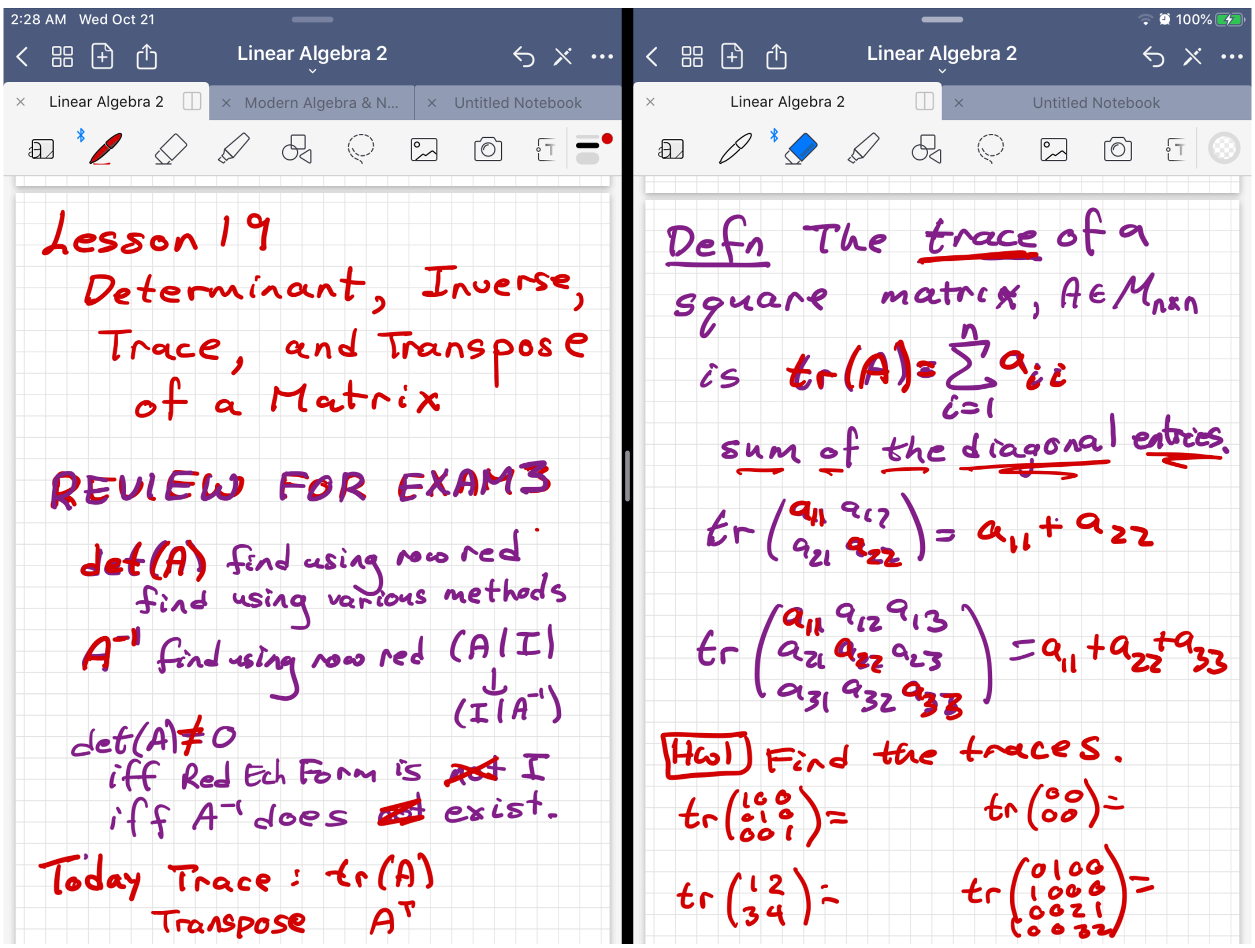
Linear Algebra
Lesson 19
Inverse, Determinant, Trace, Transpose, and Adjoint of a Matrix

