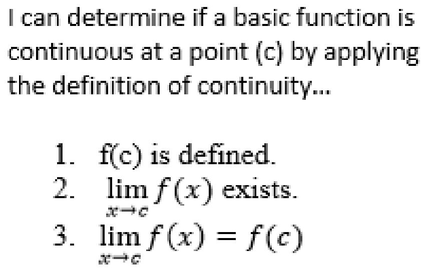
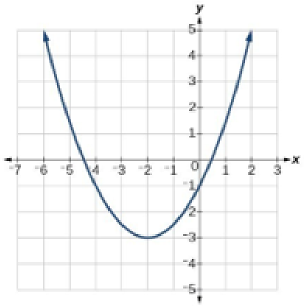
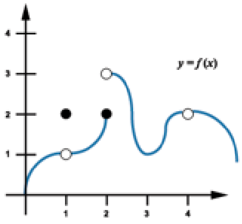
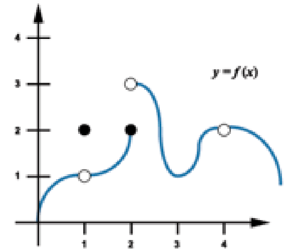
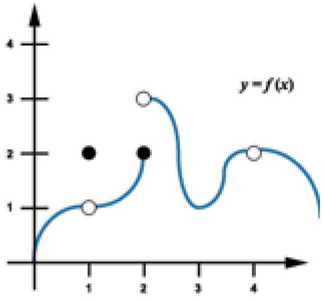
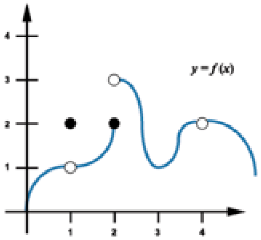
Limits and Continuity - I can analyze functions for intervals of continuity and interpret/determine limits of functions.

Beginning	Developing	Proficient	Advanced
I can determine the limit from the graph of a continuous function as the input values approach a specific number. I can use direct substitution to evaluate the limit of a basic function. I can determine if a function is continuous by looking at its graph.	I can determine from a graph of any function if a limit exists and what the limit is as the input values approach a specific number. I can recognize where discontinuities by looking at the graph of a function.	I can evaluate a limit numerically from a table of values, using direct substitution, using factoring technique, rationalizing, finding a common denominator. I can apply limits at infinity to find horizontal asymptotes of rational functions.  I can determine if a basic function is continuous at a point (c) by applying the definition of continuity… 1. f(c) is defined. 2. $\lim_{x \to c} f(x)$ exists. 3. $\lim_{x \to c} f(x) = f(c)$	I can apply the limit to the definition of the derivative and evaluate complex limits using the definition of the derivative. I can apply the definition of continuity to answer questions about continuous and/or discontinuous functions.

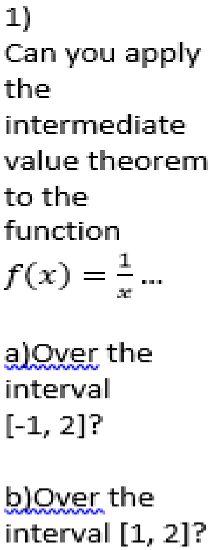
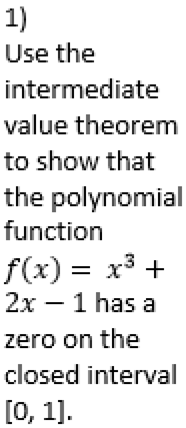
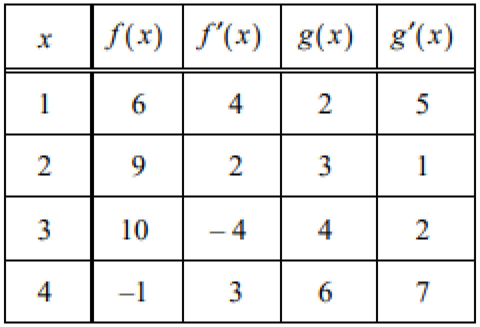
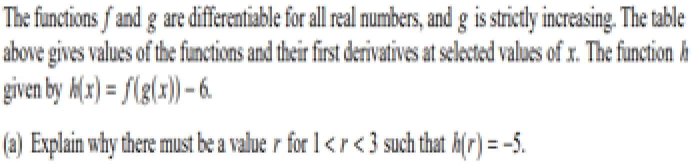
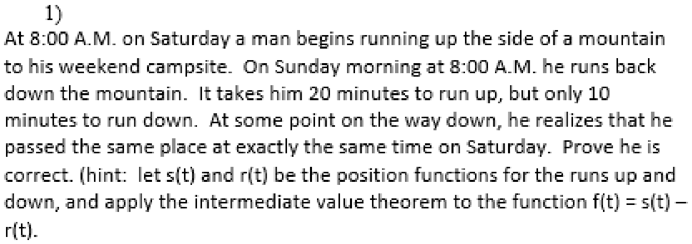
Example Problems

