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B.Sc. (Hons.) Math (Semester – 5th)

LINEAR ALGEBRA

Subject Code: BMATS1521

Paper ID: 22131220

Time: 03 Hours **Maximum Marks: 60**

Instruction for candidates:

1. Section A is compulsory. It consists of 10 parts of two marks each.
2. Section B consist of 5 questions of 5 marks each. The student has to attempt any 4 questions out of it.
3. Section C consist of 3 questions of 10 marks each. The student has to attempt any 2 questions.

Section – A **(2 marks each)**

Q1. Attempt the following:

a) Let T:$V(F)\rightarrow U(F)$ be a linear transformation. Show that Range space of T is a subspace of $U(F)$.

b) Find matrix representing the linear transformation T:$R^3 \rightarrow R^4$ defined by $T(x, y, z) = (x + y, 2z - x, 3y, 4z)$ relative to standard basis of $R^3$and R^4.

c) Prove that a real square matrix has non real eigenvalues in conjugate pair.

d) Let S be an orthonormal set of vectors in an inner product space V(F). Prove that S is a linearly independent set.

e) Define Linear Transformation.

f) Show that the similar matrices have the same characteristic polynomial and hence the same eigen values.

g) Define null space of a linear transformation. Find null space of T: $R^{2*2} \rightarrow R^2$ defined by T(A)= (b, trace(A))where A=$[a\ b\ c\ d\ ]$.

h) Let T be a linear operator on R^3which is represented in the standard ordered basis by the matrix A =$[1\ 2\ 4\ 0\ 3\ 6\ 0\ 0\ 4\ ]$. Find the minimal polynomial of T.

i) Prove that the linear operator T on R^3 defined by T(x, y, z) = (-9x+4y+4z, -8x+3y+4z, -16x+8y+7z)is diagonalizable.

j) Define Diagonalization.

Section – B **(5 marks each)**

Q2. State and prove Cayley– Hamilton theorem.

Q3. Let V and W be a finite dimensional vector space over the field F and let T: V → W be a linear transformation with dim V =dim W. Prove that T is one-one iff T is onto.

Q4. If two vector spaces are isomorphic then prove that they have same dimension.

Q5. Find the characteristic polynomial of A = $[0\ 4\ 2\ -3\ 8\ 3\ 4\ -8\ -2\ ]$.

Q6. Let T be a linear operator on a finite dimensional vector space V(F). Prove that T is onto iffT carries each linearly independent subset of V onto a L.I. subset of V.

Section – C **(10 marks each)**

Q7. Solve the system of equations: x+2y+3z=2; x-y+3z=0; 2x-3y+4z=2.

Q8. Let Vand W be vector space over the field F. Let L(V,W) be the set of all linear transformations from V intoW . Prove L(V,W)is a vector space over F.

Q9. Prove that the minimal polynomial of a linear operator is unique.