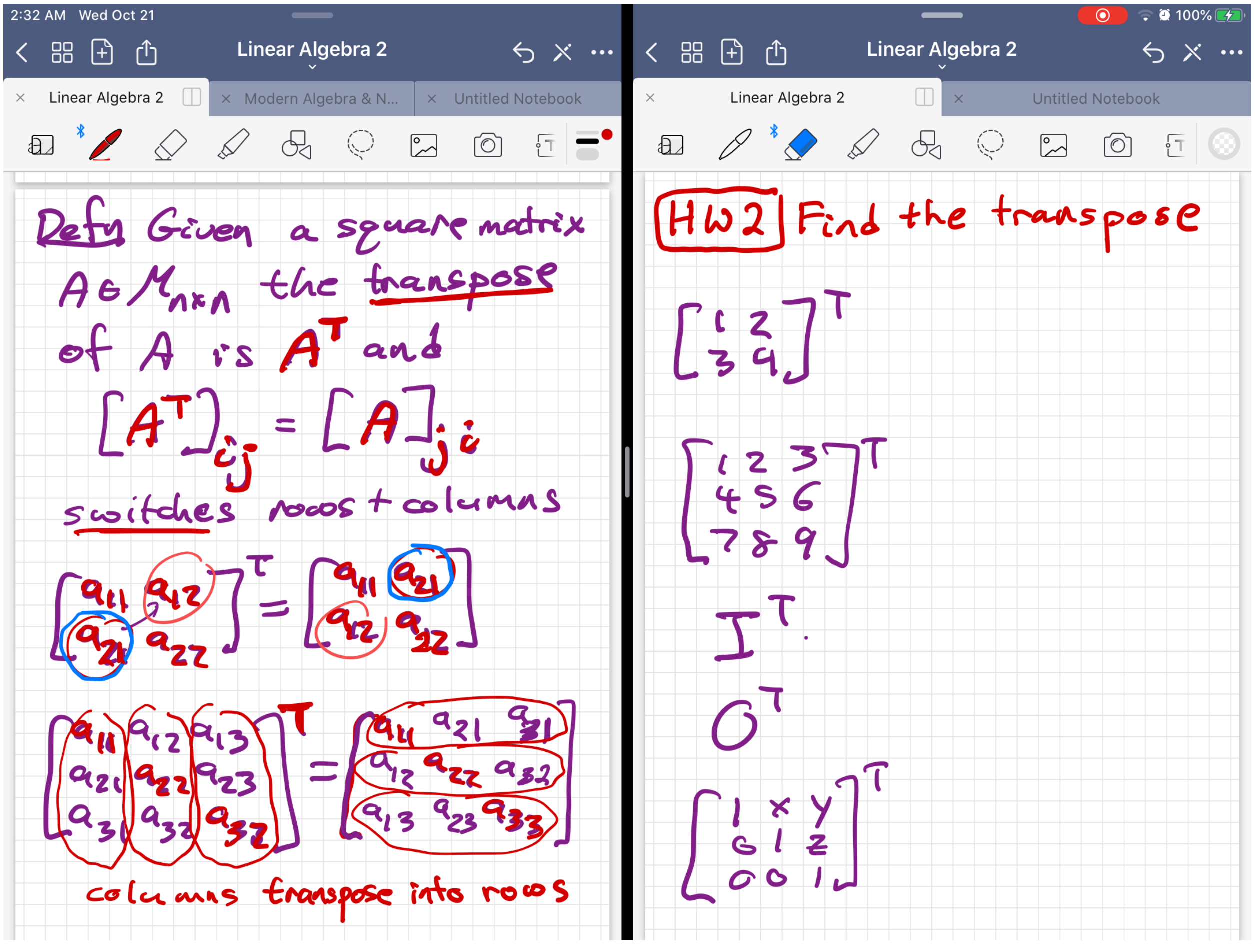
Be sure to put your notes and homework in a document:
MAT313F21-lesson19-lastname-firstname
If you have a question email me with QUESTION in the subject line.

On Exam 3 Part 1 you will be given a 4x4 square matrix and be told to find the Inverse, Determinant, Trace, and Transpose of the Matrix. Everything must be done using the methods taught in this course. You already know inverses and determinants from the prior lessons. So we begin by teaching trace and transpose and then do classwork. We then go over sample exams.

