



The
TILIAN PARTNERSHIP
Inspire to achieve

Representation Policy for Whole School Coherence

Cavendish CE Primary

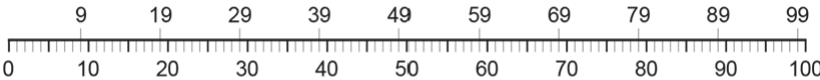
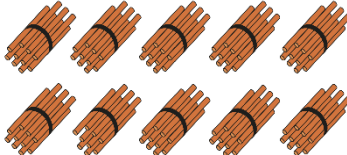
Approved by:	SGC	Date: 3 rd February 2026
Last reviewed on:	February 2026	
Next review due by:	February 2028	

At **Cavendish CE Primary School**, we are committed to ensuring that all pupils develop secure mathematical understanding through the careful and purposeful use of representations. This policy sets out a consistent whole-school approach, supporting high-quality teaching while allowing teachers the professional autonomy to respond to the needs of their pupils.

Our approach is informed by guidance from the **National Centre for Excellence in the Teaching of Mathematics (NCETM)** and aligns with the **Ready-to-Progress criteria**, which emphasise the importance of representations in securing key mathematical concepts and making the underlying structure of mathematics explicit. Representations are used to support pupils in developing fluency, reasoning, and conceptual understanding, enabling them to make connections and generalise their learning.

At Cavendish CE Primary School, representations are not used as a checklist of resources but are **carefully selected, adapted, and sequenced** to match the mathematical concept being taught and the stage of learning. Teachers make deliberate decisions about when representations are introduced, varied, and gradually withdrawn, ensuring that pupils move towards efficient and abstract thinking without over-reliance on concrete or pictorial models.

This policy reflects our commitment to a coherent, well-sequenced mathematics curriculum in which representations support progression, inclusion, and high expectations for all learners.

Year 1	Key representations	Language focus																																																																																																																																								
1NPV 1 Count forwards and backwards within 100	 Figure 6: 0 to 100 number line to support backwards counting <table><tr><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td><td>7</td><td>8</td><td>9</td><td>10</td></tr><tr><td>11</td><td>12</td><td>13</td><td>14</td><td>15</td><td>16</td><td>17</td><td>18</td><td>19</td><td>20</td></tr><tr><td>21</td><td>22</td><td>23</td><td>24</td><td>25</td><td>26</td><td>27</td><td>28</td><td>29</td><td>30</td></tr><tr><td>31</td><td>32</td><td>33</td><td>34</td><td>35</td><td>36</td><td>37</td><td>38</td><td>39</td><td>40</td></tr><tr><td>41</td><td>42</td><td>43</td><td>44</td><td>45</td><td>46</td><td>47</td><td>48</td><td>49</td><td>50</td></tr><tr><td>51</td><td>52</td><td>53</td><td>54</td><td>55</td><td>56</td><td>57</td><td>58</td><td>59</td><td>60</td></tr><tr><td>61</td><td>62</td><td>63</td><td>64</td><td>65</td><td>66</td><td>67</td><td>68</td><td>69</td><td>70</td></tr><tr><td>71</td><td>72</td><td>73</td><td>74</td><td>75</td><td>76</td><td>77</td><td>78</td><td>79</td><td>80</td></tr><tr><td>81</td><td>82</td><td>83</td><td>84</td><td>85</td><td>86</td><td>87</td><td>88</td><td>89</td><td>90</td></tr><tr><td>91</td><td>92</td><td>93</td><td>94</td><td>95</td><td>96</td><td>97</td><td>98</td><td>99</td><td>100</td></tr></table> <table><tr><td>1,000</td><td>2,000</td><td>3,000</td><td>4,000</td><td>5,000</td><td>6,000</td><td>7,000</td><td>8,000</td><td>9,000</td></tr><tr><td>100</td><td>200</td><td>300</td><td>400</td><td>500</td><td>600</td><td>700</td><td>800</td><td>900</td></tr><tr><td>10</td><td>20</td><td>30</td><td>40</td><td>50</td><td>60</td><td>70</td><td>80</td><td>90</td></tr><tr><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td><td>7</td><td>8</td><td>9</td></tr></table> Figure 5: Gattegno chart showing multiples of 1, 10, 100 and 1,000	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80	81	82	83	84	85	86	87	88	89	90	91	92	93	94	95	96	97	98	99	100	1,000	2,000	3,000	4,000	5,000	6,000	7,000	8,000	9,000	100	200	300	400	500	600	700	800	900	10	20	30	40	50	60	70	80	90	1	2	3	4	5	6	7	8	9	“...seven, eight, nine, ten, eleven, twelve, thirteen... twenty, twenty-one, twenty-two...” “...seven, eight, nine, one-ten, one-ten-one, one-ten-two, one-ten-three... two-tens, two-tens-one, two-tens two...”
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1NPV 2