Beginning	Developing	Proficient	Advanced
1)  What is the limit of the function whose graph is given above as x approaches 0? 2) Evaluate $\lim_{x\to -3} x^2 - 4$ 1)  Is the function above continuous?	 1) a) Evaluate $\lim_{x\to 1} f(x)$. b) Evaluate $\lim_{x\to 2} f(x)$. 1)  At what x-values is the function given above discontinuous?	1) a) Evaluate $\lim_{x\to 0}\frac{\sqrt{x+1}-1}{x}$ b) Evaluate $\lim_{x\to 0}\frac{\tan x}{x}$ f) Evaluate $\lim_{x\to\infty}\frac{3x^2-2x+1}{5x^2-3x+4}$ g) Evaluate $\lim_{x\to\infty}\frac{3x^3-2x+1}{5x^2-3x+4}$ 1)  Are there any removable discontinuities on the graph of the function above? If so, at what x-values? 2) Is $f(x) = \frac{1}{x-1}$ continuous over the interval [-1, 2]? If it is not, where does the discontinuity occur?	2) Evaluate $\lim_{h\to 0}\frac{\sin\left(\frac{\pi}{2}+h\right)-\sin\frac{\pi}{2}}{h}$ 1) Find the constants a and b such that the function is continuous on the entire real line. $f(x) = \begin{Bmatrix} 2, & x \le 1 \\ ax+b, & -1 < x < 3 \\ -2, & x \ge 3 \end{Bmatrix}$ 2) For what values of k is the function $f(x) = \begin{Bmatrix} 3x^2 - 11x - 4, x \le 4 \\ kx^2 - 2x - 1, x > 4 \end{Bmatrix}$ continuous at x = 4?

Intermediate Value Theorem - I can define and apply the Intermediate Value Theorem.

Beginning	Developing	Proficient	Advanced
I can determine if I can apply the intermediate value theorem to a specific problem.	I can apply the intermediate value theorem to show the existence of an output within a closed interval.	I can define and apply the intermediate value theorem to determine the existence of an output within a closed interval.	I can apply the intermediate value theorem to a complex problem to determine the existence of an output within a closed interval.

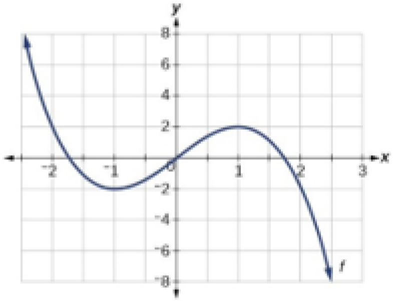
Example Problems

<table>
<tr><th>Beginning</th><th>Developing</th><th>Proficient</th><th>Advanced</th></tr>
<tr>
<td>
1)
Can you apply the intermediate value theorem to the function $f(x) = \frac{1}{x}$...
a)Over the interval [-1, 2]?
b)Over the interval [1, 2]?</td>
<td>
1)
Use the intermediate value theorem to show that the polynomial function $f(x) = x^3 + 2x - 1$ has a zero on the closed interval [0, 1].</td>
<td>1)

<table>
<tr><th>x</th><th>$f(x)$</th><th>$f'(x)$</th><th>$g(x)$</th><th>$g'(x)$</th></tr>
<tr><td>1</td><td>6</td><td>4</td><td>2</td><td>5</td></tr>
<tr><td>2</td><td>9</td><td>2</td><td>3</td><td>1</td></tr>
<tr><td>3</td><td>10</td><td>-4</td><td>4</td><td>2</td></tr>
<tr><td>4</td><td>-1</td><td>3</td><td>6</td><td>7</td></tr>
</table>

The functions f and g are differentiable for all real numbers, and g is strictly increasing. The table above gives values of the functions and their first derivatives at selected values of x. The function h given by $h(x) = f(g(x)) - 6$.
(a) Explain why there must be a value r for $1 < r < 3$ such that $h(r) = -5$.</td>
<td>
1)
At 8:00 A.M. on Saturday a man begins running up the side of a mountain to his weekend campsite. On Sunday morning at 8:00 A.M. he runs back down the mountain. It takes him 20 minutes to run up, but only 10 minutes to run down. At some point on the way down, he realizes that he passed the same place at exactly the same time on Saturday. Prove he is correct. (hint: let s(t) and r(t) be the position functions for the runs up and down, and apply the intermediate value theorem to the function f(t) = s(t) – r(t).</td>
</tr>
</table>

Identifying Derivatives - I can identify the derivative of a function as the limit of the difference quotient and apply derivatives as the slope of the tangent line to make linear approximations.

Beginning	Developing	Proficient	Advanced
I can estimate the derivative of the graph of a function at a given point. I can recognize the different notations that represent the derivative. $y', \quad f'(x), \quad \frac{dy}{dx}, \; \frac{d}{dx}[f(x)]$	I can estimate the derivative of the graph of a function at a given point. I can recognize the different notations that represent the derivative. $y', \quad f'(x), \quad \frac{dy}{dx}, \; \frac{d}{dx}[f(x)]$ I can find the derivative of a polynomial function using the limit of the difference quotient.	I can estimate the derivative of the graph of a function at a given point. I can recognize the different notations that represent the derivative. $y', \quad f'(x), \quad \frac{dy}{dx}, \; \frac{d}{dx}[f(x)]$ I can find the derivative of a polynomial function using the limit of the difference quotient. I can find the equation of the tangent line of a function through a specific point on the function. I can apply the definition of the derivative to evaluate complex limits.	I can estimate the derivative of the graph of a function at a given point. I can recognize the different notations that represent the derivative. $y', \quad f'(x), \quad \frac{dy}{dx}, \; \frac{d}{dx}[f(x)]$ I can find the derivative of a polynomial function using the limit of the difference quotient. I can apply the definition of the derivative to evaluate complex limits. I can find the equation of the tangent line and use the tangent line to make linear approximations.

