A. Sethuraman**
4. **Discrete Mathematics and its Applications by K.H. Rosen**
5. **A Walk Through Combinatorics by B. Miklos**
6. **Combinatorics Theory and Applications by V. Krishnamurthy**
7. **Combinatorics Schaum's Outlines by V.K. Balakrishnan**
8. **Elementary Number Theory by D.M. Burton**
9. **Elementary Number Theory with Applications by T. Koshy**

1. Real Analysis

Syllabus:

1. **The real numbers as an ordered field:** Number systems, ordered fields. Formal properties: the real numbers as a complete, Archimedean, ordered field. Construction (via Cauchy sequences OR Dedekind cuts, sketch). Absolute value and distance in the reals; triangular inequality. Supremum and infimum. Roots in the reals. Rational exponentiation. Introduction to complex numbers: norm, complex conjugate.
2. **Special subsets**: Open, closed, bounded sets. Properties. Intervals. Limit points; closure, interior, boundary; isolated points. Examples.
3. **Sequences**: Introduction, convergence, algebra of limits (sum, product, quotient). Complex sequences: reduction to real. Cesàro summation. Subsequences, limit points, lim-sup and lim-inf, Bolzano-Weierstrass theorem. Monotonic sequences; monotonic subsequence theorem. Cauchy sequences and completeness. Real exponentiation. Examples.
4. **Compact subsets of R:** Compact subsets. Heine-Borel theorem.
5. **Infinite series:** Introduction and definition. Complex series: reduction to real. Cauchy criterion. Absolute and conditional convergence, Convergence tests (comparison, root, ratio, Cauchy condensation, alternating), summations and rearrangements, Cauchy products. Riemann rearrangement theorem. Examples. Infinite Series and Infinite Products, Abel’s limit theorem, Tauber’s theorem.
6. **Cardinality of the real numbers:** Recap of cardinality. Decimal expansions. Uncountability of the real numbers.
7. **Functions of a real variable:** Definition. Examples, including polynomials, trigonometric functions, exponentials, logs etc.
8. **Limits:** Definition of a limit (one-sided, two-sided, at +-infinity; real-valued, +- infinite). Algebra of limits (sum, product, quotient). Sequential characterisation of limits. Examples.
9. **Continuity:** Definition of continuity at a point and on a subset. Algebra of continuous functions (sum, product, quotient, composition). Sequential characterisation of continuity. Theorems: Intermediate value theorem; every function on a closed and bounded domain is bounded and attains its bounds, Uniform continuity, proof that every function on a closed and bounded domain is uniformly continuous. Examples
10. **Differentiability:** Definition of differentiability, basic properties. Algebra of differentiable functions (sum, product, quotient). Chain rule. Rolle’s theorem; generalised mean-value theorem. L'Hospital's rule. Higher-order derivatives. Maxima and minima. Taylor series. Examples.
11. **Riemann integral**: Partitions: definition and refinements. Definition of upper and lower integrals for a bounded function defined on a bounded subset; definition of Riemann integral. Integrability of monotonic and (piece-wise) continuous functions. Algebra of Riemann integrable functions (sum, product, quotient). Relationship between integration and differentiation; fundamental theorem of calculus. Integration by parts, integration by substitution. Indefinite integrals. Functions of bounded variation. Rectifiable curves. Absolute Continuity, Riemann Stieltjes Integral, Integration over infinite domains. Maclaurin–Cauchy convergence test for an infinite series. Examples

12. **Convex functions:** Definition of convex functions. Continuity of convex functions on open sets. Characterisation in terms of derivatives. Jenson's inequality. Applications: arithmetic-geometric inequality.
13. **Spaces of functions:** Sequence of functions. Uniform vs pointwise convergence. Sup norms. Uniform convergence and continuity, differentiability, and integrability. Infinite series of functions. Weierstrass M-test. Weierstrass approximation theorem.
14. **Power series:** Definition. Radius of convergence. Abel's theorem, multiplication of power series. Exponentials and logarithms. Trigonometric functions.
15. **Metric spaces**: Definition. Examples: real and complex Euclidean spaces, spaces of sequences, spaces of functions. Various metrics on real Euclidean space, comparison. Discrete metrics, modification of metrics. Equivalent metrics. Product spaces.
16. **Special subsets:** Open, closed. Properties. Closure, interior. Boundary. Limit points. Definition of sequence convergence in metric spaces. Distance between subsets and points, subsets and subsets. Diameter of a subset. Denseness, definition of separable metric spaces.
17. **Continuous functions**: Continuous functions between metric spaces: e-d vs open set definition. Continuity at a point vs global continuity. Homeomorphisms. Properties of real/complex valued continuous functions. Uniform continuity, Lipschitz continuity, contraction mappings.
18. **Completeness:** Definition, examples. Cantor's intersection theorem. Banach fixed point theorem. Examples.
19. **Connectedness:** Definition, examples. Connectedness as a subspace vs connectedness as a subset. Connectedness of closure. Continuity and connectedness. Definition of path connectedness. Examples.
20. **Compactness:** Definition, examples, closed set formulation. Compactness and continuity. Compactness in Euclidean space. Equivalence of compactness, sequential compactness, completeness and total boundedness in metric spaces. Lebesgue numbers. Compactness and uniform continuity. Examples.
21. **Functions between Euclidean spaces:** Norm and distance on R^n. Open, closed, bounded, compact sets. Heine-Borel theorem. Continuous functions from R^n to R^m. Linear transformations between Euclidean spaces: norm of a linear operator, continuity of linear operators. Recall sequences and series of functions, compact subsets of continuous functions (Arzela Ascoli Theorem with applications).
22. **Differentiability I:** Definition of a differentiable function between R^m and R^n. Jacobians. Properties and uniqueness. Examples, including the derivative of a linear operator. Chain rule. Partial derivatives, representative of the derivative of a differentiable function by partial derivatives, directional derivative. Gradient of a function from R^{n} to R. Critical points. Examples.
23. **Differentiability II:** Continuously differentiable functions. Characterisation in terms of continuous partial derivatives. Higher-order partial derivatives. Clairaut's theorem. Hessians. Extrema and saddle points: criteria. Taylor expansions. Inverse function theorem, Implicit function theorem.
24. **Integration in 2 and 3 dimensions:** Jordan measure, integrability, properties of integrals, simple domains, Fubini's theorem, interchanging order of integration, change of variables(with proof), polar and spherical coordinates, improper integrals.

