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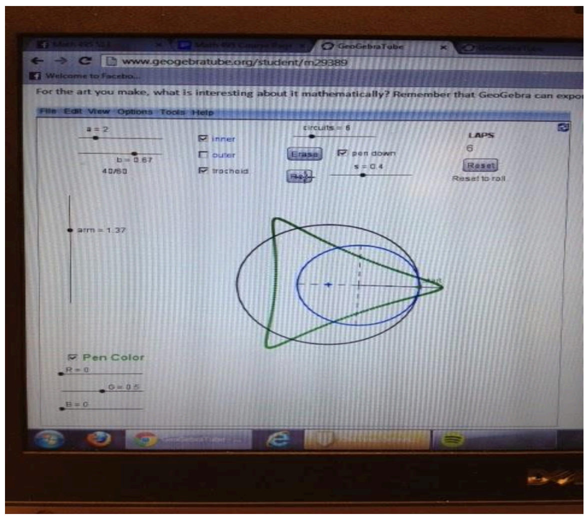
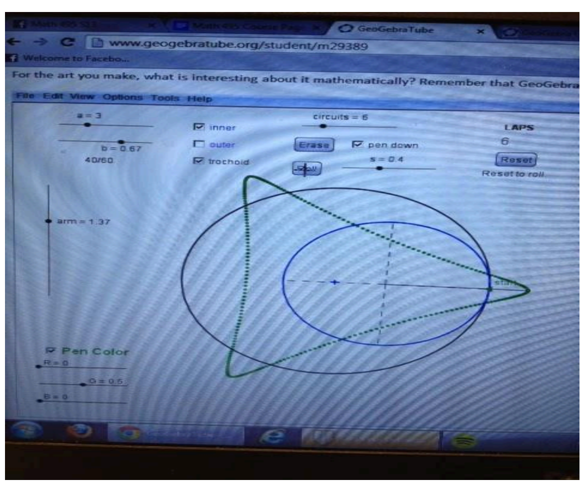
6/20/13
Doing Math-Hypocycloids

As a kid one of my toys was a spiral art set and I remember how cool it was to be able to create different patterns with various colors. Of course at the age I was I had no idea it involved math whatsoever. When we talked about Cardano and his interest in hypocycloids I also became interested in exploring hypocycloids. I chose to play around with the link to the student worksheet that was made available on our course google document.

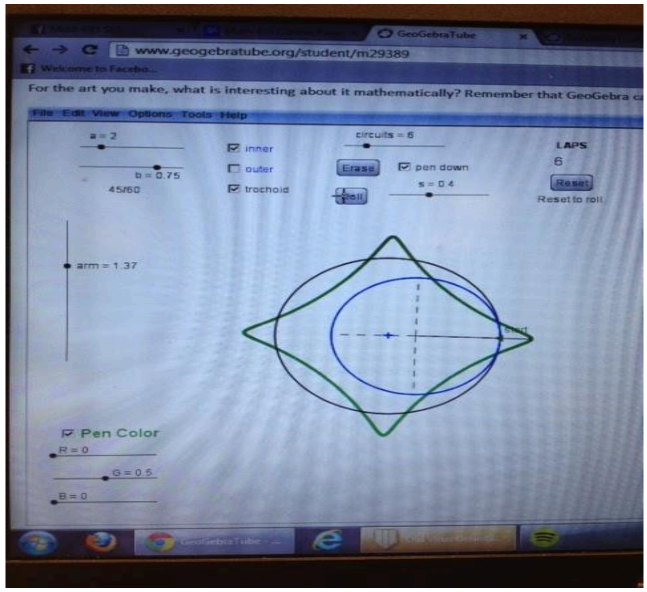
On the student worksheet it asks “What do the controls do”, so I decided to start my exploring with this question. Leaving the controls at their initial values I clicked the button “roll” to see the image it would create. Below is the image the initial settings created.



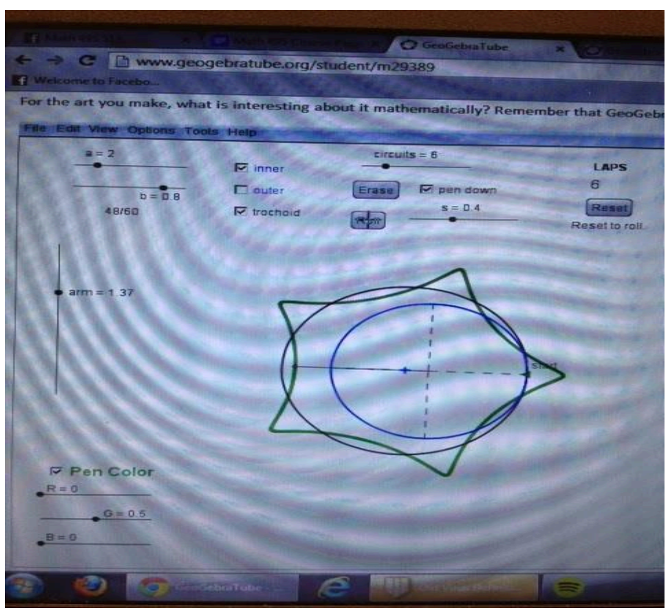
I then speculated that the value for *a* would make the inner circle bigger. What I found was that it just made the ratio between the inner/outer circle bigger. You can see this in the picture below.