Example Problems

Beginning	Developing	Proficient	Advanced
1)  Estimate the slope of the tangent line through the point (.5, 1).	1) Find $f'(x)$ for $f(x) = x^2 - 2x + 3$ by using the limit process with the difference quotient.	1) Evaluate $\lim_{h\to 0} \frac{5(\frac{1}{2}+h)^4 - 5(\frac{1}{2})^4}{h}$. 2) Find the equation of the tangent line to the graph of f at the point given. $f(x) = 2x^3 + x$ at the point (1, 3).	1) Find the equation of the tangent line for the function $f(x) = x^2 + 1$ through the point (2, 5). Then find the linear approximation for f(3).

Finding Derivatives - I can apply the general power rule, sum and difference rule, product rule, and quotient rule to find the derivative of a function.

Beginning	Developing	Proficient	Advanced
I can apply the general power rule and constant multiple rule for derivatives to an expression.	I can rewrite an expression to apply the general power rule to an expression. I can apply the sum and difference rule for derivatives to a polynomial expression.	I can apply the general power rule, sum and difference rule, product rule, and quotient rule to find the derivative of a function and use the derivative to find the equation of the tangent line through a point.	I can rewrite complex functions; including ones with rational exponents, and apply the general power rule, sum and difference rule, product rule, and quotient rule to find the derivative of a function. I can simplify the derivative expression to fit a variety of forms. I can evaluate the derivatives of functions given a table of values representing function values and their derivatives.

Example Problems

<table>
<tr><th>Beginning</th><th>Developing</th><th>Proficient</th><th>Advanced</th></tr>
<tr>
<td>1)Find the derivative of f(x) = x^3</td>
<td>1) Find the derivative
$f(x) = -2x^3 - 4x^2 + 6x$
2)Find the derivative
$$f(x) = \frac{-3}{2x^2}$$
3) Find the derivative
$$f(x) = 2x + \frac{1}{x^3}$$</td>
<td>1)Multiple Choice:
Find an equation for the tangent line to the graph of $f(x) = -3x^2 - 2x + 3$ at the point where x = 2.
a) y = -2x + 4 b) y = -14x + 15
c) y = -6x – 2 d) -2x + y = 14
e) None of these.
2) $$f(x) = \frac{x^4 - 4x + 1}{3x^2 - 1}$$ Find f'(x).
3) $$f(x) = (x^3 - 2x^2 + 1)(x^2 - 3)$$ Find f'(x).</td>
<td>1) Multiple Choice:
Find $f'(x)$: $f(x) = \frac{x^2-4x}{\sqrt{x}}$.
a) $f'(x) = x^{3/2} - 4x^{1/2}$ b) $f'(x) = \frac{2x-4}{\sqrt{x}}$
c) $f'(x) = \frac{2x-4}{1/(2\sqrt{x})}$ d) $f'(x) = \frac{3}{2}x^{1/2} - \frac{2}{x^{1/2}}$
2)
Suppose functions f and g and their derivatives have the following values at x =2.
$f(3) = 2$ $f'(3) = 1/4$
$g(3) = -2$ $g'(3) = -4$
Find the value of $\frac{d}{dx}[g(x)/f(x)]$ when x = 3.
3)
<table>
<tr><th>x</th><th>$f(x)$</th><th>$f'(x)$</th><th>$g(x)$</th><th>$g'(x)$</th></tr>
<tr><td>1</td><td>–6</td><td>3</td><td>2</td><td>8</td></tr>
<tr><td>2</td><td>2</td><td>–2</td><td>–3</td><td>0</td></tr>
<tr><td>3</td><td>8</td><td>7</td><td>6</td><td>2</td></tr>
<tr><td>6</td><td>4</td><td>5</td><td>3</td><td>–1</td></tr>
</table>
The functions f and g have continuous second derivatives. The table above gives values of the functions and their derivatives at selected values of x.
(a) Let $k(x) = f(g(x))$. Write an equation for the line tangent to the graph of k at $x = 3$.
(b) Let $h(x) = \frac{g(x)}{f(x)}$. Find $h'(1)$.</td>
</tr>
</table>

Chain Rule/Trig Derivatives - I can apply the trigonometric rules and the chain rule for derivatives.

Beginning	Developing	Proficient	Advanced
I can apply the rules for derivatives to a sine, cosine, or tangent, expression. (non-composite)	I can apply the chain rule to a variety of basic composite functions, including trigonometric functions and radicals. I can use a graphing calculator to evaluate a derivative at a specific value.	I can apply the chain rule multiple times in a composite function	I can apply the chain rule to complete more complex functions.

Example Problems

Beginning	Developing	Proficient	Advanced
1) f(x) = sin x. Find f'(x). 2) f(x)= cos x. Find f'(x). 3) f(x) = tan x. Find f'(x).	1) $f(x) = x + \sin 2x$ Find *f'(x)*. 2) $y = \sqrt{x^2 - 2x}$ Find *dy/dx*. 3) $y = (3x - 2)^4$ Find dy/dx.	1) $f(x) = \left(\frac{2x-1}{2x^2+3}\right)^3$ Find *f'(x)*. 2) $f(t) = \cos^3(2t)$ Find *f'(t)*. 3) f(x) = csc x. Find f'(x). 4) f(x) = sec x. Find f'(x). 5) f(x) = cot x. Find f'(x).	1) 2)

Position/Velocity/Acceleration - I can analyze the relationship between the position, velocity, and acceleration of an object mathematically and graphically.

