UNIT: V

SINGLE PHASE TRANSFORMERS

TOPICS:-

1.1 Introduction **1.2 Working principle and Construction**

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1.5 E.M.F equation **1.6 Minimization of hysteresis and eddy current losses**

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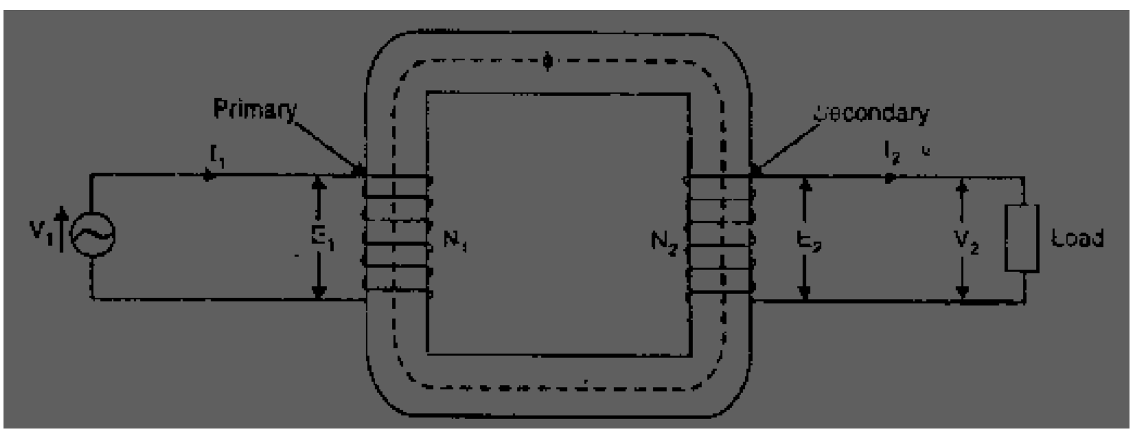
❖ *1.1 Introduction: -*

The main advantage of alternating currents over direct currents is that, the alternating currents can be easily transferable from low voltage to high or high voltage to low. Alternating voltages can be raised or lowered as per requirements in the different stages of electrical network a generation, transmission, distribution and utilization. This is possible with the static device called transformer. The transformer works on the principle of mutual induction. It transfers an electrical power from one circuit to other when there is no electrical connection between the two circuits.



Thus we can define transformer as:

> ***Transformer is a static piece of apparatus, which transfers power from one circuit to other circuit with desired change in voltage and current, without any change in frequency.***

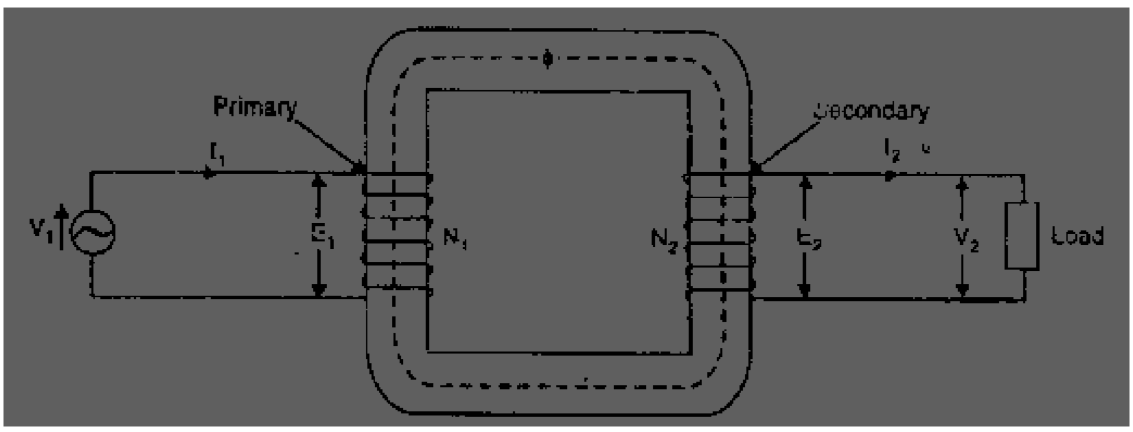
❖ *Basic definitions:-*

Mutual flux: - When current passes in primary winding of transformer a magnetic flux is produced. A pat of this flux links with the secondary winding by mutual induction effect between them which is called *mutual flux*.

Leakage flux: - The total flux produced by primary winding does not link with the secondary winding or the primary winding itself. Some of the flux is lost in the air gaps. This flux is called as *magnetic leakage* or *leakage flux*.

Magnetizing reactance: - The flux present in the transformer unit creates the reactance effect and the reactance of the conductor used for the winding is called *magnetizing reactance*.
Leakage reactance: - As the transformer is wound with two inductive coils, which have resistance as well as reactance. The reactance offered by the winding is called *leakage reactance*, X_1 is the leakage reactance of primary winding and X_2 is the leakage reactance of the secondary winding. Total reactance may be read either from primary side or secondary side.

- ***1.2 Working principle and Construction*** *:-*



Principle of Working*: -*

Transformer works on the principle of **Mutual Induction** between two coupled coils. The principle of mutual induction states that when two coils are inductively coupled and if current in one coil is changed uniformly then an emf gets induced in the other coil. The emf drives a current, when a closed path is provided to it.

The physical basis of transformer is, it consists of two coils having mutual induction between them. One coil is connected to the source of AC supply and the other is connected to the load. The alternating flux is developed in the first coil and in the second coil also due to the principle of *Mutual induction*.

The winding which is connected to AC supply is called *primary winding* and the winding which is connected to the load is known as *secondary winding*.

The primary winding has N_1 number of turns and the secondary winding has N_2 number of turns. When the primary winding is excited by an alternating voltage, it circulates an alternating current. This current produces an alternating flux (Φ) which completes its path through common magnetic core as shown in dotted line. Thus an alternating flux links with the secondary winding. As the flux is alternating according to faradays law of an electromagnetic induction, mutually induced emf gets developed in the secondary winding. If load is connected to the secondary winding, this emf drives a current through it.

❖ In brief, a transformer is a device that

1. Transfers electric power from one circuit to another
2. It does so without a change of frequency
3. It accomplishes this by electromagnetic induction and
4. Where the two electric circuits are in mutual inductive influence of each other.

❖ Can **Dc** supply be used for transformers?

The Dc supply cannot be used for transformers. The transformer works on the principle of mutual induction, for which current in one coil must change uniformly. If dc is given, the current will not change due to constant supply and transformer will not work.

For Dc the inductive reactance X_L is zero as dc has no frequency. So total impedance of winding is very low for dc. Thus winding will draw very high current if dc supply is given to it. This may causes the burning of windings due to extra heat generated and may cause permanent damage to the transformer.

❖ ***Construction:-***

The main components of transformer are:

1. Core	2. Windings	3. Conservator tank
4. Bushings	5. Breather	6. Radiator

✔ **1.Core**:-

In all types of transformer the core is constructed of transformer sheet field laminations assembled to provide a continuous magnetic path with a minimum of air gap included. The core of a transformer is made up of sheet steel doped with 4% of silicon, which is also called as silicon sheet. The silicon sheets of thickness 0.35mm to 5mm depending upon the requirement. Since the core is cut by the alternating flux produced by the flow of alternating currents, this induces an emf in the core, which leads to the flow of eddy currents. This eddy current circulates in the core and gives rise to I2R loss in the core. To reduce the eddy current loss caused by the flow of eddy current, the core is built up of laminations. The purpose of the core is to provide low reluctance path between the two windings, so that the flux caused by one winding will fully link the other coil.