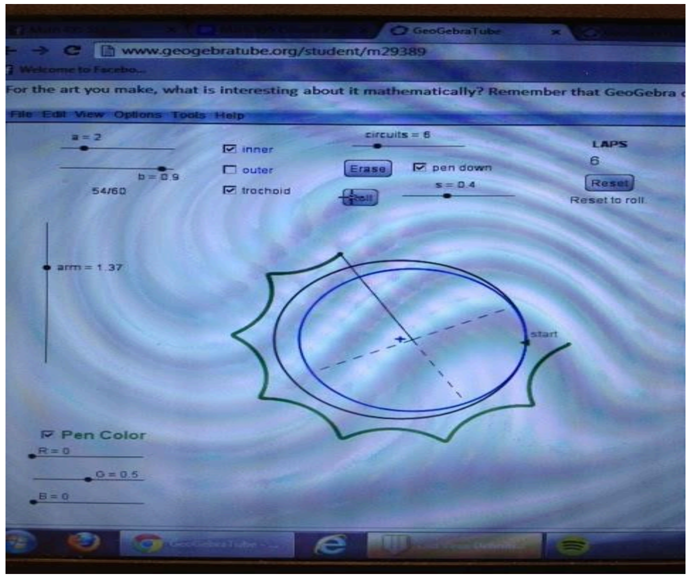
I then wondered what would happen when I changed the value for *b* so I changed it from .67 to .75 leaving the value for *a*=2(its original value).Pictured below is the image from that. As you can see, the shape of the image drawn changed from a 3 pointed figure to a 4 pointed figure.



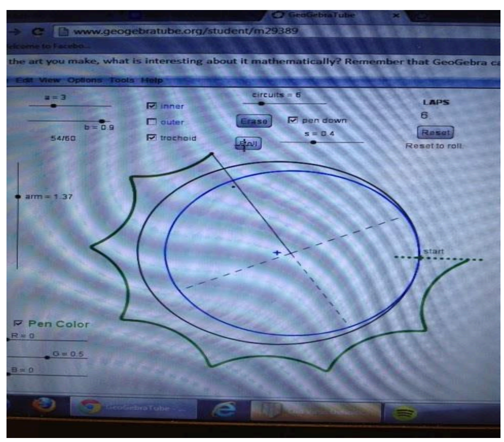
I was curious if this image would change if I again change the value for *b* to .8. Pictured below is a star that the value created.



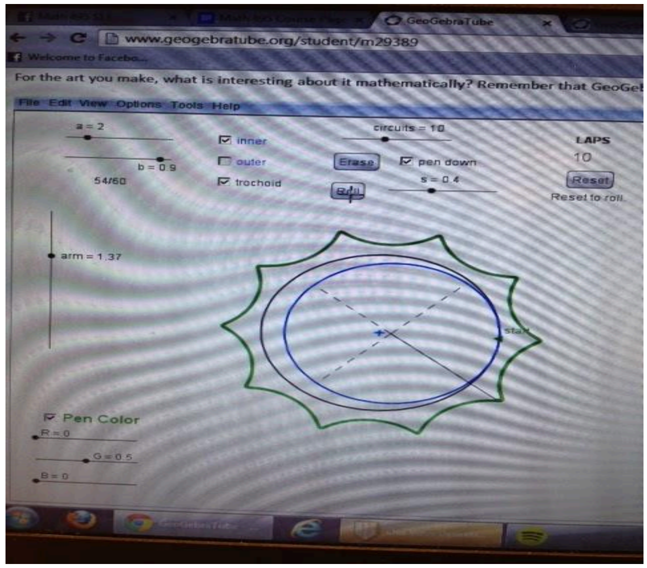
When b=.9 and the other controls are set to the original settings the result is shown below. As you can see the image is incomplete at lap 6.