Numbers to 20 in the linear number system

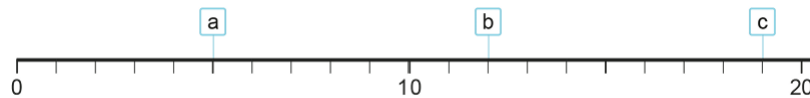


Figure 7: identifying 5, 12 and 19 on a marked 0 to 20 number line

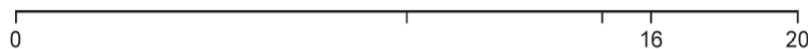


Figure 8: placing 16 on an unmarked 0 to 20 number line

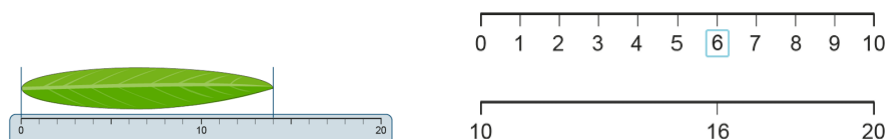


Figure 9: placing 16 on an unmarked 10 to 20 number line

“a is 5 because it is halfway between 0 and 10.” “b is 12 because it is 2 more than 10.” “c is 19 because it is one less than 20.”

1NF-1

Fluently add and subtract within 10

+	0	1	2	3	4	5	6	7	8	9	10
0	0+0	0+1	0+2	0+3	0+4	0+5	0+6	0+7	0+8	0+9	0+10
1	1+0	1+1	1+2	1+3	1+4	1+5	1+6	1+7	1+8	1+9	
2	2+0	2+1	2+2	2+3	2+4	2+5	2+6	2+7	2+8	2+9	
3	3+0	3+1	3+2	3+3	3+4	3+5	3+6	3+7	3+8	3+9	
4	4+0	4+1	4+2	4+3	4+4	4+5	4+6	4+7	4+8	4+9	
5	5+0	5+1	5+2	5+3	5+4	5+5	5+6	5+7	5+8	5+9	
6	6+0	6+1	6+2	6+3	6+4	6+5	6+6	6+7	6+8	6+9	
7	7+0	7+1	7+2	7+3	7+4	7+5	7+6	7+7	7+8	7+9	
8	8+0	8+1	8+2	8+3	8+4	8+5	8+6	8+7	8+8	8+9	
9	9+0	9+1	9+2	9+3	9+4	9+5	9+6	9+7	9+8	9+9	
10	10+0	10+1	10+2	10+3	10+4	10+5	10+6	10+7	10+8	10+9	

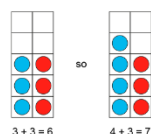


Figure 10: tens frames with counters showing derivation of a 'near-double' addition calculation

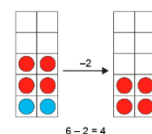
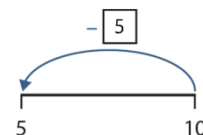
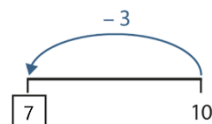
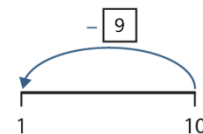
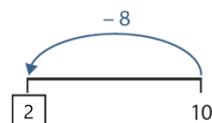


Figure 11: tens frames with counters showing that subtracting 2 from an even number gives the previous even number



I know that $2 + 8$ is equal to 10 so I know 10 minus 8 is equal to 2.

“I know that double 3 is equal to 6, so 4 plus 3 is equal to 7.”

“If I subtract 2 from an even number I get the previous even number, so 6 minus 2 is equal to 4.”