25. **Fourier Series and Fourier Integrals:** Fourier Series, Riesz Fischer Theorem, The convergence and representation problem of trigonometric series, Riemann Lebesgue Lemma, The Dirichlet Integrals, Integral Representation of partial sums of a Fourier Series, Dirichlet and Fejer Kernels, Riemann Localization Lemma, The Weierstrass Approximation Theorem, Fourier Transforms, Convolutions, The Poisson Summation Formula.

Recommended books:

1. **Principles of Mathematical Analysis by Walter Rudin**
2. **Mathematical Analysis by Tom M. Apostol**
3. **Real Analysis by H.L. Royden and P.M. Fitzpatrick**
4. **Analysis I by Terence Tao**
5. **Analysis II by Terence Tao**

2. Ordinary Differential Equations

Syllabus:

1. **Modelling with differential equations:** Introduction and formal definition of differential equations. Art of mathematical modelling with differential equations, taxonomy of differential equations (first-order ODE’s, first order systems of ODEs, linear differential equations, nonlinear differential equations).
2. **First order differential equations:** Qualitative Solutions, analytical solutions (method of separation of variables), Numerical techniques: Euler’s Method, Existence and Uniqueness of Solutions, Equilibria and the Phase Line, Bifurcations, Linear Equations, Integrating Factors for Linear Equations
3. **Second order equations:** solving second order constant coefficient ODE, solving inhomogeneous equations via variation of constants, Wronskians, Abel’s formula
4. **First order systems of differential equations:** Geometry of First-order systems, Damped Harmonic Oscillator, Additional Analytic Methods for Special Systems, Euler’s Method for Systems, Existence and Uniqueness of Solutions, popular models: SIR model for an epidemic, Lorenz equations.
5. **Linear systems:** Properties of Linear Systems and the Linearity Principle, Straight-Line Solutions, Phase Portraits for Linear Systems with Real Eigenvalues, Complex Eigenvalues, Second-Order Linear Equations, The Trace-Determinant Plane.
6. **Nonlinear systems:** Equilibrium Point Analysis, Qualitative Analysis, Hamiltonian Systems, Dissipative Systems.

Recommended books:

1. **Theory of Ordinary Differential Equations by E.E. Coddington and N. Levinson**
2. **Ordinary Differential Equations by V.I. Arnold**
3. **Ordinary Differential Equations by E.L. Ince**
4. **Differential Equations, Dynamical Systems, and Introduction to Chaos by M.W. Hirsch, S. Smale, and R.L. Devaney**
5. **Differential Equations by Shepley Ross**

3. Partial Differential Equations

Syllabus:

1. **Classification of PDEs** : PDEs may be classified according to order, linearity, homogeneity, whether they are one equation or a system of equations. What are the solutions? What are the initial value and boundary value problems? What is well-posedness?
2. **First Order PDEs** : Method of characteristics, Local Solutions, Global Solutions.
3. **Second Order PDEs** : Classification using characteristics, Cauchy Kowalewski Theorem.
4. **Wave Equations in one space dimension** : Derivation and Physical Interpretation, D'Alembert's formula, Causality, Domain of dependence and domain of influence, Duhamel Principle, Conservation of Energy, Well-posedness, Ill-posedness, Initial-Boundary value problems – Solving Wave Equation in a strip.
5. **Wave Equations in two and three space dimensions** : Kirchoff's formula, Causality, Huygens Principle, Method of descent, Causality in 2 Dimensions
6. **Laplace Equation** : Derivation and Physical Interpretation, Dirichlet and Neumann Problems, Well-posedness and ill-posedness, Relation to Brownian Motion, Mean Value Property, Maximum Principle, Dirichlet Principle, The notion of Distributions, Fundamental Solution, Green's function.
7. **Heat Equation** : Derivation and Physical Interpretation, Solution by Fourier transform, Heat kernel, Relation to Random Walks, Infinite Speed of Propagation, Regularity, Maximum Principle, Duhamel Principle, Counterexamples to uniqueness.
8. **The Method of Separation of Variables** : Heat and Wave Equation with Dirichlet and Neumann Boundary conditions, Laplace equation in rectangles and disks, etc.

