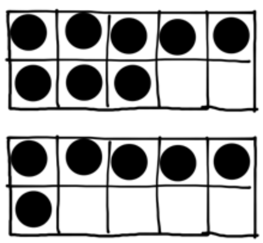
Number Talk Strategies

How to make these strategies VISUAL: http://bit.ly/MMMMMDuane

Eight Common Strategies for Addition in the K-2 Classroom

Addition Strategy: Counting All

Counting every number is an addition strategy used primarily by kindergarten and beginning first-grade students.

How many dots do you see? 	The student literally starts with 1 and counts up to 14 by counting every circle one-by-one.

Addition Strategy: Counting On

This is a transitional strategy used primarily by 1st and early 2nd grade students. The student starts with one of the numbers and counts on from this point. As the teacher, it is tempting to show or tell students this strategy in an attempt to move them to a more efficient strategy. However, if students don't construct this strategy for themselves, it becomes a magical procedure without any foundation. From an efficiency standpoint, notice whether the student counts on from the smaller or larger number.

8 + 6	The student may something like, "I put 8 in my head and then I counted up six more fingers and ended on 14: 8…9, 10, 11, 12, 13, 14."

Addition Strategy: Doubles and Near-Doubles

Beginning as early as kindergarten, students are able to recall sums for many doubles. This strategy capitalizes on this strength by adjusting one of bother numbers to make a double or near-doubles combination.

8 + 6 8 + 6 2+6+6 2+12=14	The 8 is decomposed in order to create double sixes.

Addition Strategy: Making Tens

Making tens is an important focus in the primary grades. Beginning around 2nd grade, students should be able to break numbers apart quickly to make ten.

8 + 6 8+6 8+2+4 10+4=14 Decompose the 6 to create a ten with the 8.	8 + 6 8+6 4+4+6 4+10=14 Decompose the 8 to create a ten with the 6.

Addition Strategy: Making Landmark or Friendly Numbers

Similar to Making Tens, students use decomposing and composing to create multiples of five, multiples of ten, or monetary amounts such as 25¢ or 50¢.

23 + 48 23+48 21+2+48 21+50=71 Decompose the 23 to (21 + 2) in order to create 50.	23 + 48 23+48 +2 23+50=73 73−2=71 Simply add 2 to the 48 to create 50. Then subtract by 2 at the end.

Addition Strategy: Compensation

Students will describe that they are removing a specific amount from one addend and give that exact amount to the other addend to make friendlier numbers. Compensation is very similar to making landmark numbers. Don't worry too much about whether a student's shared method is one or the other.

Make a double 7 + 9 7+9 −1 8+8=16 Move 1 from the 9 to create double eights.	Make a multiple of ten 18 + 23 18+23 −2 20+21=41 Move 2 from the 23 to create a multiple of ten.

Addition Strategy: Breaking Each Number into Its Place Value

Each addend is broken into expanded form and like place value amounts are combined. In contrast with the standard algorithm, students typically add from left to right.

24 + 38 24 + 38 20 + 30 = 50 4 + 8 = 12 50 + 12 = 62	24 is decomposed to 20 + 4 38 is decomposed to 30 + 8 Then the multiples of ten are added together to get 50. The ones are added to get 12.

Addition Strategy: Adding Up in Chunks

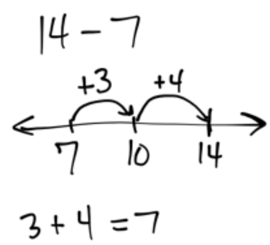
This is similar to Breaking Each Number into Its Place Value. The difference is that the student will keep one addend whole and breaking up the second addend into easy-to-use chunks.