✔ **2.Windings**:-

A single phase transformer has two windings. The winding connected to Ac source is called primary winding and the one connected to load is called secondary winding.

✔ **3. Conservator tank**:-

The oil in the transformer main tank is subjected to expand or contract due to the variations in load. While undergoing expansion and contraction, the oil is subjected to heat. The function of the conservator tank is to help the oil in the tank to settle down by expansion whenever heavy loads appear. In the absence of conservator tank high pressures are developed, that may cause the bursting of transformer tank.

✔ **4. Bushings**:-

Connections from the transformer windings are brought out by means of bushings. The function of the bushings is to give proper insulation for the output leads. Bushings are fixed on the transformer tank. Bushings made up of porcelain are available up to 33Kv. Capacitor or oil filled type of bushings are used for voltage above 33Kv.

✔ **5. Breather**:-

To avoid the entry of moisture and dust particles into the conservator tank a breather is used. Silica jell is used in the breather to absorb the moisture, When the air enter into the tank. Silica jell is of two types 1. Indicating 2. Non-indicating type:

Indicating: Initially blue in color changes into pink with the absorption of moisture. It can be reconditioned by heating.

Non-indicating: White in color.

✔ **6. Radiator**:-

Thin metal structures are connected round the transformer tank which acts as a heat sink. The function of the radiator is to cool the transformer tank gradually.

❖ ***1.3 Types of Transformers*** :-

The classification of the transformers is based on the relative arrangement or Disposition of the core and the windings. There are 3main types of transformers.

Which are: 1. Core type 2. Shell type and 3. Berry type

i)Core type:-

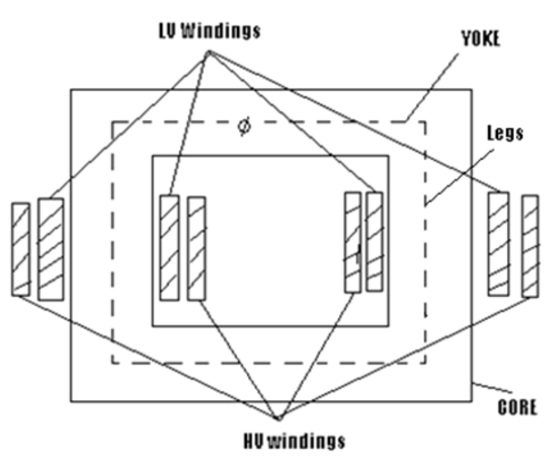
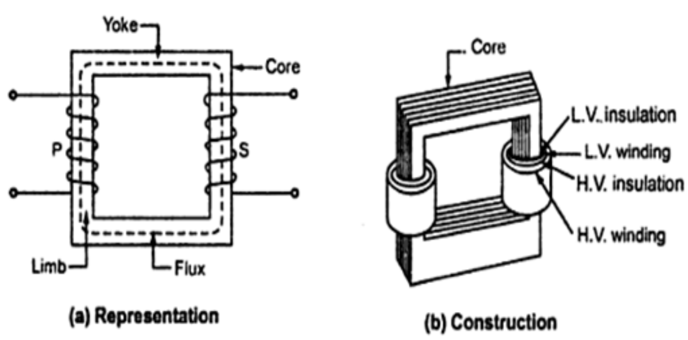


Fig: A

It has a single magnetic circuit. The core is rectangular having two limbs. The winding circles the core. The coils used are of cylindrical type. As mentioned earlier, the coils are wound in helical layers with different layers insulated from each other by paper or mica. 3oth the coils arc placed on both the limbs. The low voltage coil is placed inside near the ore while high voltage coil surrounds the low voltage coil. Core is made up of large Lumber of thin laminations.

As the windings are uniformly distributed over the two limbs the natural cooling is more effective. The coils can be easily removed by removing the laminations of the top yoke, for maintenance.



Core type transformer

ii) Shell type:-

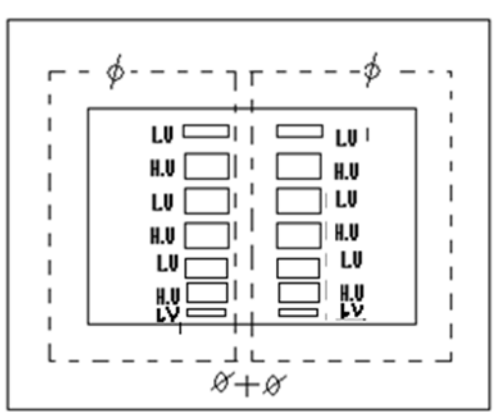
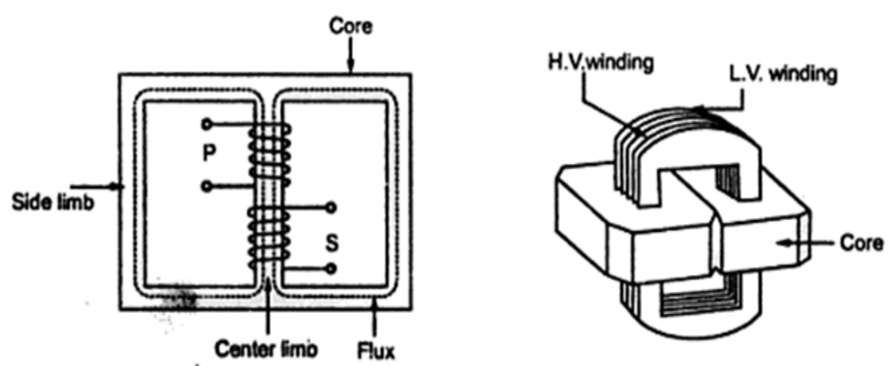


Fig: B

It has a double magnetic circuit. The core has three limbs. Both the windings are placed on the central limb. The core encircles most part of the windings. The coils used are generally multilayer disc type or sandwich coils. As mentioned earlier, each high voltage coil is in between two low voltage coils and low voltage coils are nearest to top and bottom of the yokes.

The core is laminated. While arranging the laminations of the core, the care is taken that all the joints at alternate layers are staggered. This is done to avoid narrow air gap at the joint, right through the cross-section of the core. Such joints are called overlapped or imbricate joints. Generally for very high voltage transformers, the shell type construction is preferred. As the windings are surrounded by the core, the natural cooling does not exist. For removing any winding for maintenance, large number of laminations is required to be removed.

The Fig. shows the schematic representation while the Fig (b) shows the out away view of the construction of the shell type transformer.



(a) Representation (b) Construction

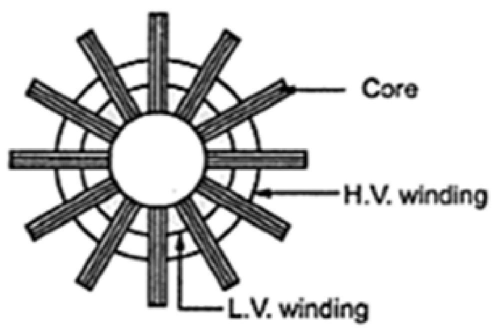
Shell type transformer

iii) Berry type:-

This has distributed magnetic circuit .The number of independent magnetic circuits are more than Its core construction is like spokes of a wheel. Otherwise it is symmetrical to that of shell type.

The transformers are generally kept in tightly fitted sheet metal tanks. The tanks are constructed of specified high quality steel plate cut, formed and welded into the rigid structures. All the joints are painted with a solution of light blue chalk which turns dark in the presence of oil, disclosing even the minutest leaks. The tanks are filled with the special insulating oil. The entire transformer assembly is immersed in the oil. The oil serves two functions: i) Keeps the coils cool by circulation and ii) Provides the transformers an additional insulation.