1NF-2

Count forwards and backwards in multiples of 2, 5 and 10

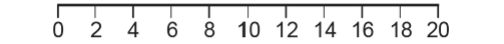


Figure 12: number line to support counting in multiples of 2

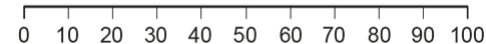


Figure 13: number line to support counting in multiples of 10

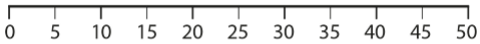


Figure 14: number line to support counting in multiples of 5

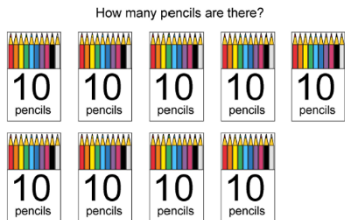


Figure 15: counting in tens - 9 packs of 10 pencils

“The pencils are in groups of 10, so we will count in tens.”

“Ten, twenty, thirty...” “1 group of 10, 2 groups of 10, 3 groups of 10...” In time, shortened to: “1 ten, 2 tens, 3 tens...”

1AS-1

Compose and partition numbers to 10

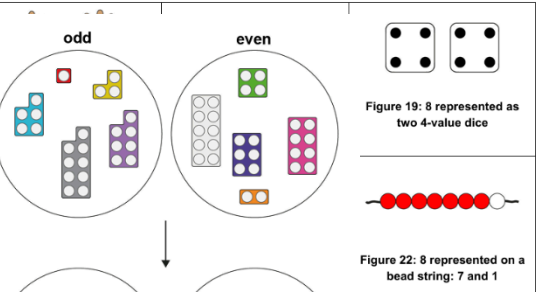


Figure 19: 8 represented as two 4-value dice

Figure 22: 8 represented on a bead string: 7 and 1

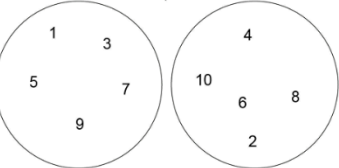


Figure 25: odd and even numbers up to 10

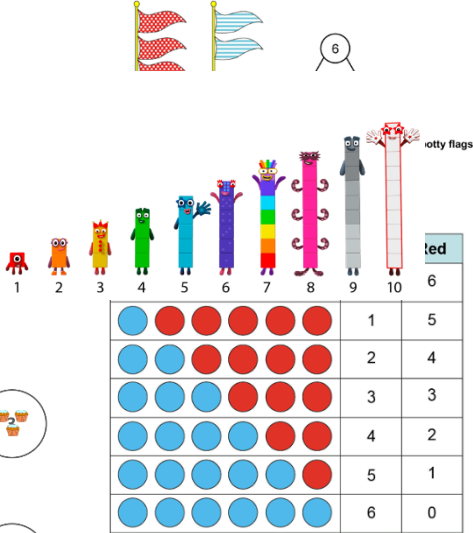


Figure 24: working systematically to partition 6

“There are 6 flags. 4 are spotty and 2 are stripy.” “6 is the whole. 4 is a part. 2 is a part.”

1AS-2

Read, write and interpret additive equations

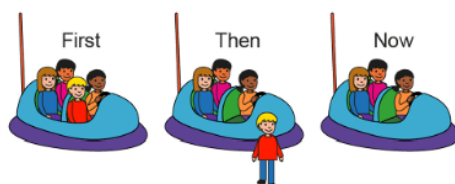
How many flowers are there altogether?



$$5 + 2 = 7$$

Figure 26: addition as aggregation

How many children are in the bumper car now?

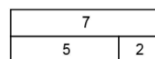


$$4 - 1 = 3$$

Figure 29: subtraction as reduction

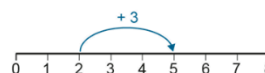


Figure 30: aggregation or partitioning context: 5 cakes altogether, 2 with cherries and 3 without



$$7 - 2 = 5$$

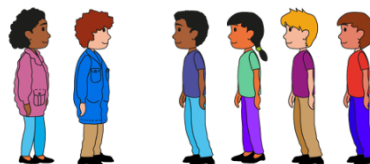
Figure 31: bar model and subtraction equation ($7 - 2 = 5$)



$$2 + 3 = 5$$

Figure 32: number line and addition equation ($2 + 3 = 5$)

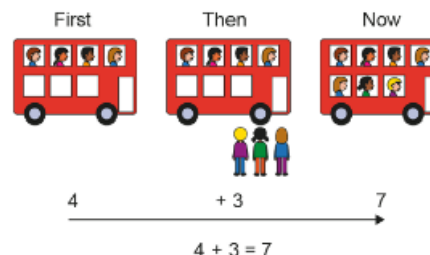
How many children are not wearing coats?



$$6 - 2 = 4$$

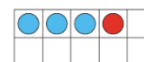
Figure 27: subtraction as partitioning

How many children are on the bus now?