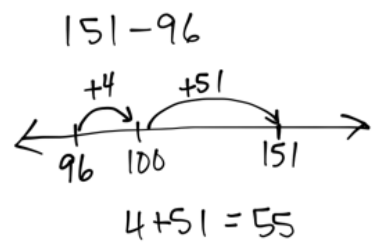
45 + 28 45 + 28 45 + 20 = 65 65 + 8 = 73 45 is kept the same and 28 is decomposed into 20 and 8. First 20 is added, then 8.	45 + 28 45 + 28 28 + 40 = 68 68 + 5 = 73 Now 28 is kept the same, but 45 is the number that is decomposed.

Three Common Strategies for Subtraction in the K-2 Classroom

Subtraction Strategy: Adding Up (Counting On)

Students use their strength with addition by adding up from the number being subtracted (subtrahend) to the whole number (minuend). As students explain the “jumps” they make, the teacher should use an empty number line to visually represent the strategy.

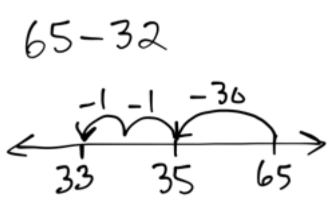
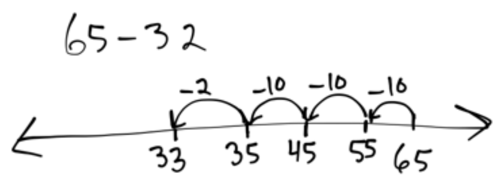
<table>
<tr>
<td>14 – 7
7 + [ ] = 14


Start with 7 and add 3 to get to 10. Then add 4 to arrive at 14.</td>
<td>151 – 96 =
96 + [ ] = 151


Start with 96 and add 4 to get to 100. Then add 51 to arrive at 151.</td>
</tr>
</table>

Subtraction Strategy: Removal or Counting Back

Students will start with the whole and then gradually subtract the subtrahend by decomposing the subtrahend into chunks. Some students will break the subtrahend into its place value parts to subtract. An empty number line is an easy way to represent student thinking.

<table>
<tr>
<td>65 – 32

</td>
<td>65 – 32

</td>
</tr>
</table>

Subtraction Strategy: Take from Ten

In this method, students make use of the fact that subtracting from 10 (or a multiple of ten) is easier than subtracting from other numbers. Students decompose the first number (minuend), subtract the subtrahend from the multiple of ten, and then add the remaining part of the minuend.

<table>
<tr>
<td>14 - 8
14 − 8
10 4
10 − 8 = 2
2 + 4 = 6</td>
<td>151 – 96
151 − 96
100 51
100 − 96 = 4
4 + 51 = 55</td>
</tr>
</table>

Eight Common Strategies for Addition in the 3-6 Classroom

Addition Strategy: Counting All

Addition Strategy: Counting On

Ideally, by the middle of 2nd grade we want students to abandon these two techniques in favor of more efficient strategies.

Addition Strategy: Doubles and Near-Doubles

Beginning as early as kindergarten, students are able to recall sums for many doubles. This strategy capitalizes on this strength by adjusting one of bother numbers to make a double or near-doubles combination.

116 + 118	116 + 118
116 + 118 −1 −3 115 + 115 = 230 230 + 4 = 234	116 + 118 +4 +2 120 + 120 = 240 240 − 6 = 234

Addition Strategy: Making Tens

Making tens is an important focus in the primary grades. Beginning around 2nd grade, students should be able to break numbers apart quickly to make ten.

116 + 118	116 + 118
116 + 118 (110 + 4 + 2) + (110 + 8) 110 + 110 = 220 2 + 8 = 10 + 4 234	116 + 118 (110 + 6) + (116 + 4 + 4) 110 + 110 = 220 6 + 4 = 10 + 4 234

Addition Strategy: Making Landmark or Friendly Numbers

Similar to Making Tens, students use decomposing and composing to create multiples of five, multiples of ten, or monetary amounts such as 25¢ or 50¢.

