

The main law of Einstein physics:

$$\frac{d}{dt} \vec{P} = \vec{F}$$

Dimension check:

$$\frac{d}{dt} (kg \cdot \frac{m}{s}) = kg \cdot \frac{m}{s^2} = N, \quad qE(A \cdot s \cdot V \cdot m^{-1}) = qE(A \cdot s \cdot \frac{kg \cdot m^2}{A \cdot s^3} \cdot m^{-1})$$

Main logic:

$$\begin{aligned} \frac{d}{dt} \frac{m\vec{v}(t)}{\sqrt{1 - \frac{v^2}{c^2}}} &= q\vec{E} + q[\vec{v}(t) \times \vec{B}] \Leftrightarrow \\ \frac{m\vec{v}(t+dt)}{\sqrt{1 - \frac{v(t+dt)^2}{c^2}}} - \frac{m\vec{v}(t)}{\sqrt{1 - \frac{v(t)^2}{c^2}}} &= q(\vec{E} + [\vec{v}(t) \times \vec{B}])dt \\ \frac{\vec{v}(t+dt)}{\sqrt{1 - \frac{v(t+dt)^2}{c^2}}} &= \frac{q}{m}(\vec{E} + [\vec{v}(t) \times \vec{B}])dt + \frac{\vec{v}(t)}{\sqrt{1 - \frac{v(t)^2}{c^2}}} \\ \vec{v}(t+dt) &= \left(\frac{q}{m}(\vec{E} + [\vec{v}(t) \times \vec{B}])dt + \frac{\vec{v}(t)}{\sqrt{1 - \frac{v(t)^2}{c^2}}} \right) \sqrt{1 - \frac{v(t+dt)^2}{c^2}} \end{aligned}$$

Denote:

$$\vec{A} = \frac{q(\vec{E} + [\vec{v}(t) \times \vec{B}])}{m}dt + \frac{\vec{v}(t)}{\sqrt{1 - \frac{v(t)^2}{c^2}}}$$

$$\vec{v}(t + dt) = \vec{A} \sqrt{1 - \frac{v(t+dt)^2}{c^2}}$$

$$\vec{v}(t + dt) = \vec{A} \left(1 - \frac{v(t+dt)^2}{c^2}\right) \Rightarrow \vec{v}(t + dt) + \vec{A} \frac{v(t+dt)^2}{c^2} = \vec{A}$$

$$\vec{v}(t + dt) \left(1 + \frac{\vec{A}}{c^2}\right) = \vec{A} \Rightarrow \vec{v}(t + dt) = \frac{\vec{A}}{1 + \frac{\vec{A}}{c^2}}$$

$$\vec{v}(t + dt) = \frac{\vec{A}}{\sqrt{1 + \frac{\vec{A}^2}{c^2}}}, \text{ where } v^2(t) = v_x^2(t) + v_y^2(t) + v_z^2(t)$$

