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B.Sc. (Hons.) Physics (Semester –1st)
BASIC MATHEMATICS-I
Subject Code: BMATH5102
Paper ID: 19131503

Time: 03 Hours **Maximum Marks: 60**

Instruction for candidates:

1. Section A is compulsory. It consists of 10 parts of two marks each.
2. Section B consist of 5 questions of 5 marks each. The student has to attempt any 4 questions out of it.
3. Section C consist of 3 questions of 10 marks each. The student has to attempt any 2 questions.

Section – A **(2 marks each)**

Q1. Attempt the following:

a. Find the values of the constants a and b such that the function $f(x) = \{x^2 - 3a\ x <- 1,\ 7\ x =- 1,\quad - x + a + 2b\ x >- 1$ becomes continuous.

b. Find the 30th derivative of $y(x) = \frac{-1}{(2x+3)^3}$.

c. Check the differentiability of the function $f(x) = |x|\ at\ x = 0$.

d. Find the value of the limit $\frac{\tan tan\ x - x}{x^2 \sin sin\ x}$.

e. Verify the Rolle's theorem for the function $y = x^2 + 2$ in the interval $[-2, 2]$.

f. Solve the integral $\int \frac{x^3}{sin^2(x^4)} dx$.

g. Find the integration of the function $\frac{1}{x^2+3x+2}$.

h. Solve the integral $\int_{-3}^{3} |x - 1| dx$.

i. Find the area enclosed by the circle $x^2 + y^2 = 25$ using the concept of integration.

j. Derive the reduction formula for $\int x^n e^{ax} dx$.

Section – B **(5 marks each)**

Q2. Find the Maclaurins series for $f(x) = \cos cos\ x$.

Q3. If $y = \left[log\left(x + \sqrt{1 + x^2}\right)\right]^2$, Prove that $\left(1 + x^2\right)y_{n+2} + (2n + 1)xy_{n+1} + n^2 y_n = 0$.

Q4. Solve the integral $\int \frac{1}{\sqrt{x^2-6x+4}} dx$.

Q5. Find the area of the region bounded by the two parabolas $y = x^2\ and\ x = y^2$.

Q6. Check the continuity and differentiability of the function $f(x) = \{x^2 sin\left(\frac{1}{x}\right),\ if\ x \neq 0,\ 0,\ if\ x = 0\ at\ x = 0$.

Section – C **(10 marks each)**

Q7. (a) Find all the asymptotes of the curve $x^3 + 3x^2y - 4y^3 - x + y + 3 = 0$.

(b) Using Taylor's theorem with Lagrange's form of remainder, show that $\log log\,(x + h) = logx + \frac{h}{x} - \frac{h^2}{2x^2} + \ldots + (-1)^{n-1}\frac{h^n}{n(x+\theta h)^n}$.

Q8. (a) Solve the integral $\int \frac{xsin^{-1}x}{\sqrt{1-x^2}}dx$.

(b) Evaluate the 100^{th} derivative of $\log\left(x^2 - 3x + 2\right)$.

Q9. (a) The circle $x^2 + y^2 = a^2$ is revolved about the $x - axis$. Find the volume of the sphere so formed.

(b) Derive the reduction formula for $\int tan^n x dx$.