About the Mathematics: The *Art* of Writing Story Problems

Elements to consider when crafting or adapting real-world problems for use in the classroom for instruction

Overview

Crafting word problems for your students is a way *to assist students in mathematizing their immediate world*. The curriculum becomes more *culturally responsive.* But crafting word problems needs to be focused and intentional. The larger *math standards become a targeted goal* around which the mathematics embedded in the problems are selected. The problems a*re intentionally designed* to draw out potential conversations in which students can engage. Below are the thought processes that many teachers use in shaping such instructional tasks. That decision-making process consists of identifying the following attributes:

- Mathematical Objectives
- Context
- Language Within A Problem (Academic language)
- Number Choices

Mathematical Objective of Word Problems

It is the teacher's choice which problem type to select. The math standards are the guidepost around which decisions are made. Reasons for which type of problem to select can be as follows:

Timing

- Explore *a current* instructional topic
    - Example: You're in a unit on multiplication and division; addition and subtraction
- Want to explore *an upcoming idea* to begin laying the groundwork
    - Example: You want to "prime the pump" for an upcoming unit on fractions; select a range of problem types that include *easy* fraction *combinations* to work with *making the mathematics accessible*

Mathematical Objective

- Want to engage students in thinking about a mathematical idea from different perspectives
    - Example: Place value – you might approach it from a multi-step multiplication and addition problem, e.g., 4x10 + 6; a measurement division scenario, e.g., 64 ÷ 10; a comparison problem, e.g., You have 124, I have 134, How many more do I have than you?… A Join change unknown, e.g., You have 34, How much to get to 50?
    - *The number choices* selected for the students will make the difference in *how the student explores the mathematical concept*
- Want student to keep past mathematical skills active
    - Example: You are in a geometry unit and you select problems for an independent station or a morning message that is intended to keep ideas fresh from the past unit on multi-digit addition and subtraction

Contexts for problems

Themes and contexts in which to cast a problem for students to solve can be derived from a variety of sources. Among them are:

- Current literature from individual student interests (sports, things they collect, toys or games, the latest craze)
- Themes occurring in other subjects that you are teaching or are being taught elsewhere in the building such as in science, social studies, literacy
- Life at-large such as the neighborhood, the school cafeteria or office, themes from nature, high-interest movies or cultural events, holidays (within culturally sensitive boundaries)
- Workplace scenarios (factory orders, store displays, bakery orders, shopping, or purchasing situations…)

The intent is to use contexts that students can immediately relate to or are already a part of their expanding world. This is one way that is meant by making curriculum culturally responsive. It allows students to visualize the scenario and to mathematize the context in a more accessible manner.

Student Names

Research has shown that simply placing a student's name into the context of a problem will increase engagement and the capacity to visualize the mathematics.

Language within a problem

Being alert to and intentional about your word choice and phrasing is surprisingly important to the various range of learners with whom you work. Context, cultural responsiveness, vocabulary range, tense usage, each contribute to the capacity of students to visualize and mathematize a scenario.

The context of a problem can be made easier or more difficult merely by altering the language used to convey the information. Look at the following two problems and think about how the language used may make the task easier or harder for a student.

> *Sarah has 12 dollars. How many more dollars does she need to get to 25 dollars?*
>
> *Sarah has 12 dollars. She got some more money for her birthday. Now she has 25 dollars. How much money did she get for her birthday?*

Both problems are identical in structure. They are both a join, change unknown problem. But for younger children or for early English Language Learners, the second version is harder as "*got some more money*" is less direct than the initial "*does she need to get*." Now consider this third version of the same scenario.

> *Sarah now has 25 dollars because her father just gave her some money for her birthday. If she had 12 dollars to start with, how much did her father give her as a present?*

This is still a join, change unknown task but the order of information is scrambled making the reading comprehension more difficult.

The language used can also be made simpler. This is a technique when re-reading/verbally re-stating a problem when working with students initially confused about a problem, English language learners, or special education students with language processing issues. The information is the same but stripped of excess vocabulary.

Sarah has 12. How many to get to 25?

