

*Generalization: The sum of any three consecutive whole numbers is divisible by three.*

**Proof by example:**  $3 + 4 + 5 = 12$  and 12 is divisible by 3  
 $2 + 3 + 4 = 9$  and 9 is divisible by 3  
 $11 + 12 + 13 = 36$  and 36 is divisible by 3



First I modelled 3 consecutive numbers. Any three consecutive whole numbers will have this stair-step shape because consecutive numbers always have 1 more than the number before. Though 3, 4, and 5 are showing, these towers could represent any three consecutive numbers.



Second I pushed them together to model the operation of adding them together.



Instead of putting them all in one tower, I moved one cube from the green tower onto the purple tower. Now you can see the sum represented as three equal towers. Since we see the total can be broken up into three equal groups, we know the total is divisible by three.

This would happen for any three consecutive numbers. You would just picture more cubes on each tower. No matter how many more cubes you added at the bottom of the towers, you'd always be able to move one cube over from the third number onto the first number, creating three equal groups.

**Representation-based proof:**

**Proof by algebraic notation and the laws of arithmetic:**

Let  $n =$  a whole number

Three consecutive numbers can be written as  $n$ ,  $n + 1$ , and  $n + 2$ .

The sum of these consecutive numbers is  $n + n + 1 + n + 2$ .

Using the commutative property of addition, this can be rearranged as  $n + n + n + 1 + 2$ .

This can then be simplified to  $3n + 3$  or  $3(n + 1)$  by combining like terms and thinking about the distributive property.

If the sum of any three consecutive whole numbers is always  $3(n + 1)$ , this means that three is always a factor of the sum, proving that the sum is divisible by 3.