PDM: 13.4: Properties of Definite Integrals

Because definite integrals can be interpreted as areas, properties of area can be used to deduce properties of these integrals.

➔ If *f* is a continuous function on the interval $[a,\ c]$ and $a < b < c$, then

$$\int_a^b f(x)\,dx + \int_b^c f(x)\,dx = \int_a^c f(x)\,dx$$

➔ If *f* is a continuous function on the interval $[a,\ c]$ and $a < b < c$, then

$$\int_a^c f(x)\,dx - \int_a^b f(x)\,dx = \int_b^c f(x)\,dx$$

1) Use subtraction to calculate the integral:

$$\int_5^8 \frac{1}{2}x\,dx$$

➔ If *f* and *g* are a continuous function on the interval from *a* to *b*, then

$$\int_a^b [f(x) + g(x)]\,dx = \int_a^b f(x)\,dx + \int_a^b g(x)\,dx$$

2) Use subtraction to calculate the integral:

$$\int_0^2 [3x - \frac{1}{2}x]\,dx$$

➔ If f is a continuous function on the interval from a to b, and c is a real number, then

$$\int_a^b c \cdot f(x)\,dx = c\int_a^b f(x)\,dx$$

3) Use multiplication to calculate the integral:

$$\int_0^2 3 \cdot (x)\,dx$$

➔ If f is a continuous function on the interval from a to b, then

$$\int_a^b f(x)\,dx = -\int_b^a f(x)\,dx$$

4) Calculate the integral: $\int_8^0 \frac{1}{2}x\,dx$

 Name:________________________

Homework

1. Suppose g is a continuous function such that $\int_{-3}^{-1} g(x)\,dx = 2$ and $\int_{-3}^{0} g(x)\,dx = 6$, and $\int_{0}^{2} g(x)\,dx = 5$. Find the following:

   a. $\int_{-3}^{2} g(x)\,dx$

   b. $\int_{-1}^{0} g(x)\,dx$

   c. $\int_{-1}^{2} g(x)\,dx$

2. Graph $f(x) = sin(x)$ and $g(x) = 5\,sin(x)$ on the the interval $[0, \pi]$ and use these graphs to explain why $\int_{0}^{\pi} g(x)\,dx = 5\int_{0}^{\pi} f(x)\,dx$.

3. Use properties of integrals to write each expression as a single integral.

a. $\int_{0}^{14} x^2 \, dx - \int_{0}^{6} x^2 \, dx =$

b. $3 \int_{a}^{b} sin(x) \, dx - \int_{a}^{b} sin(x) \, dx =$

c. $\int_{3}^{4} log(x) \, dx + \int_{3}^{4} log(x^2) \, dx =$

(Hint: use properties of logarithms)

4. Use area formulas to evaluate the following integrals:

a. $\int_{0}^{a} 7 \, dx$

b. $\int_{0}^{a} x \, dx$

c. $\int_{0}^{a} (3x + 7) \, dx$

Hint: use parts a and b to help evaluate

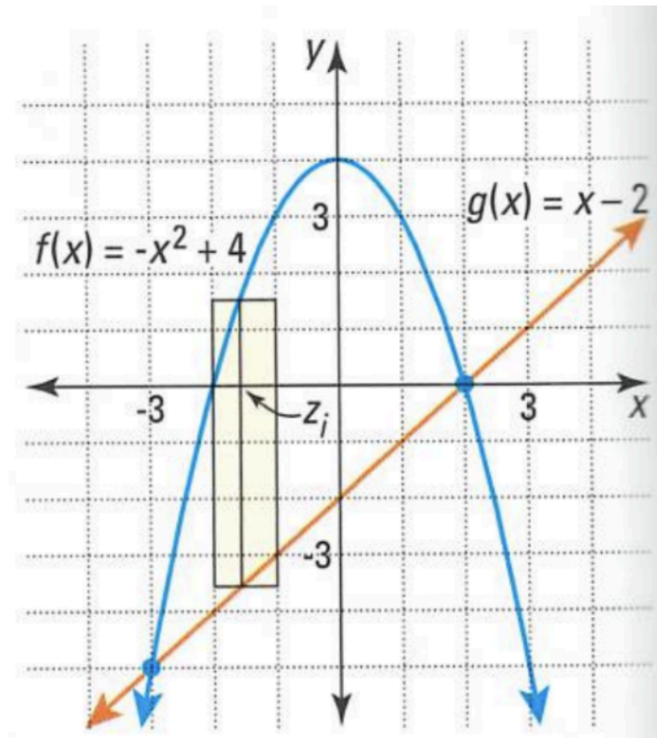
d. Verify your answer to part c by evaluating $\int_{0}^{4} (3x + 7) \, dx$

5. Fill in the blank using a single interval: $\int_a^b f(x)\,dx \;-\; \int_a^b g(x)\,dx \;=$______________

6. Interpret your answer to question #5 in terms of area.

7. Consider the region enclosed by the parabola $f(x) = -x^2 + 4$ and the line $g(x) = x - 2$.

   a. If the shaded rectangle at the right has width of Δx, write its area in terms of $f(z_i)$, $g(z_i)$, and Δx



   b. Write a Riemann sum that approximates the area of the region between the graphs of f and g.

   c. Write an integral which gives the exact area of the region.

8. Find the value of the integral to the nearest hundredth

a. $\int_{-5}^{5} \sqrt{25 - x^2}\, dx$

b. $\int_{0}^{7} \sqrt{49 - x^2}\, dx$

9. Consider the region bounded by the lines $x = 0$, $x = 5$, and $y = x - 4$.

a. Sketch the region on the axes provided.

b. Write an integral that would evaluate the area of this region.

c. Evaluate the integral.