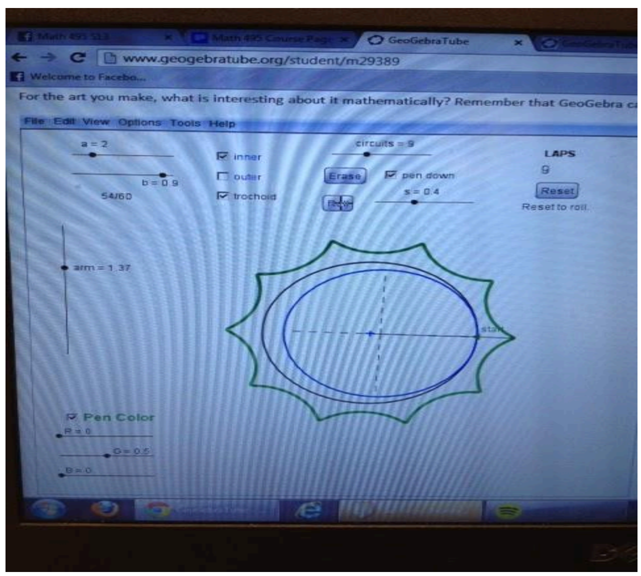
I was confused as to why the image didn't complete so I wondered if it had something to do with the value of *a* and *b.*I decided to change the controls so that a=3 and b=.9 but the image still stopped before it was complete at lap 6. As you can see the image only got bigger and still stopped at the same place.



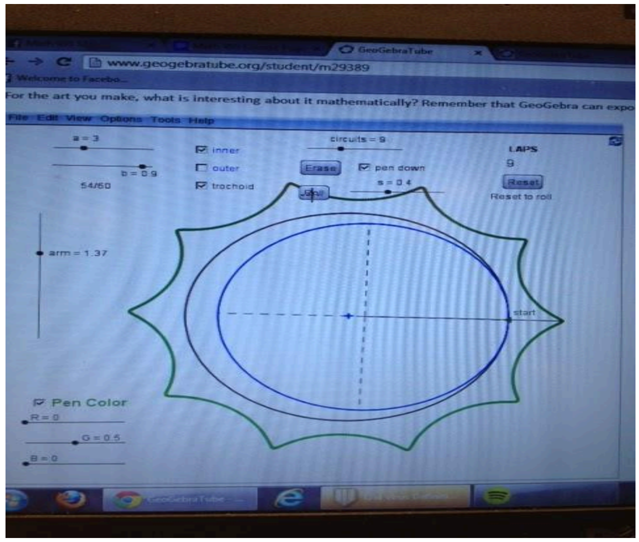
I then decided to leave a=2, b=.9 and change the circuit from 6 to 10. This time the image completed but it went one extra swoop past where it initially started. I consider a swoop to be from the top of one point to the top of another point consecutively following the first. Result is shown below.



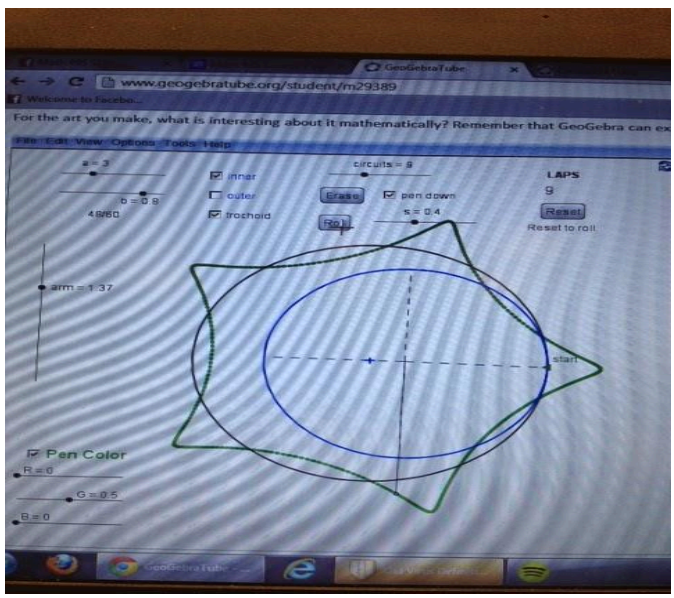
I then wondered if I changed the circuit to 9 and left everything else the same as above if it would start and stop at the same spot when I click the “roll” button. Fortunately, it did!



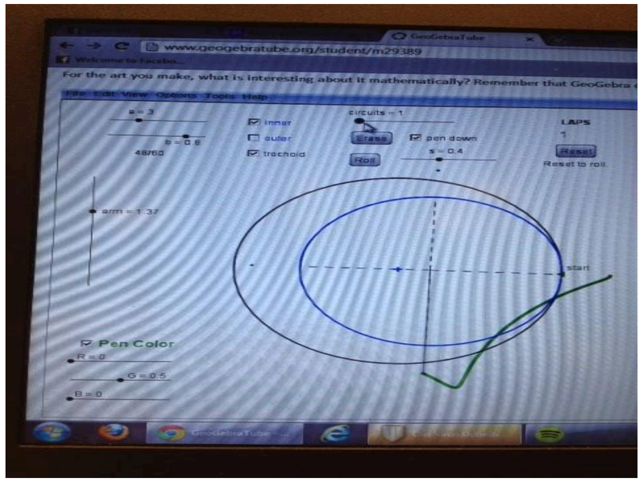
Then I set the controls a=3 and left the circuit at 9 and all the other controls the same as above to see if the size affected what number the circuit had to be set at in order for it to start and stop at the same point. Looking at the picture below you can see that changing the value for *a* had no effect on the image starting and stopping at the same point.