Recommended books:

1. **Partial Differential Equations An Introduction by Walter Strauss**
2. **Partial Differential Equations by Fritz John**
3. **Partial Differential Equations by L.C. Evans**

4. Numerical Methods

Syllabus:

1. **Mathematical Preliminaries:** Recap of sequences, limits, continuity, theorems of calculus, orders of convergence (big O, little o)
2. **Error Analysis:** Floating point representation, types of errors, significant digits, propagation of relative errors
3. **Numerical Linear Algebra:** Gauss Elimination and its variants, LU Decomposition, Iterative Methods (Jacobi, Gauss-Siedel), Solving Eigenvalue Problems (Power method). Conjugate Gradient and Krylov Subspace Methods, QR factorization, Schur decomposition, SVD.
4. **Root-finding:** Secant Method, Regula Falsi, Bisection Method, Newton-Raphson Method, Fixed Point Methods, Pitfalls and Errors.

5. **Interpolation:** Interpolation by polynomials, Newton's interpolation, Lagrange's form of interpolation, divided differences, errors, piecewise polynomial interpolation, spline interpolation.
6. **Numerical Integration and Differentiation:** Rectangle Rule, Trapezoidal Rule, Simpson's Rules, Other methods, approximations of first derivatives by different methods, errors
7. **Numerical Solution of ODEs:** Recall of existence and uniqueness theorems in ODEs, Euler's method, RK2 method and their convergence analysis. RK4 method without convergence analysis
8. **Optimization:** Basic theorems from multivariable calculus, Gradient descent, Newton's method, Conjugate Gradient Method, Constrained Optimization

Recommended books:

1. **An Introduction to Numerical Analysis by Endre Süli and David F. Mayers**
2. **Classical Numerical Analysis A Comprehensive Course by A.J. Salgado and S.M. Wise**
3. **Numerical Analysis: A Graduate Course by D.E. Stewart**

5. Measure theory and integration

Syllabus:

1. **Measures on Sigma algebras:** measurable sets, measurable functions, measures, measure spaces, Lebesgue measure, Borel measures, outer measure, Carathéodory's extension theorem, Borel-Cantelli Lemma, Nonmeasurable Sets, The Cantor Set and Cantor-Lebesgue Function, Littlewood's Three Principles, Egoroff's Theorem, Lusin's Theorem
2. **Differentiation and Integration:** integration on a measurable space, integrals of measurable functions, Countable Additivity and Continuity of Integration, Uniform Integrability: The Vitali Convergence Theorem, Convergence in Measure, Characterizations of Riemann and Lebesgue Integrability, Continuity of Monotone Functions, Differentiability of Monotone Functions: Lebesgue's Theorem, Functions of Bounded Variation: Jordan's Theorem, Absolutely Continuous Functions, Integrating Derivatives: Differentiating Indefinite Integrals
3. **Convergence theorems:** monotone convergence theorem, Fatou's lemma and dominated convergence theorem, applications
4. **Product measures:** product spaces and measures, iterated integrals, Fubini and Tonelli theorems and applications
5. **Signed measures:** Jordan and Hahn decomposition, Radon Nikodym theorem, Lebesgue decomposition theorem
6. **L^p Spaces and Weak Convergence:** Holder and Minkowski inequalities, completeness of L^p spaces, The Riesz Representation for the Dual of L^p, Weak Sequential Convergence in L^p, Weak Sequential Compactness, The Minimization of Convex Functionals
7. **Measure and Topology:** Locally Compact Topological Spaces, The Construction of Radon Measures, 4 The Representation of Positive Linear Functionals on C_c(X): The Riesz Markov Theorem, The Riesz Representation Theorem for the Dual of C(X), Regularity Properties of Baire Measures

Recommended books:

1. **Real Analysis by H.L. Royden and P.M. Fitzpatrick**
2. **Measure Theory and Probability Theory by K.B. Athreya and S.N. Lahiri**
3. **Measure Theory And Integration by G. De Barra**
4. **Real Analysis: Modern Techniques and Their Applications by G.B. Folland**
5. **Measure Theory by P. Halmos**
6. **Probability and Measure Theory by Robert Ash**

6. Probability and Statistics

Syllabus:

1. **Introduction to probability:** Probability axioms, finite and infinite sample spaces, counting, conditional probability, total probability theorem, Bayes' rule, independence.
2. **Discrete Random variables:** Distribution functions, probability mass functions, expected values and variance, expectation of a function of a discrete random variable, covariance, and correlation coefficients, moment generating functions, joint distributions of discrete random variables, marginal and conditional distributions, conditional expectation, joint moment generating functions.
3. **Continuous random variables:** Cumulative distribution functions, probability density functions, expectation and variance, expectation of a function of a continuous random variable, Distribution of a function of a single continuous random variable, joint probability density functions of several continuous random variable, marginal distributions, independence, covariance, variance of a sum of dependent jointly continuous random variables.
4. **Limit theorems:** Markov and Chebyshev Inequalities, Weak Law of Large Numbers, Central Limit Theorem
5. **Review: Summarizing Data/ Descriptive Statistics**: Introduction. Methods Based on the Cumulative Distribution Function. Histograms, Density Curves, and Stem-and-Leaf Plots. Measures of Location. Measures of Dispersion. Boxplots. Skewness and Kurtosis. Exploring Relationships with Scatterplots.
6. **Review Of Probability Theory**: Conditional Expectation and Conditional Variance. Some Probability Inequalities. Moment Generating Functions. Distribution of a Distribution Function. Problems.
7. **Distributions Derived From The Normal Distribution**: Chi-Squared, t, and F Distributions. Distribution of the Sample Mean and Sample Variance. Distribution of difference of sample means and ratio of sample variance. Problems.
8. **Survey Sampling**: Population Parameters. Simple Random Sampling. Estimation of a Ratio. Stratified Random Sampling.
9. **Some Elementary Statistical Inference**: Unbiasedness, Consistency, Order Statistics, Quantiles, Confidence Intervals of Quantiles, Tolerance Limits for Distributions, Confidence Intervals for Mean, Proportions and variance. Problems.