The oil should be absolutely free from alkalies, sulphur and especially from moisture. Presence of very small moisture lowers the dielectric strength of oil, affecting its performance badly. Hence the tanks are scaled air tight to avoid the contact of oil with atmospheric air and moisture. In large transformers, the chambers called breathers are provided. The breathers prevent the atmospheric moisture to pass on to the oil. The breathers contain the silica gel crystals which immediately absorb the atmospheric moisture. Due to long and continuous use, the sludge is formed in the oil which can contaminate the oil. Hence to keep such sludge separate from the oil in main tank, an air tight metal drum is provided, which is placed on the top of tank. This is called conservator.



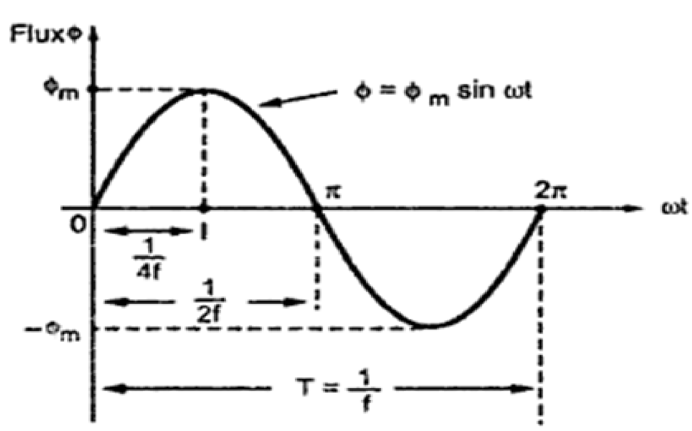
Berry type transformer

❖ **Comparison between core-type and shell-type transformers**:

Core type	Shell type
1. The core type transformer is as shown in fig **A.**	1. The shell type transformer is as shown in fig :**B**
2. In the core type transformer the windings surrounds a considerable part of steel core.	2. In the shell type the steel core surrounds a major part of windings.
3. For the given output and voltage rating it requires less iron but more conducting materials.	3. For the given output and voltage rating it requires high iron and high conducting material.

4. It is two legged transformers. 5. The vertical portion of the core is usually called LEGS or LINKS and horizontal portion are called YOKE. 6. In this type, to reduce the leakage flux placing half of the LV windings on one leg and other half are placed on second leg. 7. Concentric coils are used. 8. In order to minimize the insulation requirement LV windings is placed nearer to the steel core.	4. It is three legged transformer. 5. The vertical portion of the core is usually called LEGS or LINKS and horizontal portion are called YOKE. 6. In this type the LV and HV windings are wound over a central limb under interleaved or sand witched. 7. Sand witched coils are used. 8. Top and bottom LV coils are of half the size of the other LV coils.

***1.5 E.M.F equation**:-*



Sinusoidal flux

When the primary winding is excited by an alternating voltage V_1, it circulates alternating current, producing an alternating flux. The primary winding has N_1 number of turns. The alternating flux Φ linking with the primary winding itself induces an e.m.f. in it denoted as E_1. The flux links with secondary winding through the common magnetic core. It produces induced e.m.f. E_2 in the secondary winding. This is mutually induced e.m.f.

Let us derive the equations for E_l and E_2.The primary winding is excited by purely sinusoidal alternating voltage. Hence the flux produced is also sinusoidal in nature having maximum value of Φm as shown in the Fig.

N_1 = Number of primary winding turns

N_2 = Number of secondary winding turns

f = Frequency of the supply voltage

E_l = R.M.S. value of the primary induced e.m.f.

E_2 = R.M.S. value of the secondary induced e.m.f.

From Faraday's law of electromagnetic induction the average e.m.f. induced in each turn is proportional to the average rate of change of flux.

Average e.m.f. per turn= Average rate of change of flux

Average e.m.f. per turn= $\frac{d\Phi}{dt}$

$$\frac{d\Phi}{dt} = \frac{change\ in\ flux}{Time\ required\ for\ change\ in\ flux}$$

Consider the ¼ th cycle of the flux as shown complete cycle gets completed in 1/f seconds. In ¼ th time period, the change in flux is from 0 to Φm .

$\frac{d\Phi}{dt} = \frac{\Phi m-0}{(\frac{1}{4f})}$ as dt for 1/4th time period is 1/4f seconds

= 4 f Φm wb/sec

Average e.m.f. per turn = 4 f Φm Volts

Form Factor = $\frac{R.M.S\ Value}{Average\ Value}$ = 1.11

R.M.S Value=1.11 x Average value

R.M.S Value of Induced E.M.F per turn =1.11 x 4 f Φm

= 4.444 f Φm

There are N_1 number of primary turns hence the R.M.S. value of induced e.m.f. of primary denoted as E_1 is, E_1= N_1 x 4.44 f Φm volts

While as there are N. number of secondary turns the R.M.S value of induced e.m.f. of secondary denoted E2 is, E_2= N_2 x 4.44 f Φm volts.

The expressions of E_1 and E_2 are called e.m.f. equations of a transformer. Thus e.m.f. equations are,

E_1 = N_1 4.44 f Φm volts

E_2 = N_2 4.44 f Φm volts

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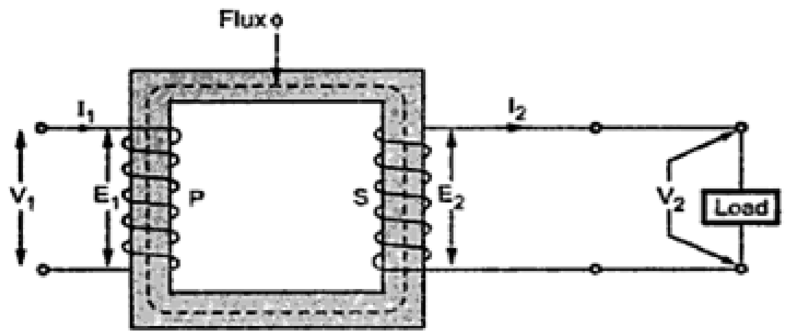
Ratios of a Transformer

Consider a transformer shown in Fig. indicating various voltages and currents.



- ***Voltage Ratio*:**

We know from the e.m.f equations of a transformer that

$E_1 = N_1\ 4.44\ f\ \Phi m$ and $E_2 = N_2\ 4.44\ f\ \Phi m$

Taking ratio of the two equations we get,

The ratio of secondary induced e.m.f to primary induced e.m.f is known as voltage transformation ratio denoted as K.

Thus $E_2 = KE_1$ where $k = N_2/N_1$

1. If $N_2 > N_1$ i.e $K>1$, we get $E_2 > E_1$ then the transformer is called step-up-transformer
2. If $N_2 < N_1$ i.e $K<1$, we get $E_2 < E_1$ then the transformer is called step-down-transformer
3. If $N_2 = N_1$ i.e $K=1$, we get $E_2 = E_1$ then the transformer is called isolation transformer or 1:1 transformer.

- ***Ideal Transformer*:**

A transformer is said to be ideal if it satisfies following properties:

i) It has no losses.

ii) Its windings have zero resistance.

iii) Leakage flux is zero i.e 100% flux produced by primary links with the secondary.

iv) Permeability of core is so high that negligible current is required to establish the flux in it.

*** For an ideal transformer, the primary applied voltage V_1 is same as the primary induced e.m.f E_1 as there are no voltage drops.