Beginning (Differential) Given the position function, I can determine the functions that represent velocity and acceleration and evaluate these functions at particular given times. From a position-time graph, I can determine the location of an object and its instantaneous velocity at any given time or from a velocity-time graph, I can determine the velocity of an object and its acceleration at any given time. -- Beginning (Integral) Given the acceleration of an object, I can find the general velocity function. Given the velocity function, I can find the general position function.	Developing (Differential) From a position-time graph or the position function, I can determine in which direction an object is moving and when an object changes direction. From a velocity-time graph or the velocity function, I can determine when an object is speeding up, moving at a constant speed, or slowing down. I can sketch a velocity graph over a given interval if given the position graph or sketch an acceleration graph over a given interval if given the velocity graph. -- Developing (Integral) Given the acceleration of an object and it's initial velocity, I can find the particular velocity function. If also given the initial position of the function, I can find the particular position function.	Proficient (Differential) From a position function, or its position-time graph, I can determine the average rate of change of the object over a specific time interval. -- Proficient (Integral) Given the velocity function and/or graph, I can find the total displacement and/or distance traveled by an object. Given the acceleration function and/or graph and an initial velocity and/or position, I can find the total displacement and/or distance traveled by an object.	Advanced (Differential) -- Advanced (Integral)

Example Problems

Beginning (Differential)

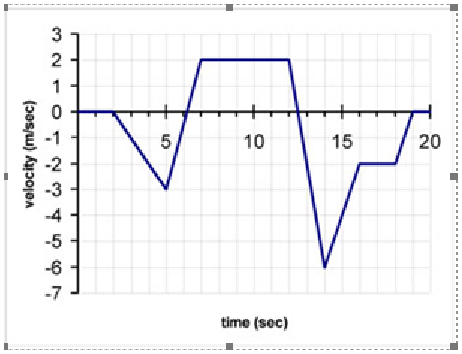
1)

Let $s(x) = x^3 - 7x^2 + 5x + 7$ describe the vertical distance a particle travels where s is in feet and x is in seconds.

a) Find the particle's velocity at 2 seconds.

b) Find the particle's acceleration at 2 seconds.

2)



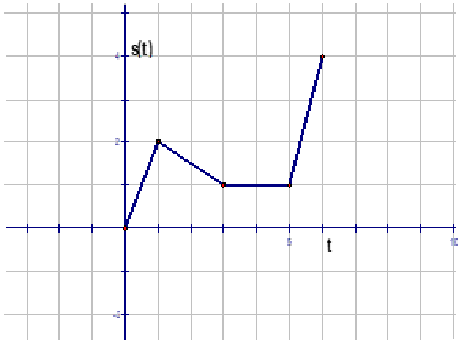
a) What is the velocity of this object after 5 seconds?

b) What is the acceleration of the object at 10 seconds?

Developing (Differential)

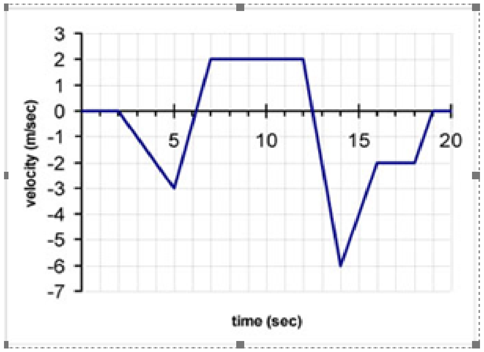
1)

The graph below represents the position of an object over a time interval from 0 to 6 seconds. Sketch the corresponding velocity function's graph next to it. (Label your axes.)



a) In the graph above, at what times does the object change direction.

b) Does the object ever stop? If so, when?

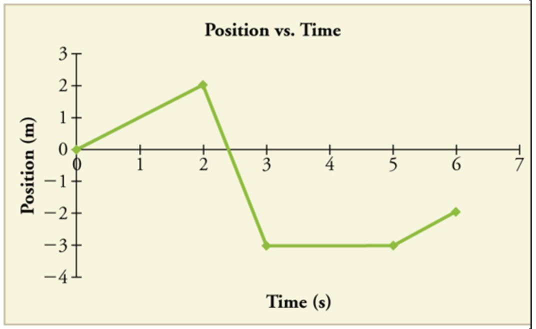


2)

a) During what time intervals is the object speeding up?

Proficient (Differential)

1)

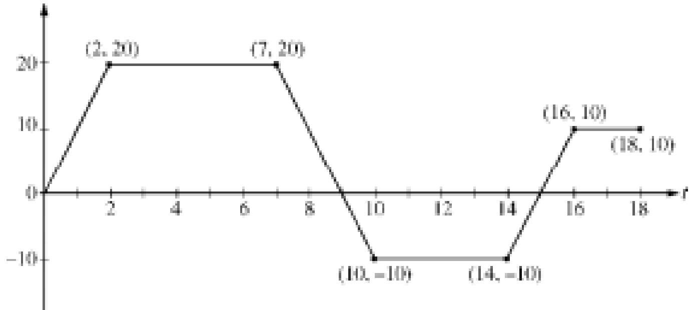


a) What is the average velocity over the interval [0, 6] seconds?

b) What is the instantaneous velocity at t = 1 second?

Advanced (Differential/Integral)

Next Page!



Graph of v

A squirrel starts at building A at time $t = 0$ and travels along a straight wire connected to building B. For $0 \le t \le 18$, the squirrel's velocity is modeled by the piecewise-linear function defined by the graph above.

(a) At what times in the interval $0 < t < 18$, if any, does the squirrel change direction? Give a reason for your answer.

(b) At what time in the interval $0 \le t \le 18$ is the squirrel farthest from building A? How far from building A is the squirrel at this time?