The language used to describe the numbers in a problem can be made more obscure. What if someone bought *three dozen eggs*? What if the same thing happened *every day for a week*? What if *everyone in the classroom* needs three of something? (Note that 'three' is in text rather than in numeral form.) To solve these tasks, I need to know that one dozen is twelve things, that a week is seven days, or how many students are currently in class, possibly even having to adjust for any absentees, and/or if the teacher or other adults in the room need to be included with the use of the phrase 'everyone in the classroom.'

Examples of Problems Written at Different Levels of Difficulty

Straightforward Writing	Simplified	More Complex Wording	Scrambled Order
Join, Change Unknown Seventeen sparrows were feeding on the ground. More sparrows flew down to join them. Now there are 25 sparrows on the ground. How many just arrived?	Seventeen sparrows. How many to have 25?	Seventeen sparrows were feeding on the ground. Suddenly, several more joined them. Now there are 25 sparrows together eating on the ground. How many sparrows flew down to join the others?	Suddenly there are 25 sparrows on the ground. Just a moment before, there were 17 sparrows eating away. How many just arrived?
Separate, Result Unknown There were 16 sparrows at the bird feeder. 7 flew away. How many are still at the feeder?	16 sparrows. 7 flew away. How many are left?	One morning I was looking out the window when I noticed sixteen sparrows were eating at the bird feeder. Suddenly seven of the sparrows had enough and flew away. How many sparrows were left eating away at the birdfeeder after the other flew away?	Seven sparrows suddenly flew away from the birdfeeder. There had been 16 sparrows eating. How many sparrows are left at the feeder?

Vocabulary

Beyond the language issues just discussed, writing problems is an opportunity to have students explore vocabulary which you wish to have them develop. This vocabulary might be mathematical; it might be out of literary contexts. There is research that shows that more elaborated story contexts are easier for students to visualize mathematically than your basic classic textbook word problems. So use a storytelling voice when crafting problems.

Action ←→ Non-Action - Verbs matter

For early learners and for learners just grappling with a new concept (including emergent English Language Learners), the more active the context the easier it is for students to visualize the mathematics. Thus your joining and separating problem structures (active contexts) are easier initially than non-active problem structures (Part-Part-Whole - Whole Unknown, Compare Problems).

Tense

Present tense is easier to visualize than past tense. Past tense is easier to visualize than future tense for early learners, those new to a concept, those new to the language. Active voice is easier than passive voice. Be intentional about what tense that you use.

Reading Comprehension

When writing, and subsequently unpacking a word problem with students before they start working, *stress reading comprehension to interpret what the scenario is asking* so students can *visualize the setting in order to mathematize the scenario.* While you may be emphasizing one problem type over others (You're in a subtraction unit so subtraction problems might predominate) students should experience enough of a mix of problem types that they truly have to stop and consider what a problem is asking rather than assuming what is being asked because the previous four problems on the worksheet where all the same problem structure. For instance, there are three types of subtraction problems that require different solution strategies. When in a subtraction unit, make sure all types of subtraction problem types are mixed together. One could also present a measurement division task as repeated subtraction is an early solution to the problem. A compare, difference unknown problem can be solved using various forms of subtraction to solve it but not necessarily. Mixing in non-subtraction problems keeps students on their toes to make sure they are comprehending the contexts before selecting a strategy.

Having students *retell* important information from the story provides you with evidence that students are comprehending the language and context in order to go about solving the problem. That way, you can decide if a child is having a mathematical issue or a language issue.

Avoid Keywords

Life does not come in keywords. It is dangerous to have kids come to rely on such habits. Many tests are specifically screened so that keywords cannot be relied upon. Take the word "gave." As a keyword, what operation would be indicated if you were relying upon keyword habits? Now consider the following problem:

> *The Rainbow Fish gave 14 of her scales away to her new friends. Then she gave 28 more scales to friends. How many scales did she give away?*

An over-reliance on 'give' meaning 'to subtract' would result in the wrong answer.

