2. *Vectors*

1	$\begin{pmatrix}3\\2\\-2\end{pmatrix}-\begin{pmatrix}2\\1\\-3\end{pmatrix}=\begin{pmatrix}1\\1\\1\end{pmatrix}$	M_1 A_1	✔exp
2	(a) (i) $4p-3q=\begin{pmatrix}10\\5\end{pmatrix}\times 1$ $P+2q=\begin{pmatrix}-14\\15\end{pmatrix}\times 4$ $4p-3q=\begin{pmatrix}10\\15\end{pmatrix}$ $4p+8q=\begin{pmatrix}-56\\60\end{pmatrix}$ $-11q=\begin{pmatrix}66\\-55\end{pmatrix}$ $q=\begin{pmatrix}-6\\5\end{pmatrix}$ $p+2\begin{pmatrix}-6\\5\end{pmatrix}=\begin{pmatrix}-14\\15\end{pmatrix}$ $p+\begin{pmatrix}-12\\10\end{pmatrix}=\begin{pmatrix}-14\\15\end{pmatrix}$ $p=\begin{pmatrix}-2\\5\end{pmatrix}$ $q=\begin{pmatrix}-6\\5\end{pmatrix}$ and $p=\begin{pmatrix}-2\\5\end{pmatrix}$ (ii) $\lvert p+2q\rvert$ $=\begin{pmatrix}-2\\5\end{pmatrix}+\begin{pmatrix}-12\\10\end{pmatrix}$ $\begin{pmatrix}-14\\15\end{pmatrix}=\sqrt{(-14)^2+(15)^2}=\sqrt{196+225}=\sqrt{421}=20.52$ (b) $\vec{AB}=\begin{pmatrix}5\\3\end{pmatrix}-\begin{pmatrix}-1\\1\end{pmatrix}=\begin{pmatrix}6\\2\end{pmatrix}$	M_1 M_1 A_1 A_1 M_1 A_1	

	$\vec{BC} = \begin{pmatrix} 11 \\ 5 \end{pmatrix} - \begin{pmatrix} 5 \\ 3 \end{pmatrix} = \begin{pmatrix} 6 \\ 2 \end{pmatrix}$ $AB = kBC$ $AB = 1BC$ B (3, 5) is common AB is a scalar multiple of BC. Hence A (1, -1), B (3,5) and C (5, 11) are collinear	B_1 B_1 B_1 A_1	Scalar 1 Correct pt B
3	i) $\underset{\sim}{P} = 2\begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix} - \frac{1}{3}\begin{pmatrix} 6 \\ -3 \\ 9 \end{pmatrix} + \begin{pmatrix} -3 \\ 2 \\ 3 \end{pmatrix}$ $= \begin{pmatrix} 2 \\ -4 \\ 2 \end{pmatrix} - \begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix} + \begin{pmatrix} -3 \\ 2 \\ 3 \end{pmatrix}$ $= \begin{pmatrix} -3 \\ -1 \\ 2 \end{pmatrix}$ ii) $\lvert P \rvert = \sqrt{9+1+4}$ $= \sqrt{14} = 3.742$	M1 A1 B1	
		3	
4.	$\underset{\sim}{PQ} = \begin{pmatrix} -2 \\ -1 \end{pmatrix} - \begin{pmatrix} -6 \\ -3 \end{pmatrix} = \begin{pmatrix} 4 \\ 2 \end{pmatrix}$ $\underset{\sim}{QR} = \begin{pmatrix} 6 \\ 3 \end{pmatrix} - \begin{pmatrix} -2 \\ -1 \end{pmatrix} = \begin{pmatrix} 8 \\ 4 \end{pmatrix}$ 2 $\underset{\sim}{PQ} = \underset{\sim}{QR}$ multiples of each other Q is common point hence PQ and R are collinear	B1 B1 B1	
		03	

1.

$Sin\ \overline{60} = \sqrt{3}/_2$ $\qquad$ $\underline{\qquad 1 \qquad}$

$\sin 45 = {}^{1}/_{\sqrt{2}}$ $\quad \frac{\sqrt{3}}{2} \quad \frac{1}{\sqrt{2}} \quad - \quad \frac{1}{\sqrt{2}}$

$$= \frac{1}{\frac{\sqrt{3}}{2\sqrt{2}} \quad \frac{-1}{\sqrt{2}}}$$

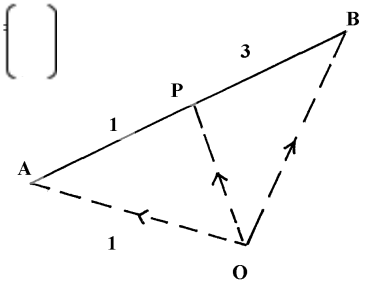
$$= \frac{\sqrt{6}}{4} - \frac{\sqrt{2}}{2}$$

$$= \frac{\sqrt{6} - 2\sqrt{2}}{4}$$

2. $OP = OA + ¼\ AB$
$= OA + ¼\ (OB – OA)$
$= OA + ¼\ OB – ¼\ OA$
$= ¾\ OA + ¼\ OB$

$= ¾\ OA + ¼\ OB$

$= \quad ¾\begin{pmatrix} \quad \end{pmatrix} + ¼\begin{pmatrix} \quad \end{pmatrix} = \begin{pmatrix} \quad \end{pmatrix} + \begin{pmatrix} \quad \end{pmatrix} \begin{pmatrix} \quad \end{pmatrix}$