(c) Find the total distance the squirrel travels during the time interval $0 \le t \le 18$.

(d) Write expressions for the squirrel's acceleration $a(t)$, velocity $v(t)$, and distance $x(t)$ from building A that are valid for the time interval $7 < t < 10$.

2)

For $t \ge 0$, a particle moves along the x-axis. The velocity of the particle at time t is given by $v(t) = 1 + 2\sin\left(\frac{t^2}{2}\right)$. The particle is at position $x = 2$ at time $t = 4$.

(a) At time $t = 4$, is the particle speeding up or slowing down?

(b) Find all times t in the interval $0 < t < 3$ when the particle changes direction. Justify your answer.

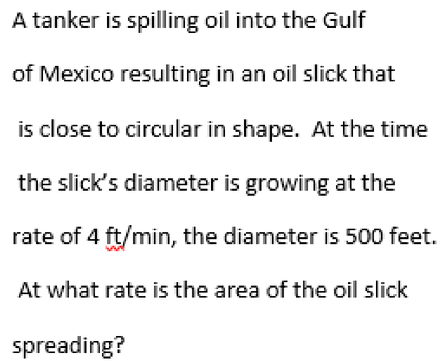
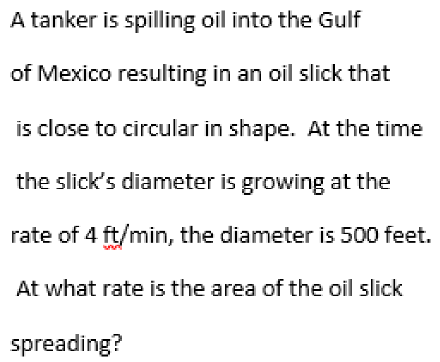
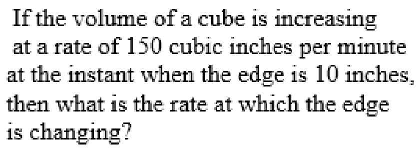
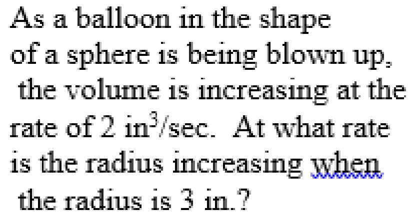
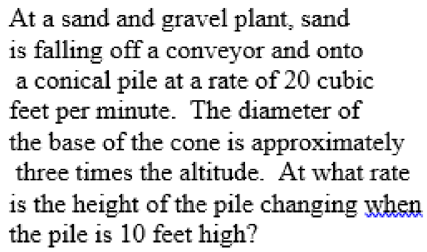
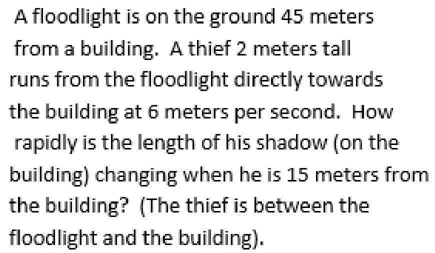
(c) Find the position of the particle at time $t = 0$.

(d) Find the total distance the particle travels from time $t = 0$ to time $t = 3$.

Implicit Differentiation - I can apply implicit differentiation to solve related rates problems.

Beginning	Developing	Proficient	Advanced
I can determine and define the known quantities and the unknown quantities in a related rate problem.	I can set up an equation/formula that relates the known quantities to the unknown quantities. I can determine what the unit label should be on the solution to a related rate problem.	Given a related rate problem, I can find a single primary equation/formula and can implicitly differentiate the equation to find an unknown in a related rate problem.	Given a related rate problem, I can use secondary equations to write a single primary equation and implicitly differentiate the equation to find the unknowns.

Example Problems

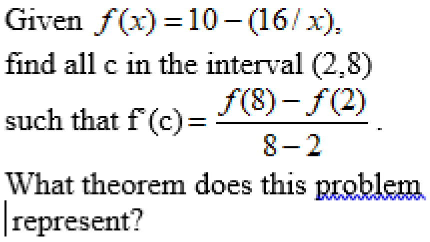
Beginning	Developing	Proficient	Advanced
1) A tanker is spilling oil into the Gulf of Mexico resulting in an oil slick that is close to circular in shape. At the time the slick's diameter is growing at the rate of 4 ft/min, the diameter is 500 feet. At what rate is the area of the oil slick spreading? a) Define the known quantities. b) Define the unknown quantities.	1) A tanker is spilling oil into the Gulf of Mexico resulting in an oil slick that is close to circular in shape. At the time the slick's diameter is growing at the rate of 4 ft/min, the diameter is 500 feet. At what rate is the area of the oil slick spreading? a) What formula/equation relates your known quantities to the unknowns? b) What will the unit label be on your answer?	1) If the volume of a cube is increasing at a rate of 150 cubic inches per minute at the instant when the edge is 10 inches, then what is the rate at which the edge is changing? 2) As a balloon in the shape of a sphere is being blown up, the volume is increasing at the rate of 2 in^3/sec. At what rate is the radius increasing when the radius is 3 in.?	1) At a sand and gravel plant, sand is falling off a conveyor and onto a conical pile at a rate of 20 cubic feet per minute. The diameter of the base of the cone is approximately three times the altitude. At what rate is the height of the pile changing when the pile is 10 feet high? 2) A floodlight is on the ground 45 meters from a building. A thief 2 meters tall runs from the floodlight directly towards the building at 6 meters per second. How rapidly is the length of his shadow (on the building) changing when he is 15 meters from the building? (The thief is between the floodlight and the building).