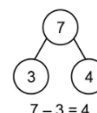
$$4 + 3 = 7$$

Figure 28: addition as augmentation



$$3 + 1 = 4$$

Figure 33: tens frame with counters and addition equation ($3 + 1 = 4$)



$$7 - 3 = 4$$

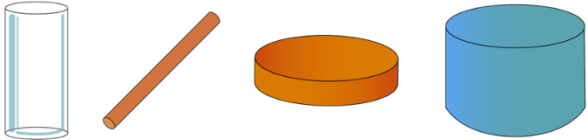
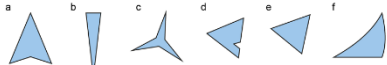
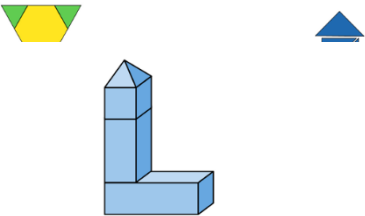
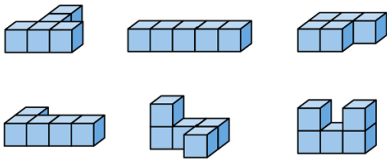
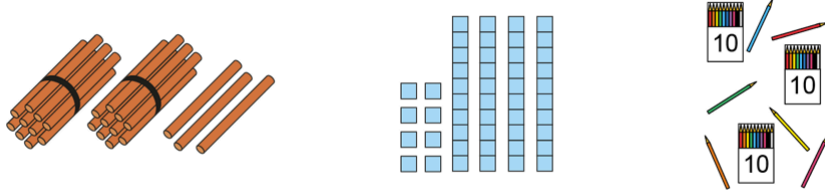
Figure 34: cherry partitioning model and subtraction equation ($7 - 3 = 4$)

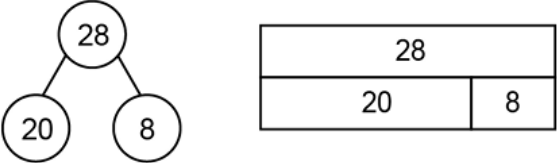
“There are 5 flowers in one bunch. There are 2 flowers in the other bunch. There are 7 flowers altogether.” “We can write this as 5 plus 2 is equal to 7.” “The 5 represents the number of flowers in 1 bunch.” “The 2 represents the number of flowers in the other bunch.” “The 7 represents the total number of flowers.”

“There are 6 children altogether. 2 children are wearing coats. 4 children are not wearing coats.” “We can write this as 6 minus 2 is equal to 4.” “The 6 represents the total number of children.” “The 2 represents the number of children that are wearing coats.” “The 4 represents the number of children that are not wearing coats.”

“First 4 children were sitting on the bus. Then 3 more children got on the bus. Now 7 children are sitting on the bus.” “We can write this as 4 plus 3 is equal to 7.” “The 4 represents the number of children that were on the bus at the start.” “The 3 represents the number of children that got on the bus.” “The 7 represents the number of children that are on the bus now.”

“First there were 4 children in the bumper car. Then 1 child got out. Now there are 3 children in the bumper car.” “We can write this as 4 minus 1 is equal to 3.” “The 4 represents the number of children that were in the car at the start.” “The 1 represents the number of children that got out of the car.” “The 3 represents the number of children that are in the car now.”

