

“Students with conceptual understanding know more than isolated facts and methods. They understand why a mathematical idea is important and the kinds of contexts in which it is useful. They have organized their knowledge into a coherent whole, which enables them to learn new ideas by connecting those ideas to what they already know. Conceptual understanding also supports retention. Because facts and methods learned with understanding are connected, they are easier to remember and use, and they can be reconstructed when forgotten.”

“Knowledge that has been learned with understanding provides the basis for generating new knowledge and for solving new and unfamiliar problems. When students have acquired conceptual understanding in an area of mathematics, they see the connections among concepts and procedures and can give arguments to explain why some facts are consequences of others. They gain confidence, which then provides a base from which they can move to another level of understanding.”

“Students need to be efficient and accurate in performing basic computations with whole numbers ($6+7$, $17-9$, 8×4 , and so on) without always having to refer to tables or other aids. They also need to know reasonably efficient and accurate ways to add, subtract, multiply, and divide multi-digit numbers, both mentally and with pencil and paper.”

“It is important for computational procedures to be efficient, to be used accurately, and to result in correct answers. Both accuracy and efficiency can be improved with practice, which can also help students maintain fluency. Students also need to be able to apply procedures flexibly.”

“Procedural fluency and conceptual understanding are often seen as competing for attention in school mathematics. But pitting skill against understanding creates a false dichotomy. As we noted earlier, the two are interwoven. Understanding makes learning skills easier, less susceptible to common errors, and less prone to forgetting. By the same token, a certain level of skill is required to learn many mathematical concepts with understanding, and using procedures can help strengthen and develop that understanding.”