Similarly the secondary induced e.m.f E_2 is also same as the terminal voltage V_2 across the load. Hence for an ideal transformer we can write

$$\frac{E_2}{E_1} = \frac{V_2}{V_1} = K$$

- ***Current ratio***:

For an ideal transformer there are no losses. Hence the product of primary voltage V_1 and primary current I_1, is same as the product of secondary voltage V_2 and the secondary current I_2.

So V_1I_1=input VA and V_2I_2=output VA

For an ideal transformer, $V_1I_1 = V_2I_2$

$$\frac{V_2}{V_1} = \frac{I_1}{I_2} = K$$

Hence the currents are in the inverse ratio of the voltage transformation ratio.

1.6 Minimization of hysteresis and eddy current losses:-

Losses in a transformer:-

Transformer, being stationary electrical equipment, does not have rotational or wind age losses. The losses which occur in a transformer are,

1. Copper losses and
2. Iron or Core losses

1. Copper losses:- These losses occur due to the resistance of the transformer windings and appear as heat, resulting in the increase of temperature. If r1 and r2 are the primary and secondary resistances and I1 and I2 are the primary and secondary currents, then the copper loss in the respective windings is given by,

Copper loss in primary=$I_1^2R_1$

Copper loss in secondary=$I_2^2R_2$

Therefore total copper loss in a transformer is given by $(I_1^2R_1+I_2^2R_2)$

2.Iron or Core losses:- Iron losses occurs when the core is subjected to alternating flux. These losses are further divided into two types, namely

a) Hysteresis loss and b) Eddy-current loss

a)Hysteresis loss:- Hysteresis loss occur due to the continuous magnetization and demagnetization of the core. In every cycle of alternating flux, i.e magnetization, the hysteresis loss appears as heat. It occurs due to the property of material.

Hysteresis loss is given by $\mathbf{W_{hys}=K_hVfB_m^X}$ watts

Where **K_h** =Proportionality constant which depends upon the volume and quantity of core material etc.

B_m = maximum flux density in wb/m^2

F= frequency

V=Volume of the magnetic core

X=Steinmetz's constant varies from 1.5 to2. Usually taken as 1.6.

Hysteresis losses cannot be eliminated. However they can be minimized by selecting the core material which posses low hysteresis coefficient. For example silicon steel and nickel-iron alloys.

b) Eddy-current loss:- Alternating flux linking with the core, induces e.m.f in it and causes circulating currents, known as eddy current to flow within the body of the transformer core. These eddy current leads to the power loss given by I^2R and are also called as eddy-current losses. These losses depend upon the length of the eddy current path and the effective resistance.
Eddy current losses are given by the relation, $W_{eddy}=K_{eddy}Vf^2B_m^2$ watts

Where **K_{eddy}** =Proportionality constant which depends upon the volume and quantity of core material etc.

B_m = maximum flux density in wb/m^2

F= frequency

X=Steinmetz's constant varies from 1.5 to2. Usually taken as 1.6.

V=Volume of the core.

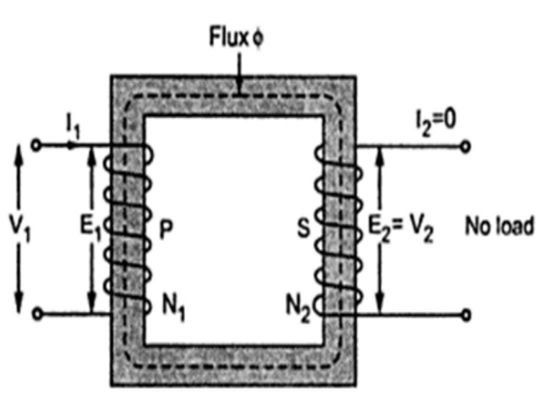
Similarly to the hysteresis losses, it is impossible to eliminate eddy-current losses. However they can be reduced by using laminations of thickness suitable to the capacity of the machine. Using laminations, effective resistance to the eddy currents can be increased and length of the circulating current path is decreased, thereby reducing the eddy-current losses.

1.7 Transformer on NO LOAD and ON LOAD and Phasor diagrams:-

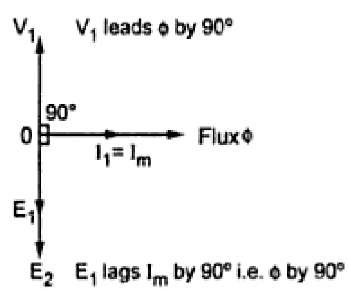
- ***Ideal Transformer on No Load :-***

Consider an ideal transformer on no load as shown in the Fig. The supply voltage is V_1 and as it is a no load the secondary current $I_2 = 0$.

The primary draws a current I_1 which is just necessary to produce flux in the core. As it is magnetizing the core, it is called magnetizing current denoted as I_{Q},. As the transformer is ideal, the winding resistance is zero and it is purely inductive in nature. The magnetizing current I_m is very small and lags V_1 by 90° as the winding is purely inductive. This I_m, produces an alternating flux Φ which is in phase with I_m .



The flux links with both the winding producing the induced e.m.f.s E_1 and E_2 in the primary and secondary windings respectively. According to Lenz's law, the induced e.m.f. opposes the cause producing it which is supply voltage V_1. Hence E_1 is in anti-phase with V1 but equal in magnitude. The induced E2 also opposes V_1 hence in anti-phase with V_1 but its magnitude depends on N_2. Thus E_1 and E_2 are in phase.
The phasor diagram for the ideal transformer on no load is shown in the Fig.



Phasor diagram for ideal transformer on no load

Practical Transformer on No Load :-

Actually in practical transformer iron core causes hysteresis and eddy current losses as it is subjected to alternating flux. While designing the transformer the efforts are made to keep these losses minimum by,

$$\frac{E_2}{E_1} = \frac{N_2}{N_1} = K$$

1. Using high grade material as silicon steel to reduce hysteresis loss.
2. Manufacturing core in the form of laminations or stacks of thin laminations to reduce eddy current loss.

Apart from this there are iron losses in the practical transformer. Practically primary winding has certain resistance hence there are small primary copper loss present.

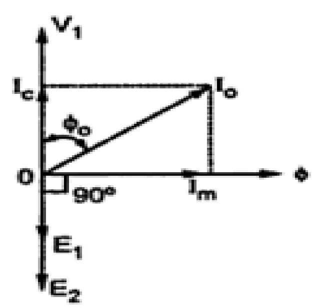
Thus the primary current under no load condition has to supply the iron losses i.e. hysteresis loss and eddy current loss and a small amount of primary copper loss. This current is denoted as I_o.

Now the no load input current I_o has two components:
1. A purely reactive component $\mathbf{I_m}$. Called magnetizing component of no load current required producing the flux. This is also called **watt less component**.

2. An active component $\mathbf{I_c}$ which supplies total losses under no load condition called power component of no load current. This is also called **wattful component** or **core loss component** of $\mathbf{I_{o.}}$

The total no load current $\mathbf{I_o}$ is the vector addition of $\mathbf{I_m}$ and $\mathbf{I_c}$.
In practical transformer, due to winding resistance, no load current $\mathbf{I_o}$ is no longer at 90° with respect to V1. But it lags V1 by angle $\mathbf{\Phi_{o\,which}}$ is less than 90^0. Thus **cos** $\mathbf{\Phi_o}$ is called no load power factor of practical transformer.