Mean Value Theorem - I can define and apply the Mean Value Theorem.

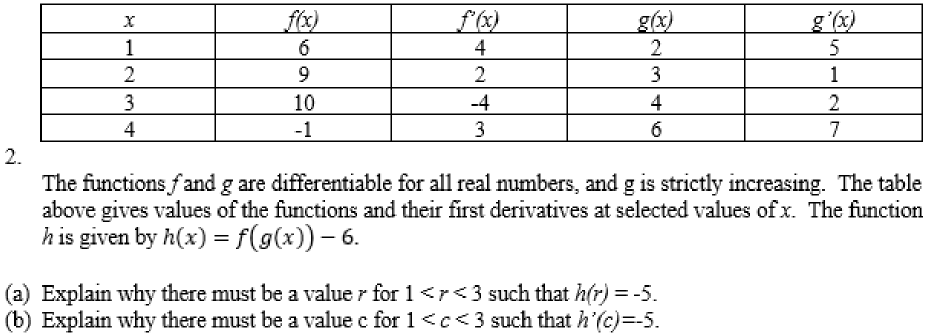
Beginning	Developing	Proficient	Advanced
I can write out what requirements must be met to apply the mean value theorem of derivatives.	I can give the total definition of the mean value theorem of derivatives and explain what it represents in multiple ways.	I can apply the mean value theorem of derivatives to a function and find a point that satisfies the theorem on the graph of the function.	I can apply the mean value theorem of derivatives to prove the existence of a point or set of values within a given interval.

Example Problems

<table>
<tr><th>Beginning</th><th>Developing</th><th>Proficient</th><th>Advanced</th></tr>
<tr>
<td>What conditions must be met in order to apply the mean value theorem to a function?</td>
<td>What is the mean value theorem for derivatives?

In your own words, describe in at least two different ways what the mean value theorem for derivatives represents.</td>
<td>

Given $f(x) = 10 - (16/x)$, find all c in the interval (2,8) such that $f'(c) = \frac{f(8) - f(2)}{8 - 2}$.

What theorem does this problem represent?</td>
<td>

<table>
<tr><th>x</th><th>f(x)</th><th>f'(x)</th><th>g(x)</th><th>g'(x)</th></tr>
<tr><td>1</td><td>6</td><td>4</td><td>2</td><td>5</td></tr>
<tr><td>2</td><td>9</td><td>2</td><td>3</td><td>1</td></tr>
<tr><td>3</td><td>10</td><td>-4</td><td>4</td><td>2</td></tr>
<tr><td>4</td><td>-1</td><td>3</td><td>6</td><td>7</td></tr>
</table>

2. The functions f and g are differentiable for all real numbers, and g is strictly increasing. The table above gives values of the functions and their first derivatives at selected values of x. The function h is given by $h(x) = f(g(x)) - 6$.

(a) Explain why there must be a value r for $1 < r < 3$ such that $h(r) = -5$.
(b) Explain why there must be a value c for $1 < c < 3$ such that $h'(c) = -5$.</td>
</tr>
</table>

Function Analysis - I can analyze a function and its derivatives to determine what a graph will look like.

Beginning	Developing	Proficient	Advanced
I can locate intercepts and points on the graph of the function. I can graph a function on my graphing calculator.	I can locate vertical, horizontal, and slant asymptotes on the graph of the function. I can determine the domain of the function. I can locate zeros on my graphing calculator.	I can determine end behavior of a polynomial function. I can determine possible critical values, extrema (absolute and relative) and intervals of increasing and decreasing behavior, concavity, and inflection points on the graph of the function and sketch the graph of the function. I can locate the derivative on a graph on a graphing calculator and locate extrema on an interval.	Given information in a word problem, I can apply the analysis of a functions graph to answer questions in the context of the word problem. I can determine unknown coefficients of a function given information on the functions extrema and/or concavity.

Example Problems

Beginning	Developing	Proficient	Advanced
1) $f(x) = 3x^2 - 12x$. Find the intercepts of the function given above.	1) $f(x) = \dfrac{4x}{x^2 - 1}$. a)Identify the asymptotes of the function given above. b)What is the domain of the function given above?	$f(x) = x^3 - 12x$. [0, 4] 1) a)Determine the critical values of the function over the given interval. b) Determine the absolute extrema of the function and the x-value at which it occurs over the given interval. c) Determine the end-behavior of the function over all real numbers. d) Over what intervals is the function increasing? e)Over what intervals is the function decreasing? f) Identify any inflection points over the domain of all real numbers. g)Determine the concavity of the function over the domain of the entire function. h) Sketch a graph of the function given.	1) Find a, b, c, and d such that the cubic $f(x) = ax^3 + bx^2 + cx + d$ satisfies the conditions: Relative Maximum: (3, 3) Relative Minimum: (5, 1) 1 Inflection Point: (4, 2)) 2) Multiple Choice The value of c for which $f(x) = x + \frac{c}{x}$ has a local minimum at x = 4 is a) -4 b) 8 c) -2 d) 4 e) 16 3) Grass clippings are placed in a bin, where they decompose. For $0 \le t \le 30$, the amount of grass clippings remaining in the bin is modeled by $A(t) = 6.687(0.931)^t$, where $A(t)$ is measured in pounds and t is measure in days. (a) Find the average rate of change of $A(t)$ over the interval $0 \le t \le 30$. Indicate units of measure. (b) Find the value of $A'(15)$. Using correct units, interpret the meaning of the value in the context of the problem. (c) Find the time t for which the amount of grass clippings in the bin is equal to the average amount of grass clippings in the bin over the interval $0 \le t \le 30$. (d) For $t > 30$, $L(t)$, the linear approximation to A at $t = 30$, is a better model for the amount of grass clippings remaining in the bin. Use $L(t)$ to predict the time at which there will be 0.5 pound of grass clippings remaining in the bin. Show the work that leads to your answer. (only parts a, c, d)

Optimization - I can solve optimization problems.