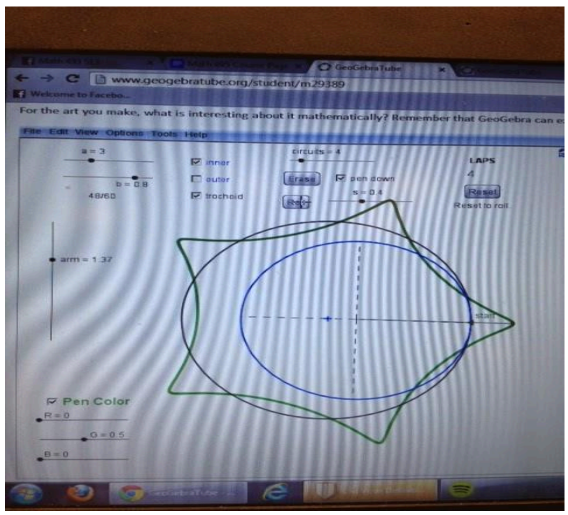
I then changed *b* to .8 to see if *b* related circuit and laps. I found that there was no correlation.



What I did notice though was that the circuit refers to how many times the inner circle center rotates around the outer circles' center. After this discovery I then asked myself, "Now what"? So I decided to change the circuit to 1 to see if I could predict how many circuits it takes for the cycle to stop where it started. As you can see in the picture above when b=.8 and everything else stayed the same, the image didn't start and stop in the same place. After seeing what 1 circuit created I estimated that if the circuit was set to 4 that the image would start and stop in the same place. Looking at the picture below, notice that my prediction was correct.

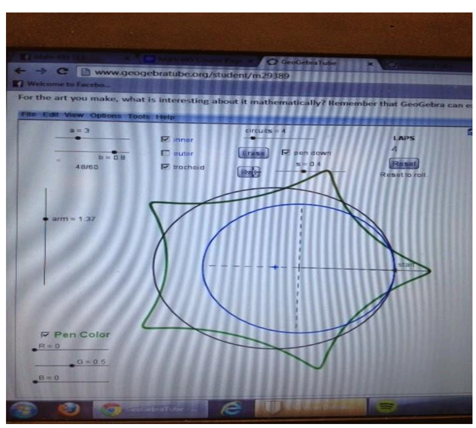


This is the image created when the circuit is set to 1.



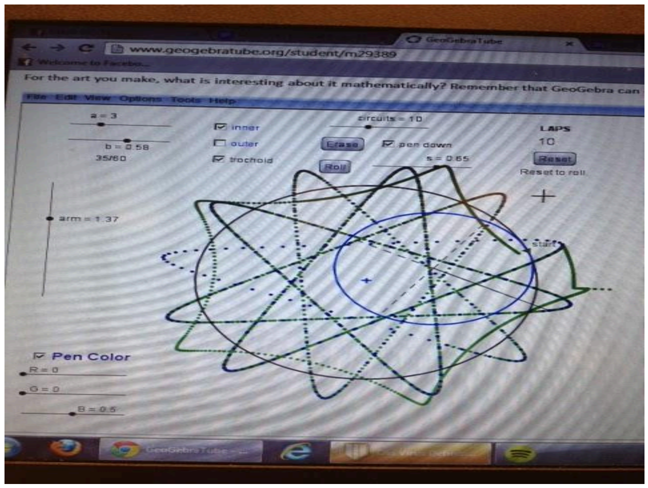
This is the image created when the circuit is set to 4.

After the previous investigation I decided to explore the value of *s*. In the previous images s=.4 so I changed it to .5. I didn't notice a change from the previous image to the next which is shown below.

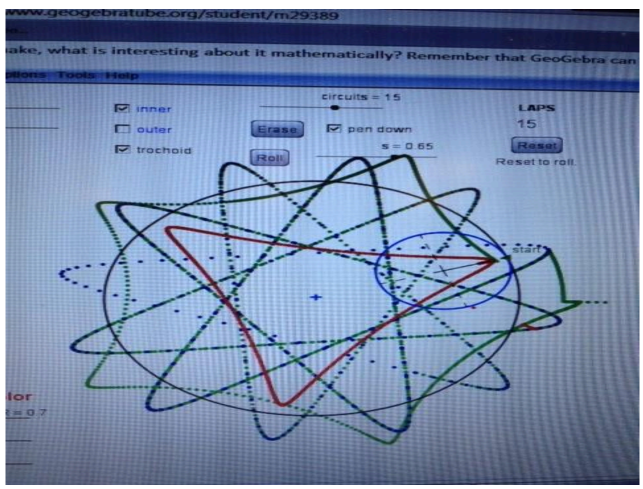


I then changed the control so that s=.9 and noticed that the speed of the circuits increased and more dots appeared along the path. When s=.1 the circuit gets very slow. I then realized that at s=0 the circuit will stop, which makes sense since the speed is 0. I took pictures of these steps but they are not easy to see the dots along the line of the image and obviously you are unable to see speed in a still picture of this.