116 + 118	116 + 118
116 + 118 +2 116 + 120 = 236 236 − 2 = 234	116 + 118 +4 120 + 118 = 238 238 − 4 = 234

Addition Strategy: Compensation

Students will describe that they are removing a specific amount from one addend and give that exact amount to the other addend to make friendlier numbers. Compensation is very similar to making landmark numbers. Don't worry too much about whether a student's shared method is one or the other.

Make a double	Make a multiple of ten
116 + 118 116 + 118 -1 117 + 117 234 Move 1 from the 9 to create doubles.	116 + 118 116 + 118 +2 114 + 120 234 Move 2 from the 116 to create a multiple of ten.

Addition Strategy: Breaking Each Number into Its Place Value

Each addend is broken into expanded form and like place value amounts are combined. In contrast with the standard algorithm, students typically add from left to right.

116 + 118 116 + 118 (100+10+6)+(100+10+8) 100+100 = 200 10+10 = 20 6+8 = 14 200+20+14 = 234	116 + 118 116 + 118 (100+10+6) + (100+10+8) 200 + 20 + 14 234

Addition Strategy: Adding Up in Chunks

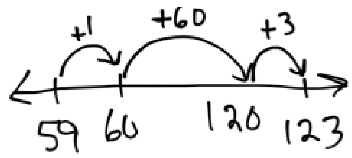
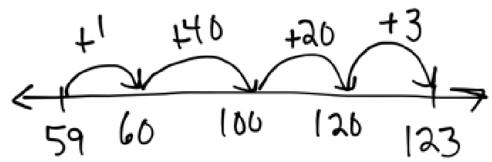
This is similar to Breaking Each Number into Its Place Value. The difference is that the student will keep one addend whole and breaking up the second addend into easy-to-use chunks.

116 + 118 116 + 118 116 + 100 = 216 216 + 10 = 226 226 + 8 = 234 116 is kept the same and 118 is decomposed into 100, 10, and 8. Each is then added to 116.	116 + 118 116 + 118 118 + 100 = 218 218 + 10 = 228 228 + 6 = 234 Now 118 is kept the same, and 116 is the number that is decomposed.

Five Common Strategies for Subtraction in the 3-6 Classroom

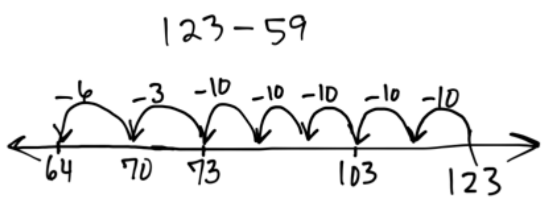
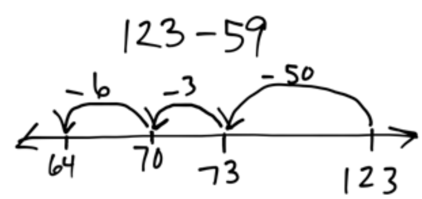
Subtraction Strategy: Adding Up

Students use their strength with addition by adding up from the number being subtracted (subtrahend) to the whole number (minuend). As students explain the “jumps” they make, the teacher should use an empty number line to visually represent the strategy.

123 – 59 59 + [ ] = 123  1 + 60 + 3 = 64 Start with 59 and add 1 to get to 60. Then add 60 to arrive at 120. Finally, adding 3 more gets us to 123.	123 – 59 = 59 + [ ] = 123  1 + 40 + 20 + 3 = 64

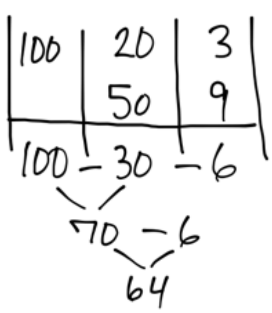
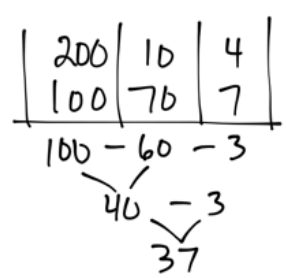
Subtraction Strategy: Removal or Counting Back

In this method, students will start with the whole and then gradually subtract the subtrahend by decomposing the subtrahend into chunks. Some students will break the subtrahend into its place value parts to subtract. An empty number line is an easy way to represent student thinking.