1G-1 Recognise common 2D and 3D shapes	 Figure 35: non-similar cylinders  Figure 36: distinguishing triangles from plausible distractors	Shape a: “This is not a triangle because it has 4 sides.” Shape b or e: “This is a triangle because it has 3 straight sides.” Shape c or d: “This is not a triangle because it has 6 sides.” Shape f: “This is not a triangle because some sides are curved.”
1G-2 Compose 2D and 3D shapes from smaller shapes	 Figure 40: example compound shape composed of 4 different 3D shapes  Figure 39: example compound 3D shapes composed of cubes	
Year 2		
Ready to Progress	Representations	Language Focus
2NPV-1 Place value in two-digit numbers	Varied representations - include straws, base 10 and obvious representations showing clear groups of 10 and additional ones  Figure 41: varied representations of two-digit numbers as groups of ten and additional ones	This is the number 42. The 4 shows we have 4 groups of ten. The 2 shows we have 2 extra ones.”

	Developed into partitioning - Part Whole models  Figure 42: partitioning 28 into 20 and 8 $20 + 8 = 28$ $28 - 20 = 8$ $8 + 20 = 28$ $28 - 8 = 20$ $28 = 20 + 8$ $8 = 28 - 20$ $28 = 8 + 20$ $20 = 28 - 8$	
2NPV-2 Two-digit numbers in the linear number system	Number lines - Marked and unmarked  Figure 43: identifying 36 and 79 on a marked 0 to 100 number line  Figure 44: placing 60 on an unmarked 0 to 100 number line	“a is 36 because it is one more than the midpoint of 35.” “b is 79 because it is one less than 80.”
2NF-1 Fluently add and subtract within 10	Addition facts - grid	

+	0	1	2	3	4	5	6	7	8	9	10
0	0+0	0+1	0+2	0+3	0+4	0+5	0+6	0+7	0+8	0+9	0+10
1	1+0	1+1	1+2	1+3	1+4	1+5	1+6	1+7	1+8	1+9	
2	2+0	2+1	2+2	2+3	2+4	2+5	2+6	2+7	2+8		
3	3+0	3+1	3+2	3+3	3+4	3+5	3+6	3+7			
4	4+0	4+1	4+2	4+3	4+4	4+5	4+6				
5	5+0	5+1	5+2	5+3	5+4	5+5					
6	6+0	6+1	6+2	6+3	6+4						
7	7+0	7+1	7+2	7+3							
8	8+0	8+1	8+2								
9	9+0	9+1									
10	10+0										

**2AS–1 Add
and subtract
across 10**

Tens frames and partitioning diagram

Addition

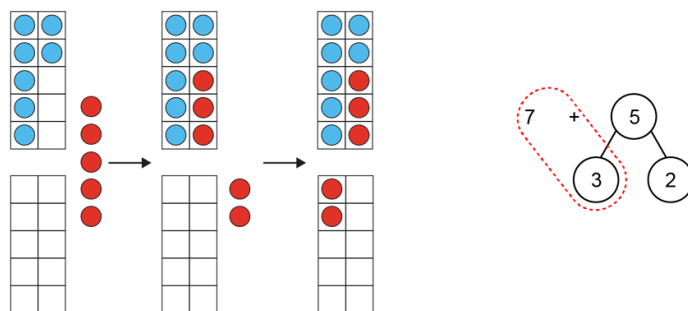


Figure 45: tens frames with counters, and a partitioning diagram, showing $7 + 5 = 12$

Subtraction

First, pupils should learn to add three one-digit numbers by making 10, for example, $7 + 3 + 2 = 10 + 2$. They can then relate this to addition of two numbers across 10, by partitioning one of the addends, for example $7 + 5 = 7 + 3 + 2$.

the 'subtracting through 10' strategy (partitioning the subtrahend) – part of the subtrahend is subtracted to reach 10, then the rest of the subtrahend is subtracted from 10

the 'subtracting from 10' strategy (partitioning the minuend) – the subtrahend is subtracted from 10, then

	Figure 46: using the 'subtracting through 10' strategy to calculate 15 minus 9 Figure 47: using the 'subtracting from 10' strategy to calculate 15 minus 9	the difference between the minuend and 10 is added
2AS–2 Solve comparative addition and difference problems	Bar models and numberlines Figure 48: finding the difference – bar model and number line	Pupils should be able to recognise contextual problems involving finding a difference, phrased as ‘find the difference’, ‘how many more’ and ‘how many fewer’,
2AS–3 Add and subtract within 100 – part 1	Tens frames/Dienes	“4 plus 3 is equal to 7. So 4 tens and plus 3 tens is equal to 7 tens.” “10 minus 3 is equal to 7. So 30 minus 3 is equal to 27.”