3. $m \quad \begin{matrix} 4 \\ 3 \end{matrix} \quad + \quad n \quad \begin{pmatrix} \quad \end{pmatrix} \quad = \quad \begin{pmatrix} \quad \end{pmatrix}$

$4m – 3n = 5$ …….. (i) x 2
$3m + 2n = 8$ ……..(ii) x 2
$8m – 6n = 10$
$9m + 6n = 24$
$17m \quad = 34$
$m = 2$
$4 \text{ x } 2 – 3n = 5$
$-3n = -3$
$n = 1$
$\therefore m = 2, n = 1$

4. (a) (i) $BM = \frac{2}{5}a – b = \frac{1}{5}(2a – 5b)$

(ii) $AN = \frac{2}{3}b – a = \frac{1}{3}(2b – 3a)$

(b) $BX = \frac{t}{5}(2a – 5b)$

$AX = \frac{h}{3}(2b – 3a)$

$OX_1 = OB + BX = b + t\,(\frac{2}{5}a - 5b)$

$= (-t)b + \frac{2}{5} + a$

$OX = OA + AX = a + h\,(2b - 3a)$

$= (1-h)a + \frac{2}{3}hb$

(c) $OX_1 = OX_2$

$\frac{2}{5} + a + (\frac{1}{3} - t)b = (1-h)a + 2hb$

$\frac{2}{5}t = 1-h$ ....(i)

$(1 - t) = ¾\ h$ ....(ii) $\quad t = \frac{5 - 5h}{2}$

$1 - (\frac{5 - 5h}{2}) = \frac{2}{3}h = 11h = 9$

$h = \frac{9}{11}$

$t = \frac{5 - 5}{2}\left[\ \ \right] = \frac{5}{11}$

(i) $BX : XM = 1:10$

(ii) $AX: XN = 3:8$

5. a) i) $MA = ½\ a$

ii) $AB = a$

iii) $AC = a + c$

iv) $AX = {}^{2}/_{7}\ AC = {}^{2}/_{7}\ (-a + c)$

b) $MA = ½\ a$

$AX = {}^{2}/_{7}\ c - {}^{2}/_{7}\ a$

$MX = ½\ a + {}^{2}/_{7} - {}^{2}/_{7}\ a$

$= {}^{3}/_{14}\ a + {}^{2}/_{7}c$

Co-ordinates of $P = (\frac{1 + 3}{2}, \frac{6 + 0}{2}, \frac{8 + 4}{2})$

$= (2, 3, 6)$

$/OP/ = \sqrt{2^2 + 3^2 + 6^2}$

$= \sqrt{4 + 9 + 36}$

$= \sqrt{49} \qquad = 7$ units

c) Co-ordinates of O (0,0,0)

Co-ordinates of A (1, 6, 8)

Mid points of $AO = (\frac{1 + 0}{2}, \frac{6 + 0}{2}, \frac{8 + 0}{2})$

$= (0.5, 3, 4)$

6. a) $AB = DC \Rightarrow 1 - x = 2 \Rightarrow x = -1$

$6 - y = 4 \Rightarrow y = 2$

$\therefore D = \underline{(-1, 2)}$

b) (i) $\overrightarrow{RQ} = Q\begin{pmatrix} \\ \end{pmatrix}q - \frac{1}{2}p$

$= \begin{pmatrix} \\ \end{pmatrix} - p\begin{pmatrix} \\ \end{pmatrix} p - q$ ✓

(ii) $\overrightarrow{PR} = \frac{3}{2}q - \frac{1}{2}p - P$ ✓

$= \frac{3}{}\begin{pmatrix} \\ \end{pmatrix}$

$\frac{3}{2}q = -\frac{1}{2}$ Also $-\frac{3}{2}p = \frac{1}{2}kp$

$\Rightarrow k = -3$ $\Rightarrow k = -3$

Hence P, Q, R, Q Collinear.