Watch [Video 313F20-19-1] for the definition of trace:



Watch [Video 313F20-19-2] for the definition of transpose:



****
Sample Exams
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Try these samples and then watch videos with solutions:

25 minutes per part.   Part 1=80% +10%EC Part 2: 20%+10%EC

3:09 AM Wed Oct 21

Linear Algebra 2

SAMPLE EXAM 3

Part I:

Let $A = \begin{pmatrix} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 4 \\ 1 & 0 & 1 & 2 \end{pmatrix}$

1. Find A^{-1} using row reduction
2. Find $A^{-1} \times A$ and $A \times A^{-1}$
3. Find $\det A$ using above row actions
4. Find $\det A$ using minors
5. Find $tr(A)$
6. Find A^T

Part II For $A, B \in M_{2\times 2}$

1. Prove $tr(A+B) = tr(A) + tr(B)$
2. Find two matrices such that $tr(A \times B) \neq trA \times trB$

Linear Algebra 2

SAMPLE EXAM 3

PART I

Let $A = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 2 \\ 0 & 2 & 0 & 2 \end{pmatrix}$

1. Find A^{-1} using row red
2. Find $A^{-1} \times A$ and $A \times A^{-1}$
3. Find $\det A$ using row actions
4. Find $\det A$ using minors
5. Find A^T
6. Find trA

Part II: for $A, B \in M_{2\times 2}$

1. Prove $\det(AB) = \det A \det B$
2. Find two matrices such that $A \times B \neq B \times A$.

The purple sample's solutions are explained in [Playlist 313F20-19-3to6]

SAMPLE EXAM 3

Part I:

Let $A = \begin{pmatrix} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 4 \\ 1 & 0 & 1 & 2 \end{pmatrix}$

1. Find A^{-1} using row reduction
2. Find $A^{-1} \times A$ and $A \times A^{-1}$
3. Find det A using above row actions
4. Find det A using minors
5. Find $tr(A)$
6. Find A^T

Part II For $A, B \in M_{2\times 2}$

1. Prove $tr(A+B) = tr(A) + tr(B)$
2. Find two matrices such that $tr(A \times B) \neq trA \times trB$

Linear Algebra 2

(1)

$$\left(\begin{array}{cccc|cccc} \boxed{1} & 0 & 0 & 2 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 4 & 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 2 & 0 & 0 & 0 & 1 \end{array}\right) \xrightarrow[\rho_4 \to \rho_4 - \rho_1]{\text{skew} \quad \cdot 1}$$

$$\left(\begin{array}{cccc|cccc} \boxed{1} & 0 & 0 & 2 & 1 & 0 & 0 & 0 \\ 0 & \boxed{1} & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 4 & 0 & 0 & 1 & 0 \\ 0 & 0 & \boxed{1} & 0 & -1 & 0 & 0 & 1 \end{array}\right) \xrightarrow[\rho_4 \leftrightarrow \rho_3]{\text{switch} \quad (-1)}$$

$$\left(\begin{array}{cccc|cccc} \boxed{1} & 0 & 0 & 2 & 1 & 0 & 0 & 0 \\ 0 & \boxed{1} & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & \boxed{1} & 0 & -1 & 0 & 0 & 1 \\ 0 & 0 & 0 & \boxed{4} & 0 & 0 & 1 & 0 \end{array}\right) \xrightarrow[\rho_4 \to \frac{1}{4}\rho_4]{\text{scale} \quad (4)}$$

$$\left(\begin{array}{cccc|cccc} 1 & 0 & 0 & 2 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & -1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1/4 & 0 \end{array}\right) \xrightarrow[\rho_1 \to \rho_1 - 2\rho_4]{\text{skew} \quad (+1)}$$

$$\left(\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & 1 & 0 & -2/4 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & -1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1/4 & 0 \end{array}\right)$$

$$A^{-1} = \begin{pmatrix} 1 & 0 & -1/2 & 0 \\ 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 1 \\ 0 & 0 & 1/4 & 0 \end{pmatrix}$$

(3)

$\det(A) = (+1)(-1)(4)(+1) \det(I)$

$= -4 \cdot 1 = -4$

(2) $A^{-1} \times A = \begin{pmatrix} 1 & 0 & -1/2 & 0 \\ 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 1 \\ 0 & 0 & 1/4 & 0 \end{pmatrix} \times \begin{pmatrix} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 4 \\ 1 & 0 & 1 & 2 \end{pmatrix}$