The phasor diagram is shown in the Fig. It can be seen that the two components Io are,



Practical transformer on no load

$$\boxed{\mathbf{I_m = I_o \,Sin\Phi_o}}$$

This is magnetizing component lagging V_1 exactly by 90°.

$$\boxed{\mathbf{I_c = I_o \,Cos\Phi_o}}$$

This is core loss component which is in phase with V_1

The magnitude of the no load current is given by,

$$\boxed{\mathbf{I_0} = \sqrt{Im^2 + Ic^2}}$$

$\mathbf{\Phi_o}$ = No load primary power factor angle

While , The total power input on no load is denoted as W. and is given by,

$$\boxed{\mathbf{W_o = V_1 I_o COS\Phi_o = V_1\, Ic}}$$

It may be noted that the current Io is very small, about 3 to 5 % of the full load rated current. Hence the primary copper loss is negligibly small hence I, is called core loss or iron loss component. Hence power input W. on no load always represents the iron losses, as copper loss is negligibly small. The iron losses are denoted as P; and are constant for all load conditions.

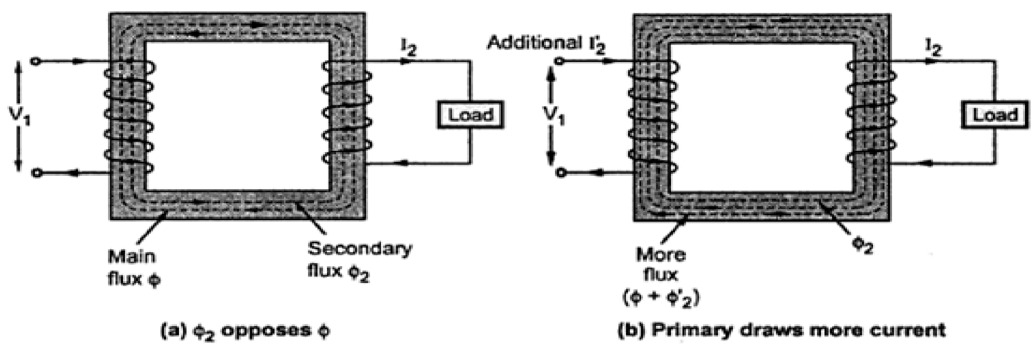
$$\boxed{\mathbf{W_o = V_1 I_o COS\Phi_o = Pi = Iron\ losses}}$$

Transformer on Load

When the transformer is loaded, the current I_2 flows through the secondary winding. The magnitude and phase of I_2 is determined by the load. If load is inductive, I_2 lags V_2. If load is capacitive,1_2 leads V_2 while for resistive load, I_2 is in phase with V_2.

There exists a secondary m.m.f. N_2I_2due to which secondary current sets up its ownFuxΦ2. This flux opposes the main flux Φ0 which is produced in the core due to magnetizing component of no load current. Hence the m.m.f. N_2I_2 is called demagnetizing ampere-turns. This is shown in the Fig.

The flux Φ_2 momentarily reduces the main flux Φo, due to which the primary induced e.m.f. E_l also reduces. Hence the vector difference V_1 - E_l increases due to which primary draws more current from the supply. This additional current drawn by primary is due to the load hence called load component of primary current denoted as I_2' as shown in the Fig.



Transformer on load

This current I_2' is in anti-phase with I_2. The current I_2' sets up its own flux Φ_2' which opposes the flux Φ_2 and helps the main flux Φ. This flux Φ_2' neutralizes the flux $\Phi_2$produced by I_2. The m.m.f. i.e. ampere turns N_1I_2' balances the ampere turns N2I2. Hence the net flux in the core is again maintained at constant level.

Key Point: *Thus for any load condition, no load to full load the flux in the core is practically constant.*

The load component current I_2' always neutralizes the changes in the load. As practically flux in core is constant, the core loss is also constant for all the loads. Hence the transformer is called constant flux machine.

As the ampere turns are balanced we can write,

$$\mathbf{N_2 I_2 = N_1 I_2'}$$

$$\mathbf{I_2'} = \frac{N2}{N1} I2 = \mathbf{KI_2}$$

Thus when transformer is loaded, the primary current I_1 has two Components:

1. The no load current Io which lags V_1 by angle $\mathbf{\Phi_o}$. It has two components $\mathbf{I_m}$ and $\mathbf{I_c}$.

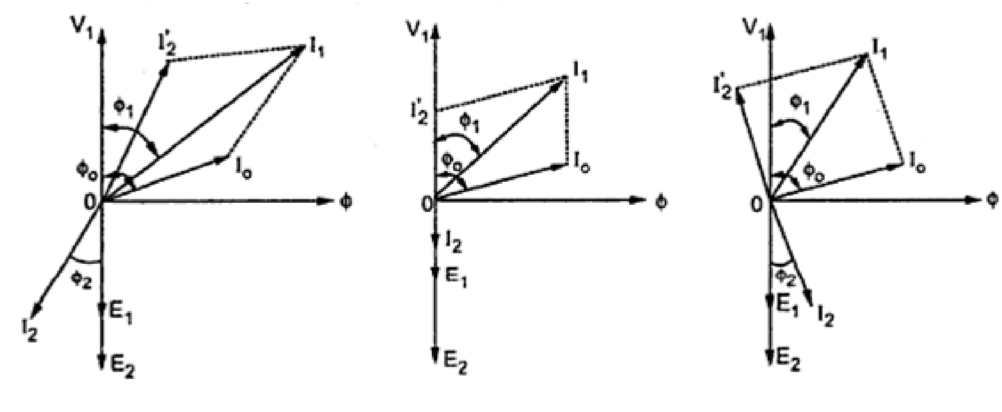
2. The load component $\mathbf{I_2'}$ which is in anti-phase with $\mathbf{I_2}$. And phase of $\mathbf{I_2}$ is decided by the load.

Hence primary current $\mathbf{I_1}$ is vector sum of $\mathbf{I_o}$ and $\mathbf{I_2}$.

$$\overline{I_2} = \overline{I_0} + \overline{I_2'}$$

*Assume inductive load, I_2 **lags** E_2 **by** Φ_2, the phasor diagram is shown in the Fig. (a).
* Assume purely resistive load, I_2 **in phase with** E_2, the phasor diagram is shown in the Fig. (b).
* Assume capacitive load, I_2 **leads** E_2 **by** Φ_2, the phasor diagram is shown in the Fig.(c).

Note : that $I_2'$is always in anti-phase with I_2.



(a) Inductive load (b) Resistive load (c) Capacitive load

Effect of winding resistance:-

Practical transformer windings possess some resistances which not only cause the power losses but also the voltage drops. Let us see, what is the effect of winding resistance on the performance of the transformer?

Let R_1=primary winding resistance in ohms
R_2=secondary winding resistance in ohms

Now when current I_1 flows through primary, there is voltage drop I_1R_1 across the winding. The supply voltage V_1 has to supply this drop. Hence primary induced e.m.f E_1 is the vector difference between $V_1$and I_1R_1.

$\therefore \overline{E_1} = \overline{V_1} - \overline{I_1R_1}$ -------- (1)

Similarly the induced e.m.f in secondary is E_2. When load is connected, current I_2 flows and there is voltage drop I_2R_2. The e.m.f E_2 has to supply this drop. The vector difference between E_2 and I_2R_2 is available to the load as a terminal voltage V_2.

$\therefore \overline{V_2} = \overline{E_2} - \overline{I_2R_2}$ ------------- (2)

The drops I_1R_1 and I_2R_2 are purely resistive drops.

Equivalent Resistance

The resistance of the two windings can be transferred to any one side either primary or secondary without affecting the performance of the transformer. The transfer of the resistances on any one side is advantageous as it makes the calculations very easy. Let us see how to transfer the resistances on any one side.