(iii) $\overrightarrow{PQ} = q - p$, $QR = \frac{1}{2}(q - P)$

$PQ : QR = 2 : 1$

7. (a) $PQ = PO + OQ = -p + q$

Or $= OP + PR = P + 2/3\ PQ$

$= P + 2/3\ (-p+q)$

$= \frac{1}{3}p + \frac{2}{3}q$

$QT = QO + OT = -q + \frac{1}{2}\ OR$ since $OT = TR$

$= -q + \frac{1}{2}\ (\frac{1}{3}p - \frac{2}{3}q)$

$= \frac{1}{6}p - \frac{2}{3}q$ OR $\frac{1}{6}\ (p-4q)$

(b) $TS = TO + OS = -\frac{1}{2}\ OR + \frac{1}{4}\ OP$

$= -\frac{1}{2}\ (\frac{1}{3}p + \frac{2}{3}q) + \frac{1}{4}\ p = \frac{-1}{6}p - \frac{1}{3}q + \frac{1}{4}\ p$

$= \frac{1}{12}p - \frac{1}{3}q$ or $\frac{1}{12}(p-4q)$

$QT: TS = \frac{1}{6}(p-4q): \frac{1}{12}(p-4q) = \frac{1}{6}:\frac{1}{12} = 2:1$

$\therefore QT = 2TS$ $OT//TS$ but T is a common point hence Q, T, S are collinear

(c) Vector OT can be expressed in 2 ways

1^{st} $OT = \frac{1}{2}\ OR$ given

$= \frac{1}{2}\ (\frac{1}{3}\ P + \frac{2}{3}q) = \frac{1}{6}q + 1/3q$……..(i)

2^{nd} using OPT

$OT = OP + PT = P + \frac{5}{6}PM$

But $PM = PO + OM = -P + KOQ = -P + Kq$

$OT = P + \frac{5}{6}\ (-P + kq)$

$= P - \frac{5}{6}kq$

$= \frac{1}{6}p + n\frac{5}{5}kq$……(ii)

Aqn (i) and (ii) represent the same vector OT

$\frac{1}{6}p + \frac{1}{3}q = \frac{1}{6}p + \frac{5}{6}kq$……..(iii)

Comparing coefficients of q in eqn (iii) have $\frac{5}{6}k = \frac{1}{3}$

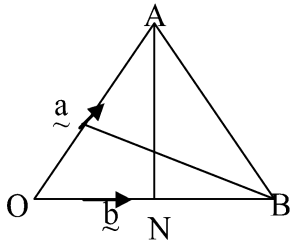
$15k = 6$

8. $3a = 3(-3) = (-9)$
$2 \quad 6$
$\frac{1}{2} b = \frac{1}{2} (4) = (2)$
$-6 \quad -3$
$\frac{1}{10}c = \frac{1}{10} (5) = (0.5)$
$-10 \quad -1$
$P = (-9) - (2) + 0.5)$
$6 \quad -3 \quad -1$
$= (-10.5)$
8
$/P/ = \sqrt{(-10.5)^2 + 8^2}$
$= \sqrt{110.25} = 64$
$= \sqrt{174.25}$
$= 13.20037878$
$= 13.20$ (2 d.p)

9. (i) $BM = BO + OM$
$= \frac{2}{5}a - b$
(ii) $AN = AO + ON$
$= \frac{2}{3} b - a$

(b) $OX = OB + BX$
$= b + k(2a - b)$
$= \frac{2}{5} ka + b(1- k)$
$OX = OA + AX$
$= a + h(\frac{2}{3}b - a)$
$= a (1-h) + 2hb$
$= a(10h) 2hb$

(c) $\frac{2}{5} a = a (1-h)$ also $b(1-k) = 2hb$
$2k = 1-h \quad 1-k = 2h$
$k = \frac{5}{2} - \frac{5}{2}h$
$\therefore 1 - \frac{5}{2} + \frac{5}{2}h = \frac{2}{3}h$
$\frac{5}{2}h - \frac{2}{3}h = \frac{5}{2} - 1$
$1 \frac{5}{6}h = \frac{3}{2}$
$h = \frac{3}{2} \times \frac{6}{11} = 9$
$k = \frac{5}{2} - \frac{5}{2} \cdot \frac{9}{11}$
$= \frac{5}{2} - \frac{45}{22}$
$= \frac{5}{2}$



11

10. (i) $AN = AO + ON$

$= -a + \frac{4}{5} b$

(ii) $BM = BO + OM$

$= -b + \frac{2}{5} a$

(iii) $AB = AO + OB$

$= -a + b$

$AX = sAN$

$BX = tBM$

$OX = OB + BX$

$= b + tBM$

$= b + t(-b + \frac{2}{5}a)$

$= b - tb + \frac{2}{5}t$

$= b(1-t) + \frac{2}{5}t$

$OX = OA + AX$

$= a + sAN$

$= a + s(-a + \frac{4}{5}b)$

$= a - Sa + \frac{4}{5} sb$

$a(1-s) + \frac{4}{5} sb$

$b(1-t) + \frac{2}{5}ta = a(1-s) \frac{4}{5}sb$

$b(1-t) = \frac{4}{5} sb$

$1-t = \frac{4}{5}s$----------(i)