$= \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} = I$ ✓

$A \times A^{-1} = \begin{pmatrix} \ \end{pmatrix}\begin{pmatrix} \ \end{pmatrix} = I$ ✓

(3) det A using row actions
see above $\det A = -4$ ✓

(4) $\det A = \det \begin{pmatrix} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 4 \\ 1 & 0 & 1 & 2 \end{pmatrix}$ $\begin{pmatrix} + & - & + \\ - & + & - \\ + & - & + \end{pmatrix}$

$\begin{pmatrix} + & - & + & - \\ - & + & - & + \\ + & - & + & - \\ - & + & - & + \end{pmatrix}$

choose row with many zeroes

$= -0 + 1 \det \begin{pmatrix} 1 & 0 & 2 \\ 0 & 0 & 4 \\ 1 & 1 & 2 \end{pmatrix} - 0 + 0$

$= 1 \cdot (0 + 0 - 4 \det \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix})$

$= 1(-4)(1 \cdot 1 - 0 \cdot 1) = -4$ ✓

(5) $tr(A) = \begin{pmatrix} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 4 \\ 1 & 0 & 1 & 2 \end{pmatrix}$

$= 1 + 1 + 0 + 2 = 4$

(6) $A^T = \begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 2 & 0 & 4 & 2 \end{pmatrix}$

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Linear Algebra 2

Part II $A, B \in M_{2\times 2}$

Prove $tr(A+B) = trA + trB$

① LHS $= tr(A+B)$ ① by defn of $M_{2\times 2}$

$= tr\left(\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} + \begin{pmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{pmatrix}\right)$

② $= tr\left(\begin{pmatrix} a_{11}+b_{11} & a_{12}+b_{12} \\ a_{21}+b_{21} & a_{22}+b_{22} \end{pmatrix}\right)$ ② by matrix add

③ $= (a_{11}+b_{11}) + (a_{22}+b_{22})$ ③ defn tr

④ RHS $trA + trB = tr\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} + tr\begin{pmatrix} b_{11} & b_{12} \\ b_{22} & b_{21} \end{pmatrix}$ ④ defn $M_{2\times 2}$

⑤ $= (a_{11}+a_{22}) + (b_{11}+b_{22})$ ⑤ defn tr

⑥ $= a_{11} + b_{11} + a_{22} + b_{22}$ ⑥ $a+b = b+a \in \mathbb{R}$

Linear Algebra 2

Find two matrices such that

$tr(A \times B) \neq trA \times trB$

$\begin{pmatrix} 2 & 0 \\ 0 & 3 \end{pmatrix}\begin{pmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{pmatrix} = \begin{pmatrix} 2b_{11} & 2b_{12} \\ 3b_{21} & 3b_{22} \end{pmatrix}$

$tr\begin{pmatrix} 2 & 0 \\ 0 & 3 \end{pmatrix}$ $tr\begin{pmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{pmatrix}$ trying ideas $tr\begin{pmatrix} 2b_{11} & 2b_{12} \\ 3b_{21} & 3b_{22} \end{pmatrix}$

$(2+3)$ $(b_{11}+b_{22})$ $(2b_{11}+3b_{22})$

$(5)(b_{11}+b_{22})$

$A = \begin{pmatrix} 2 & 0 \\ 0 & 3 \end{pmatrix}$ $B = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ $A \times B = \begin{pmatrix} 2 & 0 \\ 0 & 3 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 2 & 0 \\ 0 & 3 \end{pmatrix}$

$trA = 2+3 = 5$ $trB = 1+1 = 2$

$(tr A) \times (tr B) = 5 \cdot 2 = 10$

but $tr(A \times B) = tr\begin{pmatrix} 2 & 0 \\ 0 & 3 \end{pmatrix} = 2+3 = 5$

$5 \neq 10$

Note that Part II (2) has many correct answers.