The total copper loss due to both the resistances can be obtained as,

Total copper loss = $I_1^2R_1 + I_2^2R_2$

$= I_1^2\left[R_1 + \frac{I_2^2}{I_1^2}R_2\right]$

$= I_1^2\left[R_1 + \frac{1}{K_2}R_2\right]$

$\frac{I_2}{I_1} = \frac{1}{K}$ Neglecting No load Current

Now the expression indicates that the total copper loss can be expressed as $I_1^2R_1 + I_1^2\frac{R_2}{K^2}$

This means $\frac{R2}{K2}$ is the resistance value of R_2 shifted to primary side which causes same copper loss with I_1 as R_2 causes with I_2. This value of resistance R_2/K^2 which is the value of R_2 referred to primary is called **equivalent resistance of secondary referred to primary**. It is denoted as **R_2'.**

$$\mathbf{R_2' = \frac{R_2}{K^2}}$$

Hence the total resistance referred to primary is the addition of R_1 and **R_2'** called equivalent resistance of transformer referred to primary and denoted as R_{1e}.

$$\mathbf{R_{1e} = R_1 + R_2' = R_1 + \frac{R_2}{K^2}}$$

This resistance $\mathbf{R_{1e}}$ causes same copper loss with I_1 as the total copper loss due to the individual windings.

Total copper loss$= \mathbf{(I_1^2R_1 + I_2^2R_2) = I_1^2R_{1e}}$

So, equivalent resistance R_{1e} simplifies the calculations as we have to calculate parameters on one side only.

Similarly it is possible to refer the equivalent resistance to secondary winding.

Total copper loss $= \mathbf{(I_1^2R_1 + I_2^2R_2) = I_2^2}\left[\frac{I_1^2}{I_2^2}R_1 + R_2\right] \mathbf{= I_2^2[K^2R_1 + R_2]}$

Thus the resistance $\mathbf{K^2\ R_1}$, is primary resistance referred to secondary denoted as **R_1'**.

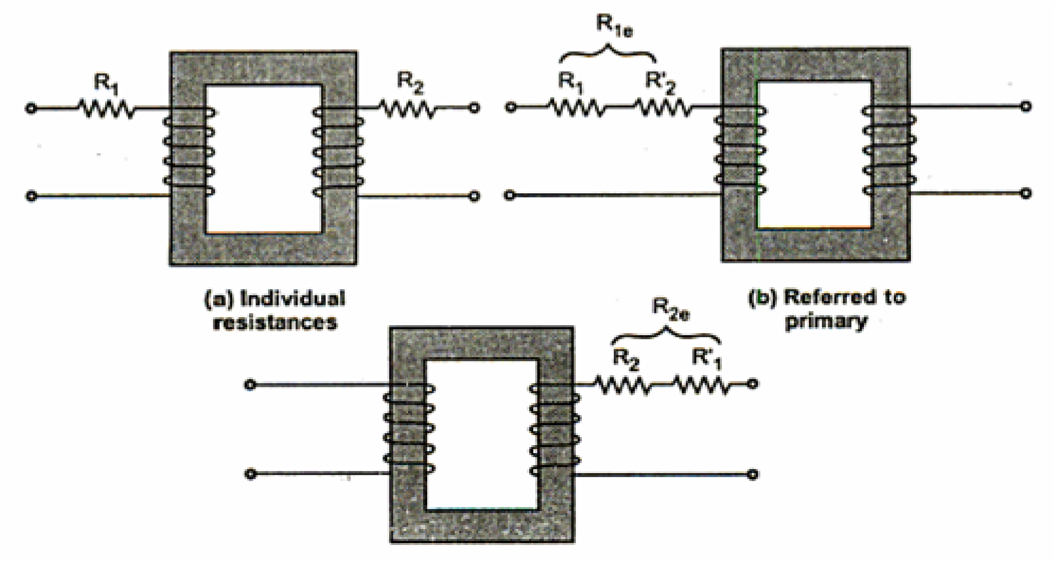
$$\mathbf{R_1' = K^2\ R_1}$$

Hence the total resistance referred to secondary is the addition of $\mathbf{R_2}$ and **R_1'** called equivalent resistance of transformer referred to secondary and denoted as $\mathbf{R_{2e}}$.

$$\mathbf{R_{2e} = R_2 + R_1' = R_2 + K^2\ R_1}.$$

Total copper loss $= \mathbf{I_2^2\ R_{2e}}$.

The concept of equivalent resistance is shown in the following figs a, b, c….



(a) Individual resistances (b) Referred to primary (c) Referred to secondary

Equivalent resistance

❖ ***IMPORTANT NOTE:***

When a resistance is to be transferred from the primary to secondary, it must be multiplied by K^2. When a resistance is to be transferred from the secondary to primary, it must be divided by K^2. Remember that K is N_2/N_1.

The result can be cross-checked by another approach. The high voltage winding is always low current winding and hence the resistance of high voltage side is high. The low voltage side is high current side and hence resistance of low voltage side is low. So while transferring resistance from low voltage side to high voltage side, its value must increase while transferring resistance from high voltage side to low voltage side, its value must decrease.

Effect of Leakage Reactance:-

Till now it is assumed that the entire flux produced by the primary links with the secondary winding. But in practice it is not possible. Part of the primary flux as well as the secondary flux completes the path through air and links with the respecting windings only. Such flux is called leakage flux.

The flux $\mathbf{\Phi_{L1}}$ is the primary flux which is produced due to primary current $\mathbf{I_1}$. The flux $\mathbf{\Phi_{L2}}$ is the secondary flux which is produced due to secondary current $\mathbf{I_2}$. Due the leakage fluxes $\mathbf{\Phi_{L1}}$ and $\mathbf{\Phi_{L2}}$ there is self induced e.m.f's $\mathbf{e_{L1}}$ and $\mathbf{e_{L2}}$. The primary voltage $\mathbf{V_1}$ has to overcome this voltage $\mathbf{e_{L1}}$ to produce $\mathbf{E_1}$, while induced e.m.f $\mathbf{E_2}$ has to overcome $\mathbf{e_{L2}}$ to produce terminal voltage $\mathbf{V_2}$.

The value of $\mathbf{X_1}$ is such that the drop $\mathbf{I_1X_1}$ is nothing but the self induced e.m.f $\mathbf{e_{L1}}$ due to the flux $\mathbf{\Phi_{L1}}$. The value of $\mathbf{X_2}$ is such that the drop $\mathbf{I_2X_2}$ is nothing but the self induced e.m.f $\mathbf{e_{L2}}$ due to the flux $\mathbf{\Phi_{L2}}$.

Equivalent Leakage Reactance:-

Similar to the resistances, the leakage reactance also can be transferred from primary to secondary or vice versa. The relation through $\mathbf{K^2}$ remains same for the transfer of reactance's as it is studied earlier for the resistances.