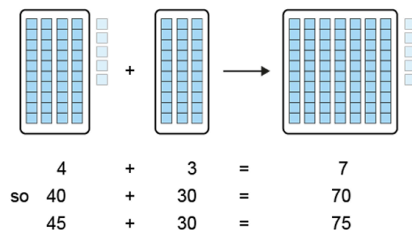


Figure 49: Dienes and equations to support adding a multiple of 10 to a two-digit number

Tens Frames and Number lines

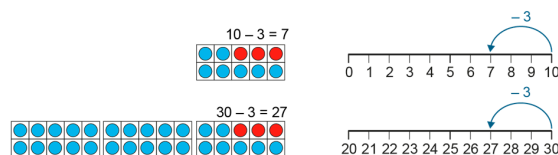


Figure 50: tens frames with counters, and number lines, to support subtracting ones from a multiple of 10

“I know that 2 plus 5 is equal to 7, so 20 plus 50 is equal to 70. There are 9 ones as well, so 20 plus 59 is equal to 79.”

2AS–4 Add and subtract within 100 – part 2

Base 10 (dienes)

- $40 + 20 + 5 + 3 = 60 + 8 = 68$
- $40 + 5 + 20 + 3 = 60 + 8 = 68$
- $45 + 23 = 60 + 8 = 68$

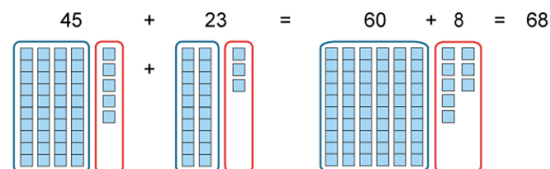


Figure 51: Dienes and an equation to support adding 2 two-digit numbers

“First I partition both numbers. Then I add the tens. Then I add the ones. Then I combine all of the tens and all of the ones.”

2MD–1 Multiplication as repeated addition

Explore equal and unequal groups in a “real life” context

“There are 3 equal groups of eggs.”
 “There are 5 eggs in each group.” “There are 3 groups of 5.”
 “The 3 represents the number of groups.”
 “The 5 represents the number of eggs in



Figure 52: recognising equal groups – 3 groups of 5 eggs

Place value counters

Pupils should be able to relate multiplication to situations where the total number of items cannot be seen, for example by representing 3 5 with three 5-value counters.

each group.” “The 15 represents the total number of eggs.”

2MD–2
Grouping problems: missing factors and division

Real life representations alongside unitised counters



Figure 53: three 5-value counters

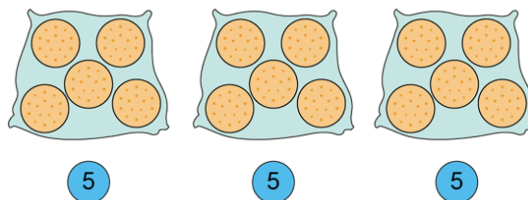


Figure 54: 3 bags of 5 biscuits alongside three 5-value counters

“The 15 represents the total number of biscuits.” “The 5 represents the number of biscuits in each bag.” “The 3 represents the number of bags.” “15 divided into groups of 5 is equal to 3.”

2G–1
Describe and compare 2D and 3D shapes

Regular and irregular shapes Because it has 6 sides it is a hexagon

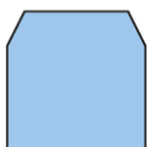


Figure 55: an irregular hexagon

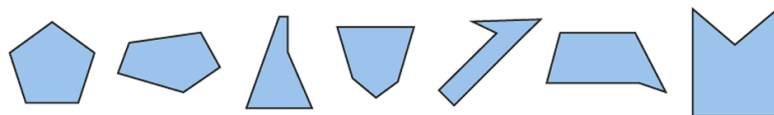
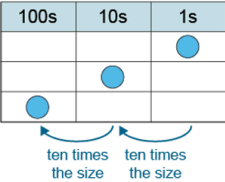
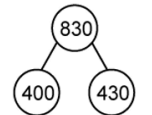


Figure 56: a variety of different pentagons (regular and irregular)

Language focus “These shapes are all pentagons because they all have exactly 5 straight sides.”