123 - 59 	123 - 59 

Subtraction Strategy: Place Value and Negative Numbers

Each number is broken apart into its respective place value using expanded notation. Like place values are groups and then subtracted. Negative numbers might result and then are recorded as an additional subtraction problem.

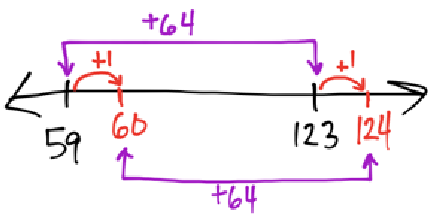
123 - 59 	214 - 177 

Subtraction Strategy: Keeping a Constant Difference

Adding or subtracting the same quantity from both the subtrahend and minuend maintains the difference between the numbers. Manipulating the numbers in this way allows the student to create a friendlier problem without changing the result.

123 - 59

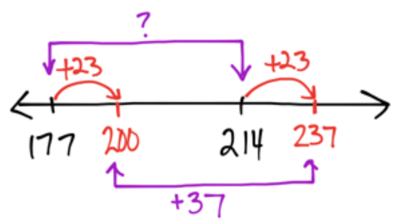
$$\begin{array}{r} 123 - 59 \\ \underline{+1} \quad \underline{+1} \\ 124 - 60 = 64 \end{array}$$



Both numbers have been adjusted by +1, which makes a problem with an easy multiple of ten.

214 - 177

$$\begin{array}{r} 214 - 177 \\ \underline{+23} \quad \underline{+23} \\ 237 - 200 = 37 \end{array}$$



Both numbers were adjusted by +23, making it easy to see their difference is 37. Therefore, the original numbers also have a difference of 37.

Subtraction Strategy: Adjusting One Number to Create an Easier Problem

Add or subtract from one number to make a multiple of ten or a friendly number. After subtracting, the difference needs to be adjusted to compensate for the adjustment at the start.

123 - 59

$$\begin{array}{r} 123 - 59 \\ \underline{+1} \\ 123 - 60 \\ 63 \\ 63 + 1 = 64 \end{array}$$

Add 1 to 59 making it a multiple of ten. After subtracting, we need to add one to the final answer because we subtracted 60 rather than merely 59.

123 - 59

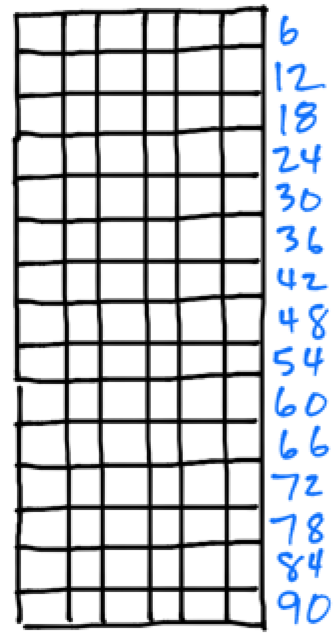
$$\begin{array}{r} 123 - 59 \\ \underline{-6} \\ 123 - 53 \\ 70 \\ 70 - 6 = 64 \end{array}$$

Subtract 6 from 59 to make both numbers end in 3. After subtracting, we need to subtract 6 more, since we initially only subtracted 53 rather than 59.

Five Common Strategies for Multiplication

Multiplication Strategy: Repeated Addition or Skip Counting

This is one of the beginning strategies for students who are just learning multiplication. Students initially use this strategy to begin the process of memorizing the multiplication facts. In addition to addition and skip counting, using arrays can make the skip counting more visual.