Let $\mathbf{X_I}$ is leakage reactance of primary and $\mathbf{X_2}$ is leakage reactance of secondary.
Then the total leakage reactance referred to primary is $\mathbf{X_1}$ given by,

$$\boxed{X_{1e} = XI + X_2' \quad \text{where } X_2' = X_2/k^2}$$

While the total leakage reactance referred to secondary is $\mathbf{X_2}$. Given by,

$$\boxed{X_{2e} = X_2 + X_1' \text{ where } X_1' = K^2 X_I}$$

And Transformation ratio

$$\boxed{K = N_2/N_I}$$

- ***Equivalent Impedance***

The transformer primary has resistance $\mathbf{R_1}$ and reactance $\mathbf{X_1}$. While the transformer secondary has resistance $\mathbf{R_2}$ and reactance $\mathbf{X_2}$. Thus we can say that the total impedance of primary winding is $\mathbf{Z_I}$ which is,

$$Z_1 = R_1 + JX_1 \text{ Ohms} \text{-------------------------} (1)$$

And the total impedance of the secondary winding is $\mathbf{Z_2}$ which is,

$$Z_2 = R_2 + JX_2 \text{ Ohms} \text{-------------------------} (2)$$

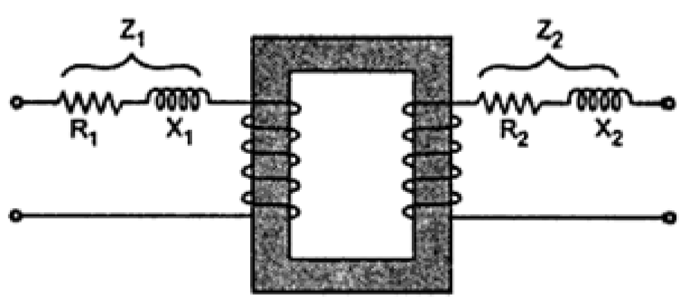


Fig. Individual Impedances

The individual magnitude of Z_1 and Z_2 are,

$$Z_1 = \sqrt{R_1^2 + X_1^2} \text{-------------} (3)$$

$$Z_2 = \sqrt{R_2^2 + X_2^2} \text{--------------} (4)$$

Similarly to resistance and reactance, the impedance also can be referred to any one side.

Let $\mathbf{Z_{1e}}$= total equivalent impedance referred to primary then

$$Z_{1e} = R_{1e} + jX_{1e}$$

$$Z_{1e} = Z_1 + Z_2' = Z_1 + \frac{Z_2}{K^2} \text{---------------} (5)$$

Similarly,

Let $\mathbf{Z_{1e}}$= total equivalent impedance referred to secondary then

$$Z_{2e} = R_{2e} + jX_{2e}$$

$$Z_{2e} = Z_2 + Z_1' = Z_2 + K^2 Z_1 \text{---------------} (6)$$

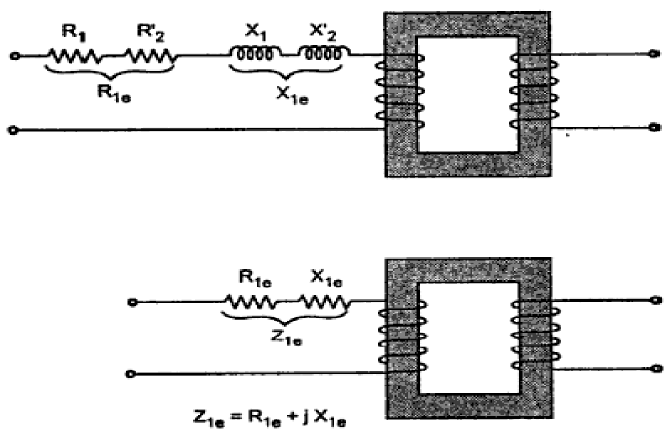
The magnitude of $\mathbf{Z_{1e}}$ and $\mathbf{Z_{2e}}$ are,

$$Z_{1e} = \sqrt{R_{1e}^2 + X_{1e}^2} \text{-------------} (7) \text{ and}$$

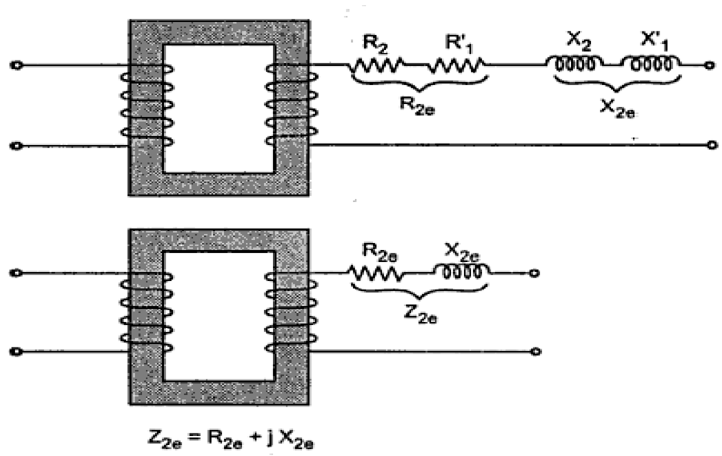
$$\mathbf{Z_{2e}}=\sqrt{R_{2e}^2 + X_{2e}^2} \text{ -------------- } (8)$$

It can be noted that,

$$\boxed{Z_{2e}= K^2 Z_{1e} \text{ and } Z_{1e}=}$$



Referred to primary



(b) Referred to secondary
Equivalent impedances

 <u>Transformer with winding resistance but no magnetic leakage</u>:-

An ideal transformer was supposed to possess no resistance, but in an actual transformer, there is always present some resistance of the primary and secondary winding. Due to this resistance there is some voltage drop in the two windings. The result is that:

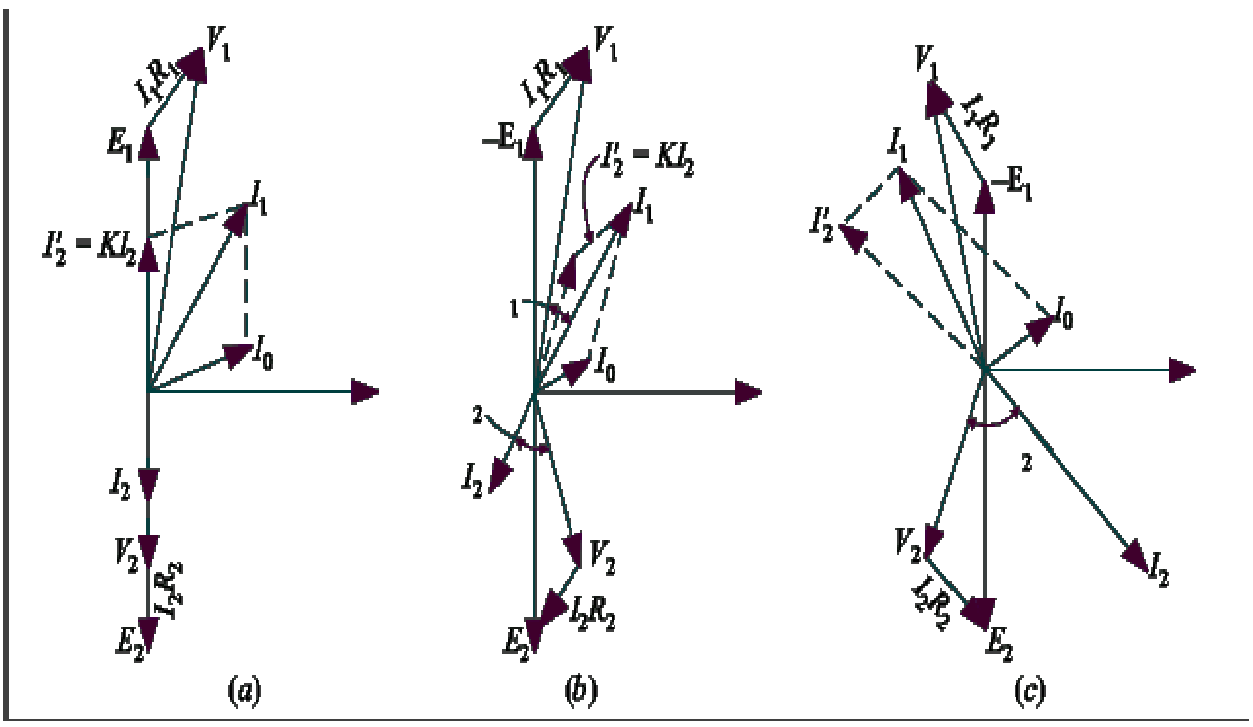
i) The secondary terminal voltage V_2 is vectorially less than the secondary induced e.m.f E_2 by an amount I_2R_2 where R_2 is the resistance of the secondary winding. Hence V_2 is equal to the vector difference of E_2 and resistive voltage drop I_2R_2.