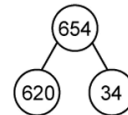
Ready to progress	Representations	Language focus
3NPV–1 Equivalence of 10 hundreds and 1 thousand	Unitised place value counters/Tens Frame  Figure 62: ten 10-value place-value counters in a tens frame Pupils must then be able to work out how many tens there are in other three-digit multiples of 10.  Figure 63: eighteen 10-value place-value counters in 2 tens frames Place value chart  Figure 64: place-value chart illustrating the scaling relationship between ones, tens and hundreds	Language focus “10 tens is equal to 1 hundred.” “18 tens is equal to 10 tens and 8 more tens.” “10 tens is equal to 100.” “So 18 tens is equal to 100 and 8 more tens, which is 180.” “100 is 10 times the size of 10.”
3NPV–2 Place value in three-digit numbers	Place value - unitised counters and part whole model comparison  Figure 65: two representations of the place-value composition of 342	

Place value/equation relationship



$$830 - 400 = 430$$

Figure 66: partitioning 830 into 430 and 400



$$620 + 34 = 654$$

Figure 67: partitioning 654 into 620 and 34

3NPV-3
Three-digit
numbers in
the linear
number
system

Number lines

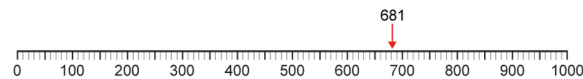


Figure 68: using a number line to identify the next and previous multiple of 100

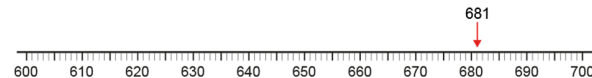


Figure 69: using a number line to identify the next and previous multiple of 10

“The previous multiple of 100 is 600. The next multiple of 100 is 700.” “The previous multiple of 10 is 680. The next multiple of 10 is 690.”

Developing onto the use of inequality symbols

$$\begin{array}{ccc} \text{previous} & & \text{next} \\ \text{multiple of} & & \text{multiple of} \\ 100 & & 100 \\ \boxed{600} & < 681 < & \boxed{700} \end{array}$$