10. **Theory Of Estimation**: Parameter Estimation. The Method of Moments. The Method of Maximum Likelihood, Efficiency and the Cramer-Rao Lower Bound. Sufficiency, Completeness. Rao-Blackwell Theorem. Problems.
11. **Testing Hypotheses And Assessing Goodness Of Fit**: The Neyman-Pearson Paradigm. MP, UMP and UMPU tests. Generalized Likelihood Ratio Tests. The Poisson Dispersion Test. Problems.
12. **Comparing Two Samples**: Two-sample inference of mean, proportion, and variance. Comparing Two Independent Samples. Comparing Paired Samples. Experimental Design.
13. **The Analysis Of Variance**: The One-Way Layout. The Two-Way Layout. The Distributions of Certain Quadratic Forms
14. **The Analysis Of Categorical Data**: The Chi-Square Test of Homogeneity/ of Independence. Goodness of fit tests.

Recommended books:

1. **An Introduction to Probability Theory and its Applications by W. Feller**
2. **Measure Theory and Probability Theory by K.B. Athreya and S.N. Lahiri**
3. **Probability and Measure Theory by Robert Ash**

7. Complex Analysis

Syllabus:

1. **Complex numbers:** Complex numbers, exponential form, roots of complex numbers, regions in the complex plane. Functions of a complex variable, mappings by the Exponential Function. Polar Coordinates.
2. **Limits, continuity and differentiability:** Definitions: Limits, Continuity, Derivatives. Differentiation Formulas, Cauchy–Riemann Equations, Sufficient Conditions for Differentiability.
3. **Analytic functions:** Definition, Entire functions, Laplace operator and equation, Harmonic functions, complex exponential and logarithmic functions with their properties, branches, branch cuts and branch points. Complex exponents; trigonometric functions, hyperbolic functions, inverse trigonometric functions of a complex variable.
4. **Integration:** Definite integral, parametrisation of a complex values curve, Jordan Curve theorem, smooth curves, contours, orientations. Contour integration and its properties, simply and multiply connected domains, Cauchy Goursat theorem, Principle of deformation of contours. Winding number of a curve, Cauchy Integral formula, Liouville's Theorem and the Fundamental Theorem of Algebra, Maximum Modulus Principle.
5. **Power series:** Convergence of Sequences, Convergence of Series, Taylor Series, Proof of Taylor's Theorem. Laurent Series, Proof of Laurent's Theorem, Absolute and Uniform Convergence of Power Series, Continuity of Sums of Power Series, Integration

and Differentiation of Power Series, Uniqueness of Series Representations, Multiplication and Division of Power Series.

6. **Singularities and Residues:** Poles and singularities. Residues, Cauchy's Residue Theorem, counting theorem and Rouche's theorem, the argument principle, applications of the residue theorem in the evaluation of integrals.
7. **Conformal maps:** Linear fractional transforms, conformal mapping and applications. Mobius transformations.

Recommended books:

1. **Functions of One Complex Variable I by J.B. Conway**
2. **Real and Complex Analysis by W. Rudin**
3. **Complex Analysis by L.V. Ahlfors**
4. **Complex Analysis by S. Lang**
5. **Complex Analysis by E.M. Stein and R. Shakarchi**
6. **Complex Analysis in One Variable by R. Narasimhan and Y. Nievergelt**
7. **Visual Complex Analysis by T. Needham**
8. **Theory of Complex Functions by R. Remmert**

8. Functional Analysis

Syllabus:

1. **Hilbert Spaces:** Elementary Properties and Examples, Orthogonality, The Riesz Representation Theorem, Orthonormal Sets of Vectors and Bases, Isomorphic Hilbert Spaces and the Fourier Transform for the Circle, The Direct Sum of Hilbert Spaces.
2. **Operators on Hilbert Space:** Elementary Properties and Examples, The Adjoint of an Operator, Projections and Idempotents; Invariant and Reducing Subspaces, Compact Operators, The Diagonalization of Compact Self-Adjoint Operators, An Application: Sturm–Liouville Systems, The Spectral Theorem and Functional Calculus for Compact Normal Operators, Unitary Equivalence for Compact Normal Operators.
3. **Banach Spaces:** Elementary Properties and Examples, Linear Operators on Normed Spaces, Finite Dimensional Normed Spaces, Quotients and Products of Normed Spaces, Linear Functionals, The Hahn–Banach Theorem, An Application: Banach Limits, An Application: Runge's Theorem, An Application: Ordered Vector Spaces, The Dual of a Quotient Space and a Subspace, Reflexive Spaces, The Open Mapping and Closed Graph Theorems, Complemented Subspaces of a Banach Space, The Principle of Uniform Boundedness.
4. **Locally Convex Spaces:** Elementary Properties and Examples, Metrizable and Normable Locally Convex Spaces, Some Geometric Consequences of the Hahn–Banach Theorem, Some Examples of the Dual Space of a Locally Convex Space, Inductive Limits and the Space of Distributions.
5. **Weak Topologies:** Duality, The Dual of a Subspace and a Quotient Space, Alaoglu's Theorem, Reflexivity Revisited, Separability and Metrizability, An Application: The Stone–Čech

Compactification, The Krein–Milman Theorem, An Application: The Stone–Weierstrass Theorem, The Schauder Fixed Point Theorem, The Ryll–Nardzewski Fixed Point Theorem, An Application: Haar Measure on a Compact Group, The Krein–Smulian Theorem, Weak Compactness.