The blue sample's solutions are explained in [Playlist 313F20-19-7to9]

SAMPLE EXAM 3

PART I

Let $A = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 2 \\ 0 & 2 & 0 & 2 \end{pmatrix}$

1. Find A^{-1} using row red
2. Find $A^{-1} \times A$ and $A \times A^{-1}$
3. Find $\det A$ using row actions
4. Find $\det A$ using minors
5. Find A^T
6. Find $\operatorname{tr} A$

Part II: for $A, B \in M_{2\times 2}$

1. Prove $\det(AB) = \det A \det B$
2. Find two matrices such that $A \times B \neq B \times A$.

Linear Algebra 2

Part I

1.
$$\left(\begin{array}{cccc|cccc} 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ \boxed{1} & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 2 & 0 & 0 & 1 & 0 \\ 0 & 2 & 0 & 2 & 0 & 0 & 0 & 1 \end{array}\right) \xrightarrow[\text{switch}\ (-1)]{\rho_1 \leftrightarrow \rho_2}$$

$$\left(\begin{array}{cccc|cccc} \boxed{1} & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & \boxed{1} & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 2 & 0 & 0 & 1 & 0 \\ 0 & 2 & 0 & 2 & 0 & 0 & 0 & 1 \end{array}\right) \xrightarrow[\substack{\rho_3 \to \rho_3 - \rho_2 \\ \rho_4 \to \rho_4 - 2\rho_2}]{(+1)\ \text{skews}}$$

$$\left(\begin{array}{cccc|cccc} \boxed{1} & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & \boxed{1} & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2 & -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 2 & -2 & 0 & 0 & 1 \end{array}\right) \longrightarrow$$

wait Red Ech Form is not I
no leader in third row.

No Inverse

2. Cannot do no inverse
3. $\det A = 0$ because $E \neq I$

SAMPLE EXAM 3

PART I

Let $A = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 2 \\ 0 & 2 & 0 & 2 \end{pmatrix}$

1) Find A^{-1} using row red
2) Find $A^{-1} \times A$ and $A \times A^{-1}$
3) Find det A using row actions
4) Find det A using minors
5) Find A^T
6) Find tr A

Part II: for $A, B \in M_{2\times 2}$

1) Prove $\det(AB) = \det A \det B$
2) Find two matrices such that $A \times B \neq B \times A$.

Linear Algebra 2

Part I

1)
$$\left(\begin{array}{cccc|cccc} 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ \boxed{1} & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 2 & 0 & 0 & 1 & 0 \\ 0 & 2 & 0 & 2 & 0 & 0 & 0 & 1 \end{array}\right) \xrightarrow[\text{switch } (-1)]{\rho_1 \leftrightarrow \rho_2}$$

$$\left(\begin{array}{cccc|cccc} \boxed{1} & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & \boxed{1} & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 2 & 0 & 0 & 1 & 0 \\ 0 & 2 & 0 & 2 & 0 & 0 & 0 & 1 \end{array}\right) \xrightarrow[\rho_3 \to \rho_3 - \rho_2,\ \rho_4 \to \rho_4 - 2\rho_2]{(+1)\ \text{skews}}$$

$$\left(\begin{array}{cccc|cccc} \boxed{1} & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & \boxed{1} & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2 & -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 2 & -2 & 0 & 0 & 1 \end{array}\right) \longrightarrow$$

wait Red Ech Form is not I
no leader in third row.

No Inverse

2) Cannot do no inverse
3) det A = 0 because E ≠ I

SAMPLE EXAM 3

PART I

Let $A = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 2 \\ 0 & 2 & 0 & 2 \end{pmatrix}$

1. Find A^{-1} using row red
2. Find $A^{-1} \times A$ and $A \times A^{-1}$
3. Find $\det A$ using row actions
4. Find $\det A$ using minors
5. Find A^T
6. Find $\operatorname{tr} A$

Part II: for $A, B \in M_{2\times 2}$

1. Prove $\det(AB) = \det A \det B$
2. Find two matrices such that $A \times B \neq B \times A$.

(4) $\det A$ using minors

$\begin{pmatrix} + & - & + & - \\ - & + & - & + \\ + & - & + & - \\ - & + & - & + \end{pmatrix}$ $A = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 2 \\ 0 & 2 & 0 & 2 \end{pmatrix}$

choose a row or column with lots of zeroes

a column of all zeroes

$\det(A) = +0 - 0 + 0 - 0 = 0$

(5) Find $A^T = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 2 & 2 \end{pmatrix}$

(6) $\operatorname{tr} A = 0 + 0 + 0 + 2 = 2$.