$$\mathbf{V_2= E_2- I_2R_2}$$

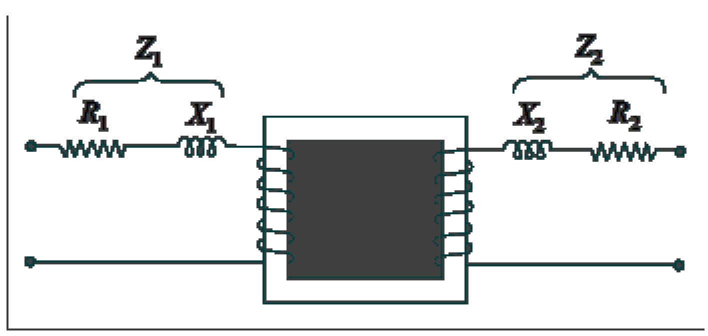
ii) Similarly, primary induced e.m.f E_1 is equal to the vector difference of $V_1$and I_1R_1 where R_1 is the resistance of the primary winding.

$$\mathbf{E_1=V_1-I_1R_1}$$

The vector diagrams for non-inductive, inductive and capacitive loads are shown in below fig:



 Transformer with resistance and leakage reactance:-



In fig shown the primary and secondary windings of a transformer with reactance taken out of the windings. The primary impedance is given by

$$\mathbf{Z_1}=\sqrt{R_1^2 + X_1^2}$$

Similarly secondary impedance is given by, $\mathbf{Z_2}=\sqrt{R_2^2 + X_2^2}$

The resistance and leakage reactance of each winding is responsible for some voltage drop in each winding. In primary, the leakage reactance drop id I_1X_1 (usually 1 or 2% of V_1)

$$\mathbf{V_1=E_1+I_1\,(R_1+jX_1)=E_1+I_1Z_1}$$

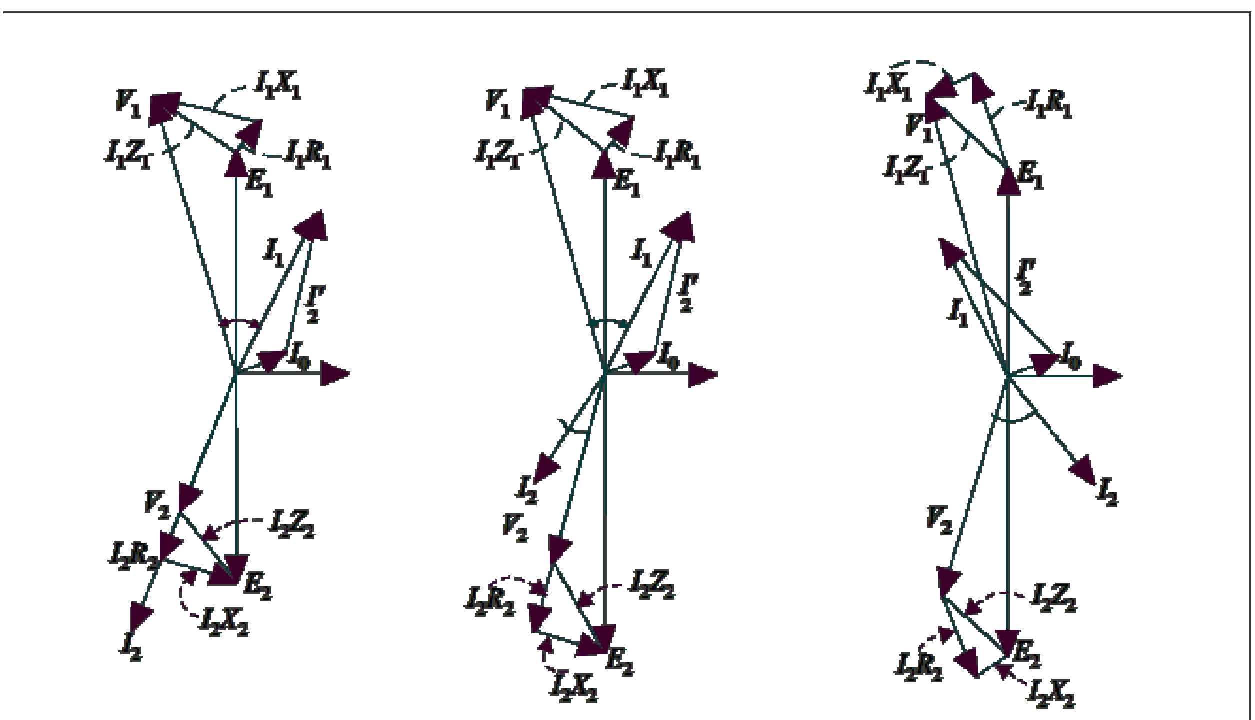
Similarly, there are I_2R_2 and I_2X_2 drops in secondary which combine with V_2 to give E_2.

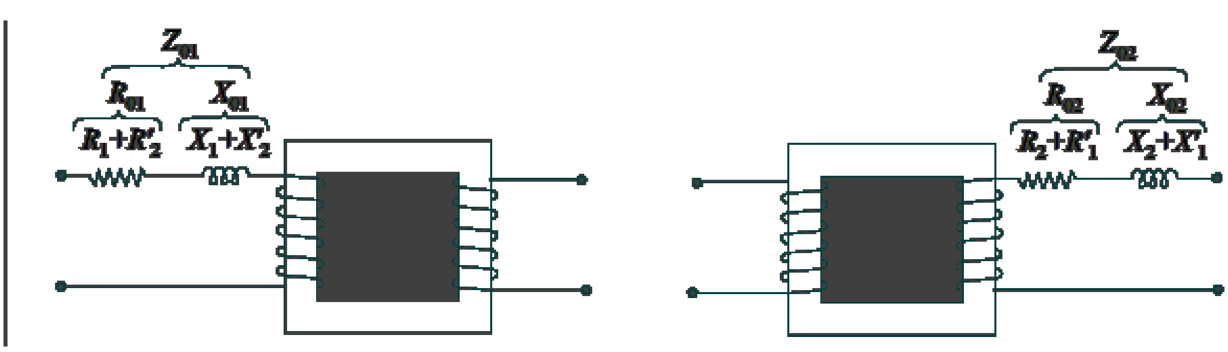
$$\mathbf{E_2=V_2+I_2\,(R_2+jX_2)=V_2+I_2Z_2}$$

The vector diagram for such a transformer for different kinds of loads is shown in below figs. In these diagrams, vectors for resistive drops are drawn parallel to current vectors whereas reactive drops are perpendicular to current vectors. The angle $\Phi_1$between V_1 and I_1 gives the power factor angle of transformer.

$$\mathbf{X_2'=X_2/K^2} \quad \textbf{and} \quad \mathbf{X_1'=X_1K^2}$$

$$\mathbf{X_{01}=X_1+X_2'=X_1+X_2/K^2} \quad \textbf{and} \quad \mathbf{X_{02}=X_2+X_1'=X_2+K^2X_1}$$





It is obvious that total impedance of the transformer as referred to primary is given by

$$\mathbf{Z_{01}}=\sqrt{R_{01}^2+X_{01}^2} \qquad \mathbf{Z_{02}}=\sqrt{R_{02}^2+X_{02}^2}$$