The image below is a result of playing around with s=0 while changing the value of b and resuming with s equal to a number larger than 0 as well as changing the circuit value.

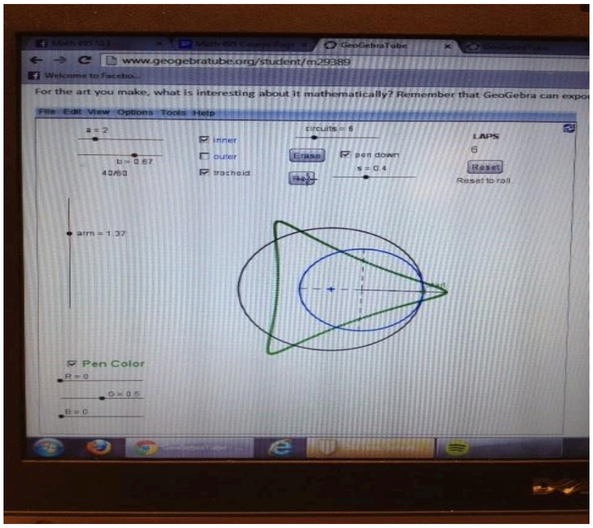


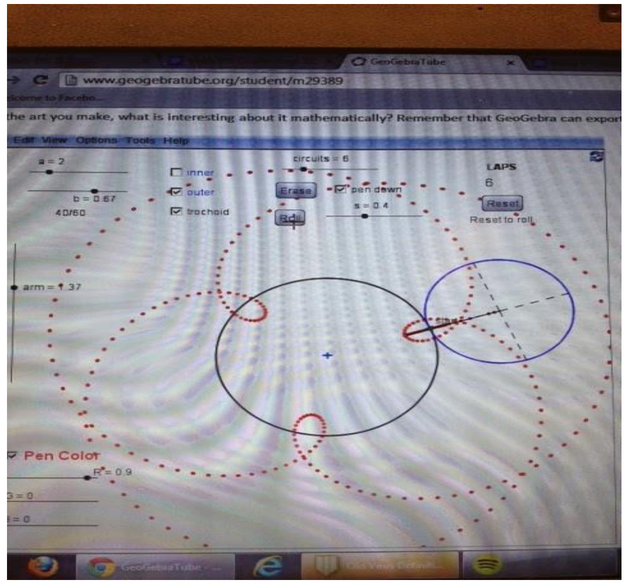
The pen down option allows you to change the arm length with ot without creating a path this is also represented in the picture above and below.



The image above is a result of change in the arm length (radius of the inner circle) and the color using the color combinations in the bottom left corner of the student worksheet.

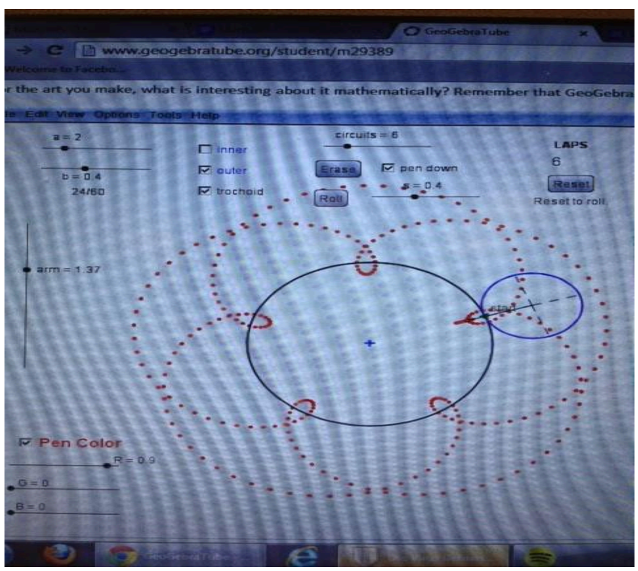
The next and last control I chose to look at was the outer option rather than the inner option that I have been using thus far. To remind you, the original settings for the program are as follows: a=2, b=.67, circuit is set to 6, s=.4 and arm is set to 1.37. The only difference between the very first image I presented in this weekly writing and the one below is that the “outer” option is selected rather than the “inner”. As you can see though, this slight change creates a completely different image than the first image.