Beginning	Developing	Proficient	Advanced
I can define the knowns and unknowns in an optimization word problem and determine what it is that I must optimize.	I can create a primary formula/equation that relates each of the variables (including using a secondary equation if necessary). I can determine a realistic domain for the solution set to be in.	In the context of a word problem, I can determine the potential critical values and end points and test each to determine if it is where an absolute extreme value occurs.	I can apply the concepts of extrema to a complex optimization word problem.

Example Problems

Beginning	Developing	Proficient	Advanced
1) A farmer has 360 ft. of fencing to enclose 2 adjacent rectangular pig pens. What dimensions should be used for each pig pen so that the enclose area will be a maximum? What are you trying to maximize in this problem?	1) A farmer has 360 ft. of fencing to enclose 2 adjacent rectangular pig pens. What dimensions should be used for each pig pen so that the enclose area will be a maximum? Define the knowns and unknowns in the problem above and create a primary equation that relates each.	1) A farmer has 360 ft. of fencing to enclose 2 adjacent rectangular pig pens. What dimensions should be used for each pig pen so that the enclose area will be a maximum? 2) The position function of a moving particle is $s(t) = 5 + 4t - t^2$ for $0 \leq t \leq 10$ where s is in meters and t is measured in seconds. What is the maximum speed in m/sec of the particle on the interval $0 \leq t \leq 10$?	1)A light source is located over the center of a circular table of diameter 4 feet. Find the height h of the light source such that the illumination I at the perimeter of the table is maximum if $I = k(sin\ \alpha)/s^2$, where s is the slant height, αis the angle at which the light strikes the table, and k is a constant. 2) In marketing a certain item, a business has discovered that the demand for the item is represented by $p = 50/(\sqrt{x})$ The cost of producing x items is given by $C = 0.5x + 500$. Find the price per unit that yields a maximum profit.

Antiderivatives - I can find the antiderivative of a function.

Beginning	Developing	Proficient	Advanced
I can find the antiderivative of 0, a constant, and a polynomial function.	I can find the antiderivative of sine, cosine, and tangent.	I can find an antiderivative of a function using u-substitution. I can apply the antiderivative to find the average value of a function over a closed interval.	I can find the particular solution to a function given its derivative and an initial condition. I can find the antiderivative of secant, cosecant, and cotangent functions.

Example Problems

Beginning	Developing	Proficient	Advanced
1. Evaluate: $\int (2x^4 + 3x^2 - 2x)dx$	$\int (sin\, 2x)\, dx$	Evaluate $\int \frac{x^2}{\sqrt{x^3 - 5}} dx$. Find the average value of $f(x) = x^2 - 2\ over\ [-1,\ 3]$.	5. Solve the initial value problem for y as a function of x given: $\frac{dy}{dx} = 2x - 6$; and an initial condition f(1) = 4. An object is thrown upward. After 1 second its velocity is 70 ft/sec. Find its velocity after 3 sec. (Hint: Because of gravity the acceleration of a falling body is 32 ft/sec). Evaluate $\int \csc(2x + 1)\, dx$

Area Approximations - I can use Riemann Sums and trapezoidal approximations to approximate areas under a curve.

Beginning	Developing	Proficient	Advanced
I can find the area of a trapezoid.	I can complete a summation in sigma notation using the shortcut rules.	I can find the area under a curve using summation rules and the limit process. I can find the area under a curve using a definite integral. I can use the trapezoidal rule to approximate the area under a curve using n trapezoids of equal height. I can use a Riemann sum to approximate the area under a curve using n rectangles of equal width.	I can apply definite integrals, Riemann Sums, and/or the trapezoidal rule to answer questions in a word problem.

Example Problems

Beginning	Developing	Proficient	Advanced
Estimate the area under the curve $f(x) = x^2$ over the interval [-2, 2] using 4 equal width right endpoint rectangles. Is this an overestimate or underestimate of the actual area?	Let $s(n) = \sum_{i=1}^{n}(1+\frac{i}{n})^2(\frac{1}{n})$. Find the limit of s(n) as $n \to \infty$.	Find the area of a region bounded by the curve $y = x^2 - 2x + 3$ and the x-axis from x = 0 to x = 2. Use the trapezoidal rule to approximate $\int_2^3 \frac{1}{(x-1)^2}dx$. Let n = 4.	See Below

Advanced

t (hours)	0	1	3	4	7	8	9
L(t) (people)	120	156	176	126	150	80	0

Concert tickets went on sale at noon (t = 0) and were sold out within 9 hours. The number of people waiting in line to purchase tickets at time t is modeled by a twice-differentiable function L for $0 \le t \le 9$. Values of $L(t)$ at various times t are shown in the table above.

(a) Use the data in the table to estimate the rate at which the number of people waiting in line was changing at 5:30 P.M. (t = 5.5). Show the computations that lead to your answer. Indicate units of measure.