$a(1-s) = \frac{2}{5}ta$

$1-s = \frac{2}{5}ta$

$s = 1 - \frac{2}{5}t$----------(ii)

$1-t = \frac{4}{5}(1 - \frac{2}{5}t)$

$1 - t = \frac{4}{5} - \frac{8}{25}t$

$-\frac{17}{25} = -\frac{1}{5}$

$t = \frac{5}{17}$

$s = \frac{15}{17}$

11. $\frac{115800}{76.84} \times \frac{97.5}{100}$

$= 1469.35$ ✓

$= 1469.35 - 270$

$= 1199.35$ ✓

$= 1199$ dollars

12. $RM = \begin{pmatrix} \\ \end{pmatrix} - \begin{pmatrix} \\ \end{pmatrix} = \begin{pmatrix} \\ \end{pmatrix}$

$|\underset{\sim}{RM}| = \sqrt{(-3)^2 + 82(-1)^2}$

$\sqrt{74} = 8.602$ units

13. (a) (i) $\underset{\sim}{OB} = \underset{\sim}{a} + \underset{\sim}{b}$

(ii) $BC = \underset{\sim}{BA} + \underset{\sim}{AO} + \underset{\sim}{OC}$

$= \underset{\sim}{-b} + \underset{\sim}{-a} + \underset{\sim}{2b}$

$= \underset{\sim}{b} - \underset{\sim}{a}$

(b) $CX = CO + OA + AB + BX$

$= 2b + a + b + hBC$

$= a - b + h(b - a)$

$= a - b + hb - ha$

$= (1 - h)a + (h - 1)b$

(c) $CX = CO + OA + AX$

$= 2b + a + KAT$

but $AT = AO + OT$

$= -a + 3b$

$CX = 2b + a + K(3b - a)$

$= a - Ka + 3Kb + 2b$

$= (1 - K)a + 3(K + 2)b$

(d) $I - h = 1 - k$ .....(i)

$h - 1 = 3k + 2$.....(ii)

from (i) $h = k$

sub in (ii) $h - 1 = 3h + 2$

$h = -3/2$

$K = -3/2$

14. $a + b = (2 - 3)i + (1 + 4)j + (-2 - 1)k$

$= -i + 5j - 3k$

$|a + b| = \sqrt{(-1)^2 + (5)^2 + (-3)^2}$

$= \sqrt{35}$

$= 5.916$

15. i) $BD = BA + AD$

$= -b + 3/5 c$

$AE = AB + BE$
$= b + ½\ BC = b + ½\ (c - b)$
$= ½\ b + ½\ c$

ii) $BF = t\ (^3/_5 c - b)$
$AF = n\ (½\ b + ½\ c) = {}^n/_2\ (b + c)$
$AF = AB + BF$
$= b + t(^3/_5 c - b) = b + {}^3/_5 tc + tb$
$= (1 - t)\ b + {}^3/5tc$
$(1 - t)b + {}^3/_5 tc = {}^n/_2 b + {}^n/_2 c$
$1 - t = {}^n/_2$; $2 - 2t = n$ …… (i)
$^3/_5 t = {}^n/_2$; $6t - 5n = 0$ …….. (ii)

Sub from équation (ii)
$6t - 5(2 - 2t) = 0$
$6t - 10 + 10t = 0$
$16t = 10$
$t = {}^{10}/_{16} = {}^5/_8$
$n = ¾$

iii) $BF = {}^5/_8\ BD$
F divides BD in the ratio 5 :3
$AF = ¾\ AE$
F divides AE in the ratio 3 :1

16. $BA =$ [ ]

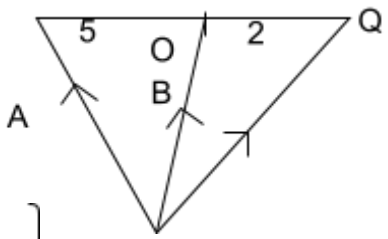
½ BC = ½ [ ] = [ ]
-4 -2

$OP =$ [ ] + [ ] ½ [ ] ½
-2 -4

Co-ordinates of P (-9 ½ , -4)

17. $OB = \underline{5}OQ + \underline{2}OA$
$\quad\ 7 \quad\ 5$
$OQ = \underline{7}OB - \underline{2}OA$
$\quad\ 5 \quad\ 5$
$OQ = \underline{7}$ [ ] - $\underline{2}$ [ ]
$\quad\ 5$ $\quad\ 5$ -7

= [ ] - [ ] = [ ] = [ ]
-15 -5



Q= (4, -3)