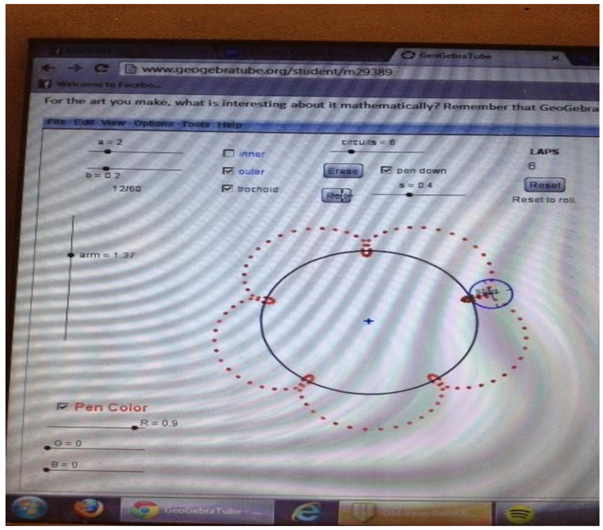


First image with the inner selection. Second image with the outer selection.

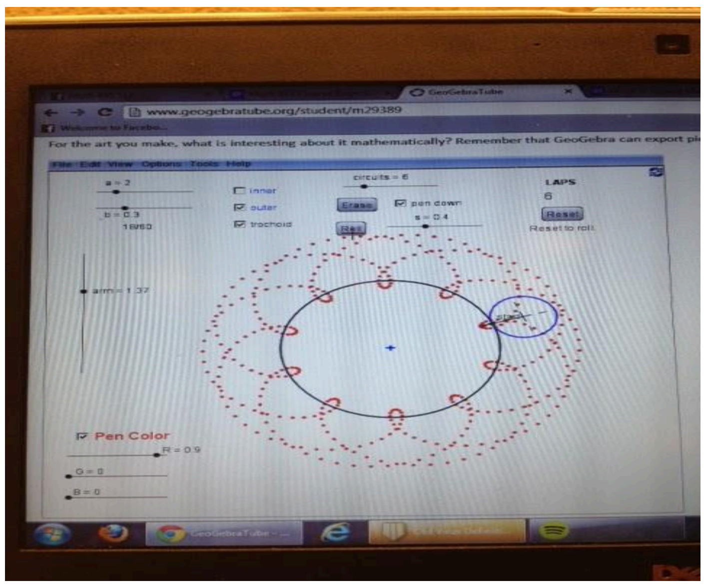
I then changed the control b=.4 and noticed that there are now 5 petals instead of 3 (shown below). I consider a petal to be one loop around the inner solid black circle.



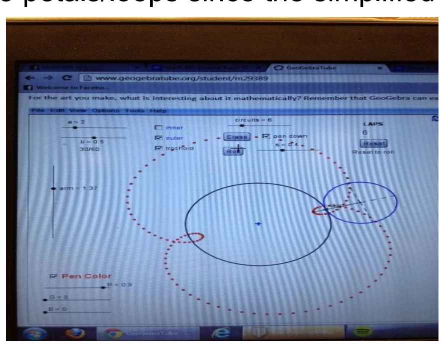
I had speculated that if b=.2 there would be 7 petals. Notice by looking at the image below that my prediction was again, incorrect. There are still 5 petals but I noticed they aren't overlapping like in the previous images.

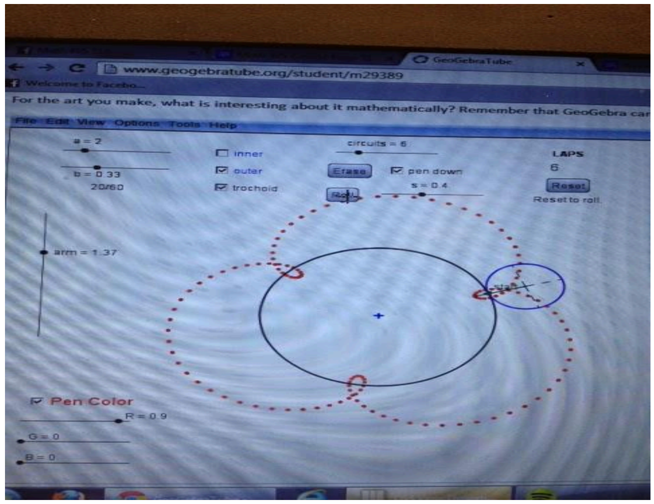


When b=.3 there are 10 petals.



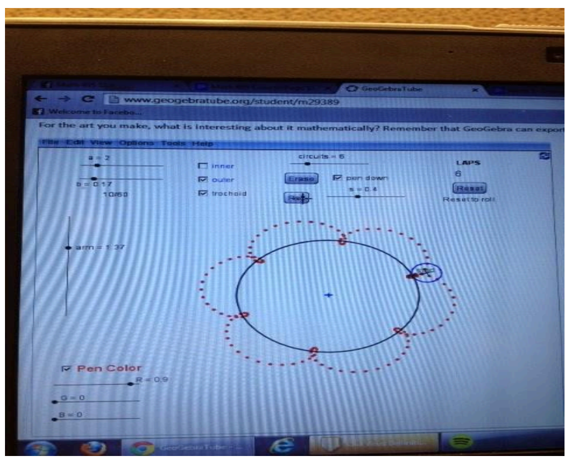
When b=.5 it makes two large petals (pictured below). I found this to be interesting and noticed that the ratio below the value of a and b was 30/60 or ½. This made me wonder if when the ratio is 20/60 if there would be 3 petals/loops since the simplified ratio is ⅓.



