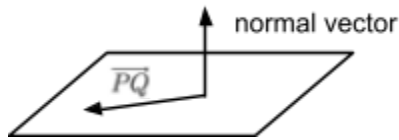


2.4 Cartesian Equation of a Plane

Another way to describe a plane is to describe the set of vectors perpendicular to a normal vector.



If $\vec{n} = (A, B, C)$ and $\overrightarrow{PQ} = (x - x_0, y - y_0, z - z_0)$ where $\overrightarrow{PQ}$ is some vector on the plane.

For any $\overrightarrow{PQ}$, $\vec{n} \cdot \overrightarrow{PQ} = 0$ and it can be shown that $Ax + By + Cz + D = 0$.

The good news is that this equation is unique. There is only one normal to a plane, and all other normal vectors will be collinear to it. If a point (x, y, z) satisfies the equation, then it is a point on the plane.

Ex Determine the equation of the plane:

(a) that contains $A(1, 2, 2)$ and has a normal $\vec{n} = (-1, 2, 6)$.

$$-x + 2y + 6z - 15 = 0$$

(b) that contains the points $A(-1, 2, 5)$, $B(3, 2, 4)$ and $C(-2, -3, 6)$.

$$5x + 3y + 20z - 101 = 0$$

(c) whose vector equation is $(x, y, z) = (1, 2, -1) + s(1, 0, 2) + t(-1, 3, 4)$

$$2x + 2y - z - 7 = 0$$