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Linear Algebra 2

Part II $A, B \in M_{2\times2}$

Prove $\det(AB) = \det(A)\det(B)$

① LHS
$$\det(AB) = \det\left(\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22}\end{pmatrix}\begin{pmatrix} b_{11} & b_{12} \\ b_{21} & b_{22}\end{pmatrix}\right)$$
by defn $M_{2\times2}$

②
$$= \det\begin{pmatrix} a_{11}b_{11}+a_{12}b_{21} & a_{11}b_{12}+a_{12}b_{22} \\ a_{21}b_{11}+a_{22}b_{22} & a_{21}b_{12}+a_{22}b_{22}\end{pmatrix}$$
by defn matrix mult

③
$$= (a_{11}b_{11}+a_{12}b_{21})(a_{21}b_{12}+a_{22}b_{22}) - (a_{11}b_{12}+a_{12}b_{22})(a_{21}b_{11}+a_{22}b_{22})$$

④ = simplify that

100%

Linear Algebra 2

⑤ RHS

$\det(A)\cdot\det(B) =$
$$= \det\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22}\end{pmatrix} \det\begin{pmatrix} b_{11} & b_{12} \\ b_{21} & b_{22}\end{pmatrix}$$
by defn of $M_{2\times2}$

⑥
$$= (a_{11}a_{22} - a_{12}a_{21})(b_{11}b_{22} - b_{12}b_{21})$$
by formula for 2x2 det
ad - bc

⑦ = simplify

← Match.

do for HW.

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SAMPLE EXAM 3

PART I

Let $A = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 2 \\ 0 & 2 & 0 & 2 \end{pmatrix}$

1. Find A^{-1} using row red
2. Find $A^{-1} \times A$ and $A \times A^{-1}$
3. Find $\det A$ using row actions
4. Find $\det A$ using minors
5. Find A^T
6. Find $\operatorname{tr} A$

Part II: for $A, B \in M_{2\times 2}$

1. Prove $\det(AB) = \det A \det B$
2. Find two matrices such that $A \times B \neq B \times A$

Part II (2)

Find 2 matrices s.t.

$$A \times B \neq B \times A$$

Try $A = \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}$ $B = \begin{pmatrix} 2 & 0 \\ 0 & 0 \end{pmatrix}$

$$A \times B = \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix} \times \begin{pmatrix} 2 & 0 \\ 0 & 0 \end{pmatrix} = \begin{pmatrix} 2 & 0 \\ 2 & 0 \end{pmatrix}$$

$$B \times A = \begin{pmatrix} 2 & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix} = \begin{pmatrix} 2 & 0 \\ 0 & 0 \end{pmatrix}$$

Don't Match.

Note that Part II (2) has many correct answers.

Watch [Video 313F20-19-10] for the next sample of Part II only:

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Linear Algebra 2

Exam III Part II

Let $A, B \in M_{2\times 2}$

Prove

$$\boxed{(A \times B)^T = B^T \times A^T}$$

or find matrices

such that

$$\boxed{(A \times B)^T \neq B^T \times A^T}$$

Linear Algebra 2

"Prove or Find a Counterexample"

Try to prove and if the proof fails find the bad matrices

True

or

False?

Try this one (pause)

Proof:

LHS

(1) $(A\times B)^T = \left(\begin{pmatrix}a_{11}&a_{12}\\a_{21}&a_{22}\end{pmatrix}\times\begin{pmatrix}b_{11}&b_{12}\\b_{21}&b_{22}\end{pmatrix}\right)^T$

(1) by defn of $M_{2\times2}$

(2) $$= \begin{pmatrix} a_{11}b_{11}+a_{12}b_{21} & a_{11}b_{12}+a_{12}b_{22}\\ a_{21}b_{11}+a_{22}b_{21} & a_{21}b_{12}+a_{22}b_{22}\end{pmatrix}^T$$

(2) by matrix mult

(3) $$= \begin{pmatrix} a_{11}b_{11}+a_{12}b_{21} & a_{21}b_{11}+a_{22}b_{21}\\ a_{11}b_{12}+a_{12}b_{22} & a_{21}b_{12}+a_{22}b_{22}\end{pmatrix}$$

RHS

(4) $B^T\times A^T = \begin{pmatrix}b_{11}&b_{12}\\b_{21}&b_{22}\end{pmatrix}^T\times\begin{pmatrix}a_{11}&a_{12}\\a_{21}&a_{22}\end{pmatrix}^T$