Repeated Addition	Adding in bunches	Using an array to skip count
6 x 15 15 + 15 = 30 30 + 15 = 45 45 + 15 = 60 60 + 15 = 75 75 + 15 = 90	6 x 15 = 15 + 15 + 15 + 15 + 15 + 15 = 30 + 30 + 30 = 90	

Multiplication Strategy: Making Landmark or Friendly Numbers

A multiplication problem can be made easier by changing one of the factors to a friendly or landmark number. Students who are comfortable multiplying by multiples of 5 or ten will often adjust factors to allow them to take advantage of this strength.

9 x 15 ↓ 10 x 15 = 150 150 − 15 = 135	9 x 15 (5 x 15) + (5 x 15) − 15 75 + 75 − 15 75 + 60 135

Multiplication Strategy: Partial Products

This is based on the distributive property. Students decompose one or both factors into the base-ten place values.

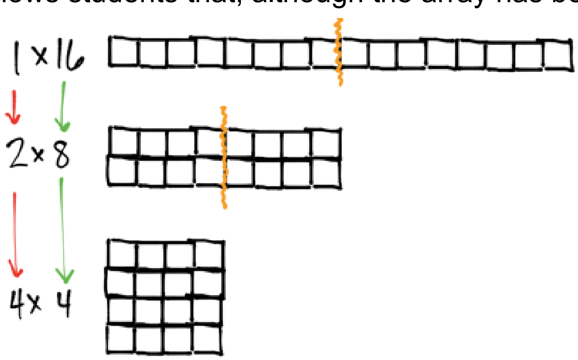
Leave 12 alone. Decompose 15.	Decompose the 12. Leave 15 alone.
12×15 $12 \times 10 = 120$ $12 \times 5 = 60$ $120 + 60 = 180$	12×15 $10 \times 15 = 150$ $2 \times 15 = 30$ $150 + 30 = 180$

Multiplication Strategy: Doubling and Halving

As students become familiar with building arrays to represent multiplication problems, they will be able to develop the new strategy or rearranging the array such that the area stays the same, but the dimensions change: one is doubled, while the other is halved. Doing so, changes the look of the array, but maintains the product.

8×25 $\downarrow \quad \downarrow$ 4×50 $\downarrow \quad \downarrow$ $2 \times 100 = 200$	12×15 $\downarrow \quad \downarrow$ 6×30 $\downarrow \quad \downarrow$ $3 \times 60 = 180$

To make sure students understand why the “doubling & halving” strategy works, it may be useful to build a 1 x 16 array and then use the “doubling and halving” strategy to turn it into a 4 x 4 array. This shows students that, although the array has been changed, the product stays the same.



Multiplication Strategy: Breaking Factors into Smaller Factors

Breaking factors into smaller factors instead of addends can be a powerful and efficient strategy for multiplication. By factoring the factors into smaller factors, students can then use the commutative and associative properties to make a more convenient problem to solve.

12×25 $3 \times 4 \times 5 \times 5$ $4 \times 5 \times 5 \times 3$ $20 \times 5 \times 3$ 100×3 300	12×25 $6 \times 2 \times 5 \times 5$ $2 \times 5 \times 6 \times 5$ 10×30 300
12×25 $3 \times 4 \times 25$ 3×100 300	

Four Common Strategies for Division

Division Strategies: Repeated Subtraction or Sharing/Dealing Out

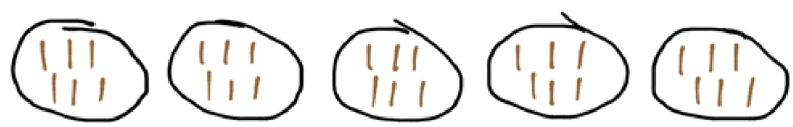
Repeated Subtraction

This is one of the least efficient division strategies, especially as the numbers get larger. Students may initially use this strategy especially if given a story-problem context in which we know the size of each group and are finding the number of groups.