6. **Linear Operators on a Banach Space:** The Adjoint of a Linear Operator, The Banach–Stone Theorem, Compact Operators, Invariant Subspaces, Weakly Compact Operators.
7. **Banach Algebras and Spectral Theory for Operators on a Banach Space:** Elementary Properties and Examples, Ideals and Quotients, The Spectrum, The Riesz Functional Calculus, Dependence of the Spectrum on the Algebra, The Spectrum of a Linear Operator, The Spectral Theory of a Compact Operator, Abelian Banach Algebras, The Group Algebra of a Locally Compact Abelian Group.
8. **C*-Algebras:** Elementary Properties and Examples, Abelian C*-Algebras and the Functional Calculus in C*-Algebras, The Positive Elements in a C*-Algebra, Ideals and Quotients of C*-Algebras, Representations of C*-Algebras and the Gelfand–Naimark–Segal Construction.
9. **Normal Operators on Hilbert Space:** Spectral Measures and Representations of Abelian C*-Algebras, The Spectral Theorem, Star-Cyclic Normal Operators, Some Applications of the Spectral Theorem, Topologies on B(H), Commuting Operators, Abelian von Neumann Algebras, The Functional Calculus for Normal Operators, Invariant Subspaces for Normal Operators, Multiplicity Theory for Normal Operators: A Complete Set of Unitary Invariants.
10. **Unbounded Operators:** Basic Properties and Examples, Symmetric and Self-Adjoint Operators, The Cayley Transform, Unbounded Normal Operators and the Spectral Theorem, Stone's Theorem, The Fourier Transform and Differentiation, Moments.

Recommended books:

1. **A Course in Functional Analysis by J.B. Conway**
2. **Functional Analysis by W. Rudin**

9. Geometry of Curves and Surfaces

Syllabus:

1. **Introduction to Euclidean spaces:** vectors (in dimensions 2 and 3), dot product, matrices, determinants of square matrices, basic features of n-dimensional Euclidean space
2. **Limits, continuity, and differentiation of functions of more than one variable:** Definitions, the partial derivative and total derivative, the derivative as a linear map, examples, basic properties of the derivative: the chain, product, and quotient rule, the gradient operator, directional derivatives, introduction to paths and curves
3. **Higher order derivatives:** iterated partial derivatives, Taylor's theorem, extrema of functions of many variables, constrained extrema and Lagrange multipliers, implicit function theorem: statement and examples.
4. **Vector values functions:** arc length, vector fields, divergence and the curl operator, examples

5. **Double and triple integrals:** over rectangles and more general regions, changing the order of integration, examples, change of variables formula
6. **Integrals over paths and surfaces:** path and line integrals, parametrized surfaces, area of a surface, integrals of scalar functions and vector fields over surfaces
7. **Integral theorems of vector analysis:** Green's theorem in the plane, Gauss' divergence theorem, Stokes' theorem, conservative vector fields, applications
8. **Geometry of Euclidean spaces:** Tangent spaces, vector fields, frame fields, isometries of Euclidean spaces with emphasis on dimensions 2 and 3.
9. **General theory of curves:** Velocity vectors, length, arc length reparameterization, acceleration vectors, curvature.
10. **Curves in plane:** Frame field, signed curvature, curvature determines planar curve up to congruence, curvature formula for regular planar curves, periodic, closed and simply closed curves, convex curves, Four Vertex Theorem, rotation number.
11. **Curves in space:** Cross product of vectors in R^3, frame field, torsion, Fresnet formulas, curvature and torsion determine space curve up to congruence, geometric interpretation of curvature and torsion using Taylor approximation, curvature and torsion formulas for regular space curves, total curvature.
12. **Preliminaries on surfaces:** Definition, level surfaces, surfaces of rotation, differentiability of transition functions, differentiability of functions to and from surfaces, tangent space to a surface, derivative of a differentiable map between surfaces, covariant derivative.
13. **Geometry of surfaces:** Shape operator, normal curvatures, principal curvatures, mean and Gaussian curvatures, geometric interpretation of Gaussian curvature, formulas for curvatures of a surface, shape operator determines surface up to congruence, isometry of surfaces, isometry invariance of Gaussian curvature.
14. **Forms on Manifolds:** Differential forms, orientation, adapted frame field and fundamental forms, Riemannian structure on 2-manifolds, Riemannian connection on a 2 manifold, Gaussian curvature of a 2-manifold, integration of 2-forms on a surface, Euler characteristic, Gauss Bonnet Theorem.

Recommended books:

1. **Lecture Notes on Elementary Topology and Geometry by I.M. Singer and J.A. Thorpe**
2. **Calculus on Manifolds by M. Spivak**
3. **An Introduction to Manifolds by Loring W. Tu**
4. **An Introduction to Differentiable Manifolds and Riemannian Geometry by W.M. Boothby**
5. **Differential Geometry of Curves and Surfaces by M.P. do Carmo**

10. Topology

Syllabus:

1. **Separation and countability axioms:** Countability axioms. Lindelof spaces. Separable spaces. First countable spaces, second countable spaces. Separation axioms and related theorems. Urysohn's lemma. Urysohn's metrization theorem. Tietze extension theorem.

