Roll No…………………….. **Total No. of Printed Pages: [01]**

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B.Sc. (Hons.) Math (Semester – 4th)

LINEAR ALGEBRA

Subject Code: BMATS1402

Paper ID: [19131217]

Time: 03 Hours **Maximum Marks: 60**

Instruction for candidates:

1. Section A is compulsory. It consists of 10 parts of two marks each.
2. Section B consist of 5 questions of 5 marks each. The student has to attempt any 4 questions out of it.
3. Section C consist of 3 questions of 10 marks each. The student has to attempt any 2 questions.

Section – A **(2 marks each)**

Q1. Attempt the following:

a. What do you mean by linear span of a set?

b. Give a basis for C.

c. Define kernel of a linear transformation.

d. If $S=\{(1,0,0),(2,0,0),(3,0,0)\}$ in $V_3(R)$ then L(S) is ....

e. Find the Eigen values of matrix A= $\begin{bmatrix} 3 & 0 & 0 \\ 5 & 4 & 0 \\ 3 & 6 & 1 \end{bmatrix}$.

f. If $f(x_1,x_2)=x_1^2-6x_1x_2-5x_2^2$, find the symmetrical matrix A whose quadratic equation is $f(x_1,x_2)$.

g. A linear transformation T: V→W is called .... if the transformation is one-to-one.

h. Let V be a vector space over F and W be a subspace of V. Then V/W is a .... of V by W.

i. In R[x], let $S=\{1,x,x^2,x^3\}$. Then L(S)= ....

j. Prove that $S=\{(1,0,0),(0,1,0),(1,1,1)\}$ is a basis for $V_3(R)$.

Section – B **(5 marks each)**

Q2. Let T: V→W be a Linear Transformation. Then prove that $\dim V = rank\,T + nullity\,T$.

Q3. Let $U=\{(x_1,x_2,x_3)\in R^3 / x_1+x_2-2x_3=0\}$ and $W=\{(x_1,x_2,x_3)\in R^3 / x_1-3x_2+2x_3=0\}$ be subspaces of R3. Find a basis and dimension of $U, W\ and\ U\cap W$.

Q4. Let V be a vector space over a field F. Let $S,T\subseteq V$, then prove that (i) $S\subseteq T\Rightarrow L(S)\subseteq L(T)$ (ii) $L(S\cup T)=L(S)+L(T)$.

Q5. Prove that subset of a Linearly independent set is Linearly independent.

Q6. Reduce the quadratic form $x_1^2+4x_1x_2+4x_1x_3+4x_2^2+16x_2x_3+4x_3^2$ to the diagonal form using Lagrange's method.

Section – C **(10 marks each)**

Q7. Prove that any two basis of a finite dimensional vector space have same number of elements.

Q8. Let V be a finite dimensional vector space over a field F. Let W be a subspace of V .

Prove that (i) $\dim W \le \dim V$ (ii) $\dim \frac{V}{W} = \dim V - \dim W$.

Q9. Prove that any vector space of dimension n over a field F is isomorphic to $V_n(F)$.