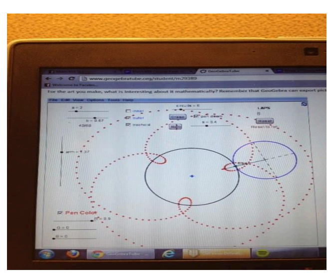


Looking at the image above you can see that my hypothesis was proven for this case; there are three petals.

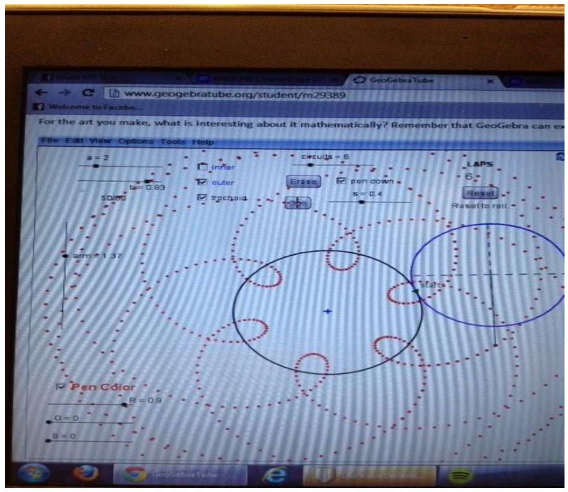
To continue my investigation I predicted that if the ratio was set to 10/60 or ⅙ there would be 6 petals, and again it held to be true.



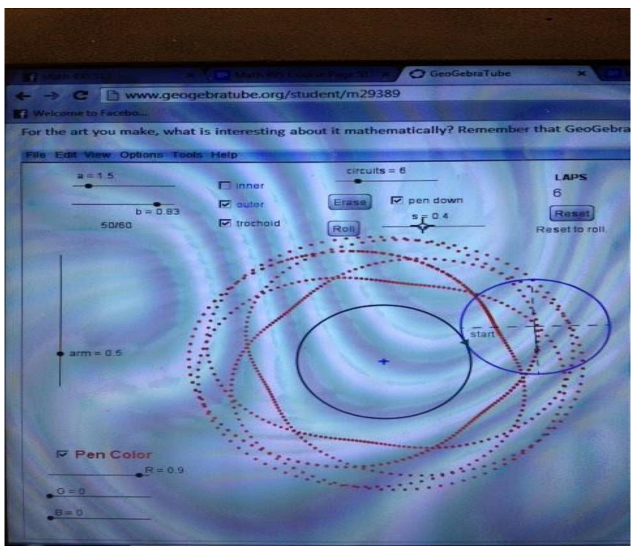
If the ratio was set to 40/60 I predicted that there would be two larger loops/petals and three smaller ones. You can see below that this was not the case.



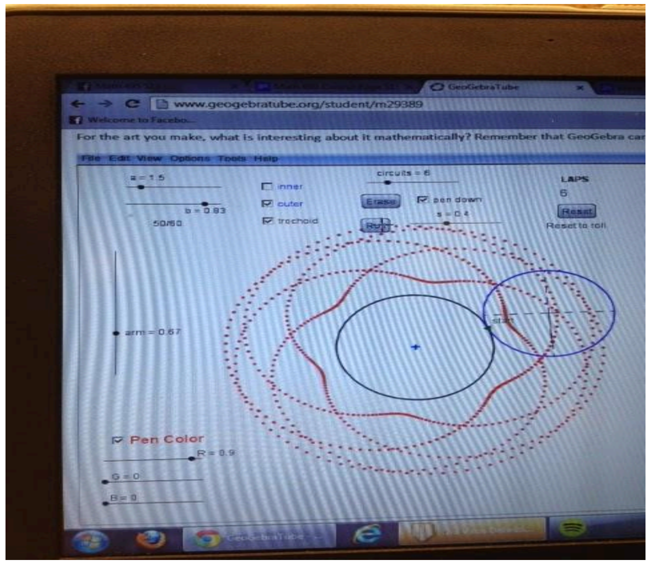
I hadn't predicted anything for when the ratio is 50/60 but the result is presented below.