A Focus on Phrases

Take the phrase 'how many.' These two words are not enough to interpret what is occurring in a story. More words are needed to comprehend. Is the story asking 'How many *altogether*?' 'How many *more than*…?' 'How many *to get*…?' Each of these phrases carries a different meaning from the other. Reading comprehension requires a more complete sense of the context than just knowing keywords.

Number Choices

Once the problem context has been written, selecting the numbers for the students to use shapes the mathematical ideas you want them to grapple with. Consider the same problem but with different numbers.

> *There are two plates of cookies on the counter. One has 6, the other 8. How many more cookies does one plate have than the other?*
>
> *There are two plates of cookies on the counter. One has 10, the other 14. How many more cookies does one plate have than the other?*
>
> *There are two plates of cookies on the counter. One has 26, the other 36. How many more cookies does one plate have than the other?*

Now consider the presentation of the same problem.

There are two plates of cookies on the counter. One has ___, the other ___. How many more cookies does one plate have than the other?

(6, 8) (10, 14) (26, 36)

Each of the number sets changes, not the problem context, but the mathematical ideas being explored simply by the choice of numbers. The first may be for those still grappling with the problem structure of comparing to find the difference between two numbers. If that is the case the numbers are accessible by being under ten and only two numbers apart. The second set provides for the exploration of place value concepts and how a multidigit number is composed and decomposed by place value terms. The third set also has a focus on place value ideas but in this instance, the focus is on comparing values in the tens place.

The fourth version of the problem above leaves where numbers occur in the scenario blank with number sets listed below. This is one way to *differentiate your lesson* by providing numbers that are *'just right'* or *challenging'* for your students. If a student looks at a set of numbers and knows the answer right away, those numbers are *'too easy.'* If a student looks at a set of numbers and doesn't know where to even start, then those numbers are *'frustrating.'* The key aspect of your decision-making process is to *know your students' capabilities* in order to select *range of number sets* that allow your students to be successful in grappling with the mathematics and, in a heterogeneous setting, *hear the strategies used by others* to gain *a sense of the next level strategy and levels of efficiency.*

The number you choose should be selected based on the following.

- Do they focus on your mathematical objective?
- If the math objective is a relatively new one, are the numbers accessible to work with in order for students to attempt solving the problem? If the objective is more familiar, do the number combinations support the students to move to the next level of complexity?
- Numbers under 10, close together
    - Examples: JRU***** 3, 4; SRU 7, 2; JCU 4, 6; CDU 6, 8
- Numbers under 10, greater span
    - Examples: JRU 2, 7; SRU 7, 5; JCU 3, 8; CDU 4, 9
- Numbers Spanning 10
    - Examples: JRU 7, 5; SRU 13, 5; JCU 8, 12; CDU 8, 14
- Numbers over 10
    - Close together, Greater span
- Numbers spanning 20
- Double-digit numbers
    - No configuring a new 10 or decomposing a 10
        - Example: JRU 24, 35; SRU 37, 26
    - Requiring configuring or decomposing a 10
        - Example: JRU 26, 35; SRU 36, 27, SRU 60, 27
    - Over/Under 100: JRU 46, 55; SRU 100, 27; SRU 125, 36
- Fractions & Decimals
    - Fractions: Common Denominators
    - Fractions: Uncommon Denominators
    - Fractions: Mixed Numbers; Improper Fractions
- Decimals
    - Mixed numbers, the same number of decimal places
        - Example: JRU $3.15, $2.38
    - Mixed numbers, differing decimal places

- Example: JRU 4.6, 10.04

*** Codes: JRU** - Join Result Unknown; **SRU** - Separate Result Unknown; **JCU** - Join Change Unknown; **SCU** - Separate Change Unknown; **JSU** - Join Start Unknown; **SSU** - Separate Start Unknown; **CDU** - Compare Difference Unknown

You are in control of selecting the numbers. The numbers will propel particular strategies as well as particular mathematical ideas. Who you have present with which number set and with which strategy is up to you in how you wish to have the public sharing conversation unfold. These are all a part of the professional instructional decision-making that is in your control.

Go forward and design!