2. **Fundamental definitions and examples:** Definitions of topological space, basis, subbasis. Examples: real line, metric spaces, Euclidean spaces, order topology, discrete and indiscrete topology, finite topological spaces, co-finite topology, co-countable topology. Lower limit topology. Product topology. Subspace topology. Comparing topologies. Closed sets. Closure and interior. Limit point. Boundary of a set. T1 and Hausdorff spaces. Convergence of sequences.
3. **Continuous functions:** Continuous functions. Projections. Inclusion. Homeomorphisms. Embeddings. Pasting lemma.
4. **Product and box topologies:** Definition. Coarsest topology on domain with respect to a collection of maps: example - projections. Characterization of continuous functions into a product space.
5. **Open and closed maps and quotient spaces.** Definitions. Quotient maps, universal property. Quotient topology. Examples: closed interval to circle. Disk to sphere. Real line to circle, plane to torus (covering maps).
6. **Connectedness:** Connected spaces, examples: intervals, real line, comb space. Path connected spaces, locally connected spaces, locally path connected spaces. Theorems involving continuous maps and connected spaces. Connected components, path connected components.
7. **Compactness:** Compact spaces, examples: closed intervals, finite sets. Tube lemma. Finite intersection property and characterization of compactness in terms of closed sets. Lebesgue number lemma for compact spaces. Limit point compactness, sequential compactness, local compactness. Theorems involving continuous maps and compact spaces. Equivalence of sequential compactness and compactness in metric spaces. One-point compactification.
8. **Advanced topics in Point-Set Topology:** Tychonoff's Theorem, Stone-Cech Compactification, Arzela-Ascoli Theorem, Embedding of compact manifolds, Partitions of unity, Normality of Quotient Spaces, Adjunction spaces, Space filling curves, Paracompactness, Nagata Smirnov Metrization Theorem, Topologies on Function Spaces, Pointwise and Compact Convergence.
9. **Simplicial and CW Complexes:** Simplicial complexes and geometric realization, Simplicial maps, Barycentric subdivision (basic properties), CW complexes and the weak topology, Attaching cells and cellular maps, Cellular approximation theorem, Examples of CW structures on spheres, tori, and projective spaces.
10. **Fundamental Group:** Paths and loops, Path homotopy, Homotopy of maps and homotopy equivalence. The fundamental group and change of basepoint, Functoriality of π_1, Computation of π_1 for basic spaces (circle, torus, sphere, projective spaces), Retractions and induced homomorphisms, Deformation retractions and contractible spaces, Fundamental group of product spaces, Simply connected spaces.
11. **Van Kampen Theorem and Applications:** Statement and proof of the Seifert–van Kampen theorem, Free products and amalgamated products, Fundamental group of wedge sums, Cell complexes and attaching maps, Computation of fundamental groups via CW structures, Fundamental group of compact surfaces.
12. **Covering Spaces:** Definition and basic examples of covering spaces, Evenly covered neighborhoods, Lifting properties for paths and homotopies, Unique path lifting theorem, Homotopy lifting property, Classification of covering spaces via subgroups of the fundamental group (for path-connected, locally path-connected, semilocally simply connected spaces), Universal covering space, Deck transformations and group of covering transformations, Relationship between deck transformations and π_1, Covering spaces of the circle and graphs,

Covering spaces of surfaces.

13. **Classification of Surfaces:** Construction of surfaces via edge identifications of polygons, Connected sum, Orientability, Euler characteristic (combinatorial definition), Classification theorem for compact surfaces (statement and proof outline using fundamental groups), Fundamental groups of orientable and non-orientable surfaces.
14. **Selected Topics:** Mapping cylinder and mapping cone, Homotopy extension property (statement and applications), Degree of maps $S^1 \to S^1$, Degree of maps $S^n \to S^n$ (low-dimensional viewpoint), Brouwer fixed point theorem in dimension 2, Borsuk–Ulam theorem (low-dimensional case and applications), Jordan Curve Theorem, Schoenflies Theorem, Triangulation of Surfaces, Standard Polygonal Representations of Surfaces, Combinatorial Classification of Surfaces using Polygonal Representations.

Recommended books:

1. **Topology, A First Course by J. Munkres**
2. **Basic Topology by M.A. Armstrong**
3. **Algebraic Topology by A. Hatcher**
4. **A Basic Course in Algebraic Topology by W.S. Massey**
5. **Topology and Geometry by G.E. Bredon**
6. **Geometric Topology in Dimensions 2 and 3 by E.E. Moise**

11. Linear Algebra

Syllabus:

1. **Linear Equations:** Fields, Systems of Linear Equations, Matrices and Elementary Row Operations, Row-Reduced Echelon Matrices, Matrix Multiplication, Invertible Matrices.
2. **Vector Spaces:** Vector Spaces, Subspaces, Bases and Dimension, Coordinates, Summary of Row-Equivalence, Computations Concerning Subspaces.
3. **Linear Transformations:** Linear Transformations, The Algebra of Linear Transformations, Isomorphism, Representation of Transformations by Matrices, Linear Functionals, Dual spaces, dual basis. Duals of subspaces, quotients, and direct sums. Double dual - canonical isomorphism. Canonical bilinear form from V \times V^{*} \to F; orthogonal subspaces with respect to Canonical bilinear form. The Double Dual, The Transpose of a Linear Transformation.
4. **Polynomials:** Algebras, The Algebra of Polynomials, Lagrange Interpolation, Polynomial Ideals, The Prime Factorization of a Polynomial.
5. **Determinants:** Commutative Rings, Determinant Functions, Permutations and the Uniqueness of Determinants, Additional Properties of Determinants, Modules, Multilinear Functions, The Grassman Ring.
6. **Elementary Canonical Forms:** Introduction, Characteristic Values, Annihilating Polynomials, Invariant Subspaces, Simultaneous Triangulation; Simultaneous Diagonalization, Direct-Sum Decompositions, Invariant Direct Sums, The Primary Decomposition Theorem.