<table>
<tr>
<td>Martha has 30 cookies. She plans to put 5 cookies in every container. How many containers will Martha need?</td>
<td>30 ÷ 5
30
− 5 ← 1.
25
− 5 ← 2.
20
− 5 ← 3.
15
− 5 ← 4.
10
− 5 ← 5.
5
− 5 ← 6.
0</td>
</tr>
</table>

Sharing/Dealing Out

This strategy will likely be used when the student is confronted with a story-problem context in which the number of groups is known and we are trying to find the size of each group.

<table>
<tr>
<td>Martha has 30 cookies. She plans to share the cookies with 5 friends. How many cookies will each friend receive?</td>
<td>
The student draws 5 circles and then places one tally in each circle, cycling through the circles until 30 tallies have been marked. Since there are six tallies in each circle, 30 ÷ 5 = 6


A slightly more efficient strategy is to use numbers larger than one to share the cookies.</td>
</tr>
</table>

Division Strategy: Multiplying Up

In this strategy students make use of the fact that a division problem can be thought of as multiplication with a missing factor. Students use smaller factors and multiples to “multiply up” until reaching the dividend.

<table>
<tr>
<td>
384 ÷ 16

<table>
<tr><td></td><td>Running Total</td></tr>
<tr><td>16 × 10 = 160</td><td>160</td></tr>
<tr><td>16 × 10 = 160</td><td>320</td></tr>
<tr><td>16 × 2 = 32</td><td>352</td></tr>
<tr><td>16 × 2 = 32</td><td>384</td></tr>
</table>

10 + 10 + 2 + 2 = 24
</td>
<td>
An open array can be used to model the student’s strategy and link multiplication, division. and area.

10 + 10 + 2 + 2 = 24
<table>
<tr><td>16</td><td>16x10=
160</td><td>16x10=
160</td><td>16x2=
32</td><td>16x2=
32</td></tr>
</table>
</td>
</tr>
</table>

Division Strategy: Partial Quotients

This strategy is similar to Multiplying Up, however, students record their thinking in a manner that begins to look like the “traditional” algorithm. Generally, students will initially use this strategy inefficiently using small multipliers. As the student chooses larger multipliers, the strategy becomes more efficient. Here are two different examples for using this strategy with the same problem.

<table>
<tr>
<td>
384 ÷ 16

16) 384
  −160 | 10
  224
  −160 | 10
  64
  −32 | 2
  32
  −32 | 2
  0
10 + 10 + 2 + 2 = 24
</td>
<td>
384 ÷ 16

16) 384
  −320 | 20
  64
  −64 | 4
  0
20 + 4 = 24
</td>
</tr>
</table>

Division Strategy: Proportional Reasoning

Since division can be written as a fraction, we can make use of this fact and "reduce" a division problem much like we "reduce" fractions. As fractions, we know $\frac{24}{8}$ can be rewritten as the equivalent fraction $\frac{6}{2}$ by dividing the numerator and denominator by 4. As a division problem this might be written as…

$$\frac{24 \div 4}{8 \div 4} = \frac{6}{2} = 3$$

$$\begin{array}{ccc} 24 & \div & 8 \\ \downarrow{\div 4} & & \downarrow{\div 4} \\ 6 & \div & 2 = 3 \end{array}$$

$$\begin{array}{ccc} 384 & \div & 16 \\ \downarrow{\div 2} & & \downarrow{\div 2} \\ 192 & \div & 8 \\ \downarrow{\div 2} & & \downarrow{\div 2} \\ 96 & \div & 4 \\ \downarrow{\div 2} & & \downarrow{\div 2} \\ 48 & \div & 2 \\ \downarrow{\div 2} & & \downarrow{\div 2} \\ 24 & \div & 1 = 24 \end{array}$$

$$\begin{array}{ccc} 384 & \div & 16 \\ \downarrow{\div 4} & & \downarrow{\div 4} \\ 96 & \div & 4 \\ \downarrow{\div 4} & & \downarrow{\div 4} \\ 24 & \div & 1 = 24 \end{array}$$