(4) by defn of $M_{2\times2}$

(5) $$= \begin{pmatrix}b_{11}&b_{21}\\b_{12}&b_{22}\end{pmatrix}\times\begin{pmatrix}a_{11}&a_{21}\\a_{12}&a_{22}\end{pmatrix}$$

(5) by defn transpose

(6) $$= \begin{pmatrix} b_{11}a_{11}+b_{21}a_{12} & b_{11}a_{21}+b_{21}a_{22}\\ b_{12}a_{11}+b_{22}a_{12} & b_{12}a_{21}+b_{22}a_{22}\end{pmatrix}$$

(6) by matrix mult

steps 3+6 match! QED

No need to find the counterexample matrices because the proof works!

Watch [Video 313F20-19-11] for the second sample of Part II only:

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Linear Algebra 2

Exam III Part II

Let $A, B \in M_{2\times 2}$

Prove

$$\boxed{\det(A+B) = \det A + \det B}$$

or find matrices such that

$$\boxed{\det(A+B) \neq \det A + \det B}$$

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Linear Algebra 2

Again start by trying to prove it.

If they match the proof works

If they don't match then find matrices

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Linear Algebra 2

Proof:

LHS

① $\det(A+B) =$

$= \det\left(\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} + \begin{pmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{pmatrix}\right)$

① by defn $M_{2\times 2}$

② $= \det\begin{pmatrix} a_{11}+b_{11} & a_{12}+b_{12} \\ a_{21}+b_{21} & a_{22}+b_{22} \end{pmatrix}$

② by Matrix addition

③ $= (a_{11}+b_{11})(a_{22}+b_{22}) - (a_{12}+b_{12})(a_{21}+b_{21})$

by defn of det

④ $= a_{11}a_{22} + b_{11}a_{22} + a_{11}b_{22} + b_{11}b_{22} - a_{12}a_{21} - b_{12}a_{21} - a_{12}b_{21} - b_{12}b_{21}$

Linear Algebra 2

RHS

① $\det A + \det B =$

$= \det\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} + \det\begin{pmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{pmatrix}$

① by defn $M_{2\times 2}$

② $= a_{11}a_{22} - a_{12}a_{21} + b_{11}b_{22} - b_{12}b_{21}$

② by defn of det.

Does it match? –

$\boxed{\text{No! Proof fails!}}$

if $b_{11}a_{22} \neq 0$

Find matrices $A = \begin{pmatrix} 0 & 0 \\ 0 & 5 \end{pmatrix}$, $B = \begin{pmatrix} 4 & 0 \\ 0 & 0 \end{pmatrix}$

LHS $\det(A+B) = \det\begin{pmatrix} 4 & 0 \\ 0 & 5 \end{pmatrix} = 4\cdot 5 - 0\cdot 0 = 20$

RHS $\det(A) + \det(B) = 0\cdot 5 - 0\cdot 0 + 4\cdot 0 - 0\cdot 0$

$= 0 + 0 = 0 \neq 20.$

HW: do these two additional sample exams

SAMPLE EXAM 3

Part I

Let $A = \begin{pmatrix} 4 & 0 & 0 & 8 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 4 & 0 & 1 & 9 \end{pmatrix}$

1) Find A^T
2) Find $tr(A)$
3) Find A^{-1} using row red.
4) Find $A^{-1} \times A$ and $A \times A^{-1}$
5) Find $\det(A)$ using row actions
6) Find $\det(A)$ using minors

Part II For $A, B \in M_{2\times2}$

Prove $\det(A) = \det(A^T)$

or find two matrices such that

$\det(A) \neq \det(A^T)$

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Linear Algebra 2

SAMPLE EXAM 3

Part I

Let $A = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 3 & 4 & 4 \end{pmatrix}$

1) Find $tr\, A$
2) Find A^T
3) Find A^{-1} using row red
4) Find $A^{-1} \times A$ and $A \times A^{-1}$
5) Find $\det(A)$ using row actions
6) Find $\det(A)$ using minors

Part II For $A, B \in M_{2\times2}$

Prove $(A \times B)^T = A^T \times B^T$

or find two matrices such that $(A \times B)^T \neq A^T \times B^T$