7. **The Rational and Jordan Forms:** Cyclic Subspaces and Annihilators, Cyclic Decompositions and the Rational Canonical Form, The Jordan Form, Computation of Invariant Factors, Summary; Semi-Simple Operators.
8. **Inner Product Spaces:** Inner Products, Inner Product Spaces, Linear Functionals and Adjoints, Symmetric, Hermitian, Unitary, Orthogonal, Normal Operators. Gram-Schmidt ortho-normalisation. Cauchy-Schwarz inequality. Spectral theorems for symmetric, Hermitian, and unitary operators. LU Decomposition, QR Decomposition, Cholesky Decomposition, Singular Value Decomposition, Schur Decomposition, Polar Decomposition.
9. **Operators on Inner Product Spaces:** Introduction, Forms on Inner Product Spaces, Positive Forms, Hermitian Forms, Spectral Theory, Further Properties of Normal Operators.
10. **Bilinear Forms:** Bilinear Forms, Symmetric Bilinear Forms, Skew-Symmetric Bilinear Forms, Structure theorems for bilinear forms, Structure theorems for symmetric and alternating forms. Change of basis. Bilinear forms over R and C; Sylvester's Law of Inertia. Structure theorems for Hermitian and Skew-Hermitian forms. Groups Preserving Bilinear Forms. Multilinear forms and Alternating forms. Characterization of the Determinant.
11. **Linear Groups:** Special Unitary Groups, Rotation Groups, Normal Subgroups of SL_2.
12. **Group Representations:** Irreducible Representations, Unitary Representations, Characters, One-Dimensional Characters, The Regular Representation, Schur's Lemma, Proof of Orthogonality Relations, Representations of SU_2.

Recommended books:

1. **Linear Algebra by K. Hoffman and R. Kunze**
2. **Algebra by M. Artin**
3. **Linear Algebra by Serge Lang**
4. **Linear Algebra Done Right by S. Axler**
5. **Finite Dimensional Vector Spaces by P.R. Halmos**

12. Group Theory

Syllabus:

1. **Introduction to groups:** Definition of a group. Examples: Cyclic groups, Dihedral groups, Symmetric groups, Matrix groups, Quaternion group, modular arithmetic, permutation groups, symmetries of the square, rectangle. Uniqueness of inverse and identity. Subgroups, subgroup criteria. Order of an element, subgroup generated by an element. Subgroup generated by a collection of elements. Product of subgroups: subgroup criterion for product. Abelian groups. Centre of a group. Examples. Lattice of subgroups of a group.
2. **Permutation groups:** Definition of permutation group on finite set. Transpositions, cycles. Every permutation is a product of disjoint cycles. Every permutation is a product of transpositions. Cycle type and sign of a permutation. Alternating groups.
3. **Cosets and Lagrange's theorem**: Definition of left/right cosets; equivalence relations induced by a subgroup. Lagrange's theorem for finite groups.

4. **Normal subgroups, quotient groups, group homomorphisms**: Normal subgroups; quotient group construction. Cauchy's theorem on abelian groups. Direct products of groups. Group homomorphisms; kernel, image. Factorisation of group homomorphisms through normal subgroups. Isomorphisms and automorphisms. Isomorphism theorems. Composition Series and the Hölder programme,
5. **Group actions**: Definition, regular representation: Cayley's theorem. Conjugation action: conjugacy classes with examples. Class equation. Cauchy's theorem. Definition of a group action: orbit stabilizer. Regular representation on cosets. Examples. P-groups: basic properties (non-trivial center; groups of order p^2 abelian). Automorphisms, Automorphism group. Simplicity of Alternating groups.
6. **P-groups, Sylow's theorems and applications:** Sylow's theorem. Examples and applications to groups of order pq, p^{2}q, pqr.
7. **Finite abelian groups**: Direct products, Structure theorem for finite abelian groups; examples and enumeration of isomorphism classes of abelian groups.
8. **Selected topics:** Semi-direct products, Groups of small order, Structure Theorem for Finitely Generated Abelian Groups, Nilpotent Groups, Solvable Groups, Free Groups, Free Product with Amalgamation, HNN Extension, Ping Pong Lemma.

Recommended books:

1. **Algebra by M. Artin**
2. **Topics in Algebra by I.N. Herstein**
3. **Abstract Algebra by D.S. Dummit and R.M. Foote**
4. **Algebra by S. Lang**
5. **Basic Algebra I by N. Jacobson**
6. **Algebra by T.W. Hungerford**

13. Ring Theory and Module Theory

Syllabus:

1. **Rings and Ring Homomorphisms:** Definition and examples of rings (commutative and noncommutative), Subrings, Ring homomorphisms, Ideals and quotient rings, First Isomorphism Theorem, Direct products of rings, Units and zero divisors, Characteristic of a ring.
2. **Ideals and Prime Structure:** Prime ideals and maximal ideals, Integral domains and fields, Nilradical and Jacobson radical, basic properties. Chinese Remainder Theorem, Spectrum of a ring, Zariski topology on Spec R. Multiplicatively closed sets, Localization.
3. **Polynomial Rings and Factorization:** Polynomial rings over a ring and over a field, Division algorithm in F[x], Irreducibility criteria (Gauss's lemma, Eisenstein criterion), Unique factorization domains, Principal ideal domains, Euclidean domains, Ring of Gaussian integers. Construction of finite fields. Formal Power Series Ring.