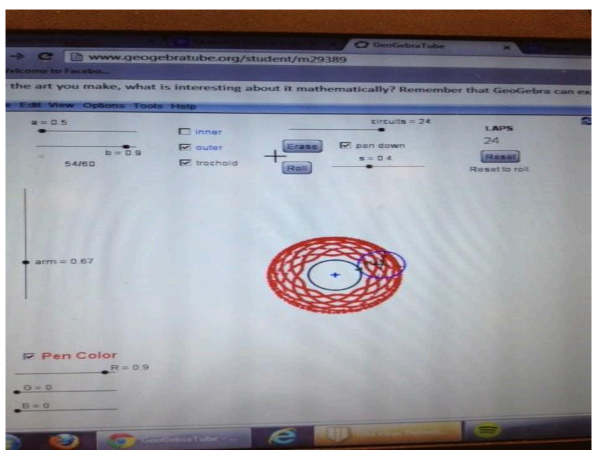
I then somewhat gave up on investigating a relationship between the ratio and the petals so I changed the arm length to .5.



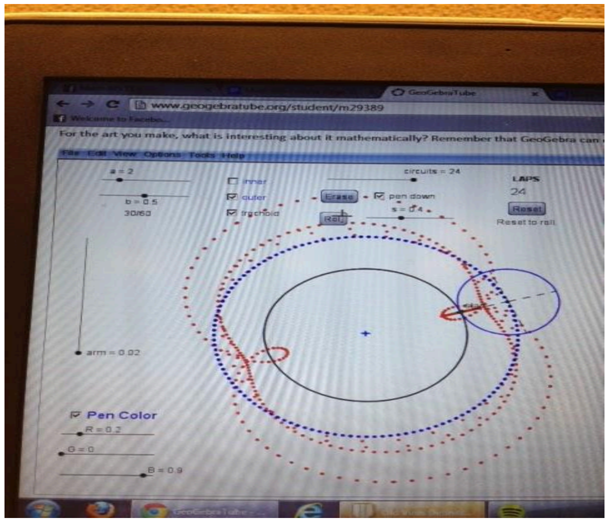
This led me to another question, when the arm length is smaller does the image become more circular? So I made the arm length bigger and set it to .67. The result is shown below. As you can see the image is still quite circular.



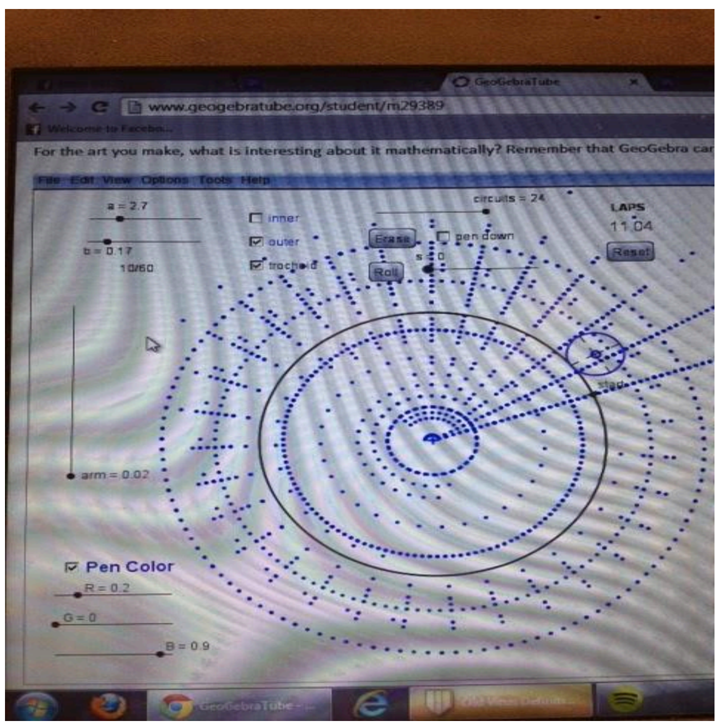
I then wondered if maybe the relationship between the values of a and b have something to do with its shape being more or less circular so I changed the controls to be a=.5, b=.9 arm=.67 and the circuits were set to 24. Notice in the result below the image is smaller and more circular.



To observe another result I changed the values of a and b to be smaller so a=2 and b=.5 and the arm length was set to .02. Observe below that the shape created in blue is more circular and the arm length is shorter than the previous image. The red lines you see are initial values which I manipulated until I reached the desired circular shape.



This concludes my investigation of hypocycloids using the student worksheet provided. I enjoyed exploring the mathematics using this creative application. It was very cool to be able to relate it to a popular childs toy and explore patterns in which I noticed or test hypothesis I came up with throughout my exploration. It's amazing how much technology has evolved and how much we have at our fingertips that allows us to explore mathematics in this non traditional pencil/paper way. The last image I created which is shown below was more of a "for fun" image, although I did have fun throughout this whole weekly writing while doing math!



**If you need to edit it to make the images large enough to see, let me know and I'll make it so you can edit it. As far as feedback goes, do you recommend another aspect of the hypocycloids that is worth investigating?