(b) Use a trapezoidal sum with three subintervals to estimate the average number of people waiting in line during the first 4 hours that tickets were on sale.

(c) For $0 \le t \le 9$, what is the fewest number of times at which $L'(t)$ must equal 0 ? Give a reason for your answer.

Fundamental Theorems of Calculus - I can apply the First and Second Fundamental Theorems of Calculus.

Beginning	Developing	Proficient	Advanced
I can define the 1st Fundamental Theorem of Calculus.	I can evaluate a definite integral using the 1st Fundamental Theorem of Calculus.	I can apply the 1st Fundamental Theorem of Calculus to find the area under a curve.	I can use the definite integral to find the average value of a function over a given interval. I can apply the 2nd Fundamental Rule of calculus.

Example Problems

Beginning	Developing	Proficient	Advanced
Define the 1st Fundamental Theorem of Calculus.	Use the 1st Fundamental Theorem of Calculus to evaluate $\int_{-2}^{1}(1-2x)dx$	If $\int_k^2 (2x-2)dx$ = -3, a possible value of k is $d/dx[\int_2^x (3t\ -\ 1)dt]$ Approximate the area under the graph of y and above the x-axis from x =0 to x = 2 for the function $f(x) = x^2 - 2x + 3$.	Find the average value of $y = 2\sin(2x)$ on the interval [0, π/6] $g(x) = \int_1^x \frac{3t}{t^3+1}dt$, then $g'(2)$

Area and Volume - I can find the area between curves and the volume of a solid of revolution.

Beginning	Developing	Proficient	Advanced
I can determine the orientation of my representative rectangles. I can write the correct definite integral to represent the volume of a solid of revolution.	I can find the area of a region bounded by two curves.	I can find the volume of a solid of revolution using the disc, washer, or shell method.	I can find the volume of a solid that has geometric cross-sections using multiple methods.

Example Problems

Beginning	Developing	Proficient	Advanced

Logarithms and Exponentials - I can differentiate and integrate logarithmic and exponential functions.

Beginning	Developing	Proficient	Advanced
I can apply the rules of logarithms and exponents to simplify an expression.	I can solve an exponential or logarithmic equation.	I can differentiate and integrate logarithmic and exponential functions. I can apply logarithms and/or exponential rules to solve compound interest formulas and exponential growth and/or decay problems.	I can solve an initial value problem using separation of variables.

Example Problems

Beginning	Developing	Proficient	Advanced
Expand $ln(x^2 y/z)$ Condense ln x - 2lny + ln z	Solve $ln\,x\ =\ 3$	$d/dx[ln(x^3 + 1)]$ $\int(2x - 3)/(2x^2 - 6x)\,dx$	Solve the equation dy /dx = (y + 1)/ (x − 1) given the boundary condition: y = 1 at x = 0

Differential Equations - I can separate variables and use directional fields to solve differential equations.

Beginning	Developing	Proficient	Advanced

Example Problems

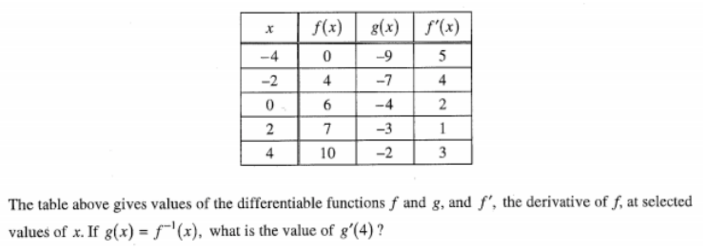
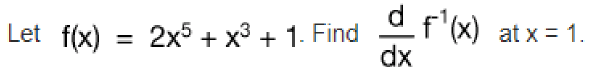
Beginning	Developing	Proficient	Advanced

Calculus Standards **St. Croix Central High School**

Inverse Functions - I can differentiate and integrate inverse functions.

Beginning	Developing	Proficient	Advanced
I can verify that two functions are inverses and determine if a function has an inverse by applying the horizontal line test.	I can find the inverse to a function that has an inverse.	I can find the derivative of an inverse function.	I can differentiate and integrate inverse trigonometric functions.

Example Problems

<table>
<tr><th>Beginning</th><th>Developing</th><th>Proficient</th><th>Advanced</th></tr>
<tr>
<td>1) Does the function $f(x) = x^2$ have an inverse over the set of all real numbers?
2) Verify that the inverse to $f(x) = x^3 + 5$ is $g(x) = (x - 5)^{1/3}$</td>
<td>3) Find the inverse to $f(x) = 2x^3 + 1$.</td>
<td>4)

<table><tr><th>x</th><th>$f(x)$</th><th>$g(x)$</th><th>$f'(x)$</th></tr><tr><td>-4</td><td>0</td><td>-9</td><td>5</td></tr><tr><td>-2</td><td>4</td><td>-7</td><td>4</td></tr><tr><td>0</td><td>6</td><td>-4</td><td>2</td></tr><tr><td>2</td><td>7</td><td>-3</td><td>1</td></tr><tr><td>4</td><td>10</td><td>-2</td><td>3</td></tr></table>The table above gives values of the differentiable functions f and g, and f', the derivative of f, at selected values of x. If $g(x) = f^{-1}(x)$, what is the value of $g'(4)$?
5)  Let $f(x) = 2x^5 + x^3 + 1$. Find $\frac{d}{dx} f^{-1}(x)$ at x = 1.</td>
<td>6) Find $d/dx[arctan(3x)]$
7) Evaluate $\int dx/(\sqrt{4 - x^2})$</td>
</tr>
</table>