4. **Modules Over a Ring:** Definition and examples of modules, Submodules and quotient modules, Module homomorphisms, Isomorphism theorems for modules, Direct sums and products, Exact sequences (short exact sequences and splitting), Free modules and bases.
5. **Finitely Generated Modules and Structure Theory:** Cyclic modules, Finitely generated modules, Presentations of modules, Structure theorem for finitely generated modules over a PID, Invariant factor and elementary divisor decompositions, Classification of finitely generated abelian groups.
6. **Projective and Injective Modules:** Direct summands and projective modules, Lifting properties, Injective modules, Splitting lemmas.
7. **Tensor Products and Hom:** Tensor product of modules, Universal property, Bilinear maps, Exactness properties of tensor product, Hom functor and adjunction between Hom and tensor.
8. **Semisimple and Artinian Theory:** Simple modules, Semisimple modules and rings, Composition series, Jordan-Hölder theorem, Artinian and Noetherian modules, Wedderburn-Artin theorem.
9. **Commutative Rings and Modules:** Chain conditions, Prime and Primary Ideals, Primary Decomposition, Noetherian Rings and Modules, Ring Extensions, Dedekind Domains, The Hilbert Nullstellensatz.
10. **Selected Topics:** Group rings and modules as representations, Modules over polynomial rings and linear transformations, Matrix rings and simple rings, Introduction to Morita equivalence, Applications to linear algebra and representation theory.
11. **Additional Topics:** Quadratic Number Fields, Rings of Algebraic Integers, Ideals in the ring of Algebraic Integers and Ideal Factorization, Prime ideals and Prime integers, Ideal Classes, Computing the Class group, Real Quadratic Fields, Lattices in Quadratic Number Fields.
12. **More Additional Topics:** Classification of Conics, Pascal's Theorem, Envelopes of Conics, Cubics, Intersection of Curves, Flexes, Singular Cubics, Addition on Cubics, Bezout's theorem, Hessians, Parametrizing Curves, Envelopes of Curves

Recommended books:

1. **Algebra by M. Artin**
2. **Basic Algebra Vol. 1 by N. Jacobson**
3. **Basic Algebra Vol. 2 by N. Jacobson**
4. **Abstract Algebra by D.S. Dummit and R.M. Foote**
5. **Algebra by S. Lang**
6. **Algebra by T.W. Hungerford**
7. **Conics and Cubics, A Concrete Introduction to Algebraic Curves by Robert Bix**

14. Field and Galois Theory

Syllabus:

1. **Basic field theory:** Fields, finite field extensions, algebraic extensions, degree of an extension, minimal polynomials, simple extensions, splitting fields, composite of fields, existence of algebraic closure, roots of unity. Examples of fields including finite fields, function fields, cyclotomic extensions. Relationship with classical straightedge and compass constructions.
2. **Theory of Symmetric Polynomials**: Symmetric polynomials in roots, Fundamental Theorem on Symmetric Polynomials, Newton's Theorem and Newtons' Identities between elementary symmetric polynomials and power sum symmetric polynomials, Discriminants. Solutions of Quadratic, Cubic, and Quartic equations. Theory of Resolvents.
3. **Separable extensions:** Separable polynomials, elements, and extensions. Formal derivative test for separability of a polynomial. Separability of algebraic extensions over fields of characteristic zero and finite fields, perfect fields. Existence and uniqueness of finite fields of prime power order.
4. **Purely Inseparable Extensions**: Algebraic extensions, separable degree, inseparable degree. Proof that separable elements in a field extension form a subfield. Structure theorem of an algebraic extension as a tower of a maximal separable extension and a purely inseparable extension above it.
5. **Homomorphisms; Normal and Galois extensions:** Extending homomorphism along algebraic extensions into algebraically closed fields, including tower law. Equivalent definitions of normal extensions (as splitting fields, splittings of irreducible polynomials, in terms of mappings into algebraic closure), examples. Definition of a Galois extension. Proof that every separable extension embeds in a Galois extension.
6. **Automorphisms and the Fundamental Theorem of Galois Theory (finite extensions):** Group of automorphisms of a field extension. Examples. Fixed fields of automorphisms groups. The fundamental theorem of Galois theory for finite extensions. Characterisation of simple extensions, primitive element theorem. Proof of Artin's lemma: examples including the action of S_n on the field of rational functions in n indeterminates. Relationship between normal subgroups of the Galois group and normal subextensions.
7. **Selected Topics:** Galois groups of polynomials of small order. Galois groups of finite fields. Galois groups of cyclotomic extensions, Abelian extensions, Cyclic extensions, Solvable extensions and solvability by radicals, Impossibility of a general Quintic equation, Kummer Extensions.
8. **Additional Topics:** Transcendental Extensions, Transcendence bases, Noether Normalization Theorem, Linearly Disjoint Extensions, Separable and Regular Extensions.

Recommended books:

1. **Algebra by M. Artin**
2. **Basic Algebra Vol. 1 by N. Jacobson**
3. **Basic Algebra Vol. 2 by N. Jacobson**
4. **Abstract Algebra by D.S. Dummit and R.M. Foote**
5. **Algebra by S. Lang**
6. **Algebra by T.W. Hungerford**