Figure 70: identifying the nearest multiple of 100

3NPV–4 Reading scales with 2, 4, 5 or 10 intervals	Bar models - show comparison 100 50 50 100 25 25 25 25 100 20 20 20 20 20 100 10 10 10 10 10 10 10 10 10 Figure 71: Bar models showing 100 partitioned into 2, 4, 5 and 10 equal parts	“Twenty-five, fifty, seventy-five, one hundred” needs to be a fluent spoken language pattern, which pupils can continue over 100.																																																																																																																																																
3NF–1 Fluently add and subtract within and across 10	Identifying core number facts: columnar addition 465 + 429 894 1 Figure 72: columnar addition of 465 and 429 Within-column calculations: 5 + 9 = 14 6 + 2 + 1 = 9 4 + 4 = 8 Identifying core number facts: columnar subtraction 61 749 - 286 463 Figure 73: columnar subtraction of 286 from 749 Within-column calculations: 9 – 6 = 3 7 – 1 = 6 14 – 8 = 6 6 – 2 = 4 Learning addition facts <table><tr><th>+</th><th>0</th><th>1</th><th>2</th><th>3</th><th>4</th><th>5</th><th>6</th><th>7</th><th>8</th><th>9</th><th>10</th></tr><tr><td>0</td><td>0+0</td><td>0+1</td><td>0+2</td><td>0+3</td><td>0+4</td><td>0+5</td><td>0+6</td><td>0+7</td><td>0+8</td><td>0+9</td><td>0+10</td></tr><tr><td>1</td><td>1+0</td><td>1+1</td><td>1+2</td><td>1+3</td><td>1+4</td><td>1+5</td><td>1+6</td><td>1+7</td><td>1+8</td><td>1+9</td><td>1+10</td></tr><tr><td>2</td><td>2+0</td><td>2+1</td><td>2+2</td><td>2+3</td><td>2+4</td><td>2+5</td><td>2+6</td><td>2+7</td><td>2+8</td><td>2+9</td><td>2+10</td></tr><tr><td>3</td><td>3+0</td><td>3+1</td><td>3+2</td><td>3+3</td><td>3+4</td><td>3+5</td><td>3+6</td><td>3+7</td><td>3+8</td><td>3+9</td><td>3+10</td></tr><tr><td>4</td><td>4+0</td><td>4+1</td><td>4+2</td><td>4+3</td><td>4+4</td><td>4+5</td><td>4+6</td><td>4+7</td><td>4+8</td><td>4+9</td><td>4+10</td></tr><tr><td>5</td><td>5+0</td><td>5+1</td><td>5+2</td><td>5+3</td><td>5+4</td><td>5+5</td><td>5+6</td><td>5+7</td><td>5+8</td><td>5+9</td><td>5+10</td></tr><tr><td>6</td><td>6+0</td><td>6+1</td><td>6+2</td><td>6+3</td><td>6+4</td><td>6+5</td><td>6+6</td><td>6+7</td><td>6+8</td><td>6+9</td><td>6+10</td></tr><tr><td>7</td><td>7+0</td><td>7+1</td><td>7+2</td><td>7+3</td><td>7+4</td><td>7+5</td><td>7+6</td><td>7+7</td><td>7+8</td><td>7+9</td><td>7+10</td></tr><tr><td>8</td><td>8+0</td><td>8+1</td><td>8+2</td><td>8+3</td><td>8+4</td><td>8+5</td><td>8+6</td><td>8+7</td><td>8+8</td><td>8+9</td><td>8+10</td></tr><tr><td>9</td><td>9+0</td><td>9+1</td><td>9+2</td><td>9+3</td><td>9+4</td><td>9+5</td><td>9+6</td><td>9+7</td><td>9+8</td><td>9+9</td><td>9+10</td></tr><tr><td>10</td><td>10+0</td><td>10+1</td><td>10+2</td><td>10+3</td><td>10+4</td><td>10+5</td><td>10+6</td><td>10+7</td><td>10+8</td><td>10+9</td><td>10+10</td></tr></table>	+	0	1	2	3	4	5	6	7	8	9	10	0	0+0	0+1	0+2	0+3	0+4	0+5	0+6	0+7	0+8	0+9	0+10	1	1+0	1+1	1+2	1+3	1+4	1+5	1+6	1+7	1+8	1+9	1+10	2	2+0	2+1	2+2	2+3	2+4	2+5	2+6	2+7	2+8	2+9	2+10	3	3+0	3+1	3+2	3+3	3+4	3+5	3+6	3+7	3+8	3+9	3+10	4	4+0	4+1	4+2	4+3	4+4	4+5	4+6	4+7	4+8	4+9	4+10	5	5+0	5+1	5+2	5+3	5+4	5+5	5+6	5+7	5+8	5+9	5+10	6	6+0	6+1	6+2	6+3	6+4	6+5	6+6	6+7	6+8	6+9	6+10	7	7+0	7+1	7+2	7+3	7+4	7+5	7+6	7+7	7+8	7+9	7+10	8	8+0	8+1	8+2	8+3	8+4	8+5	8+6	8+7	8+8	8+9	8+10	9	9+0	9+1	9+2	9+3	9+4	9+5	9+6	9+7	9+8	9+9	9+10	10	10+0	10+1	10+2	10+3	10+4	10+5	10+6	10+7	10+8	10+9	10+10	
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**3NF-2 Recall
of
multiplication
tables**

Number line and array

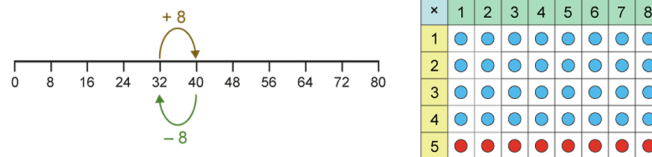


Figure 74: number line and array showing that adjacent multiples of 8 (32 and 40) have a difference of 8

When pupils commit multiplication table facts to memory, they do so using a verbal sound pattern to associate the 3 relevant numbers, for example, “six fours are twenty four”. It is important to provide opportunities for pupils to verbalise each multiplication fact as part of the process of developing fluency.

“4 times 5 is 20, so 20 divided by 5 is 4.”

**3NF-3
Scaling
number
facts by 10**

Tens frame with 10 value counters

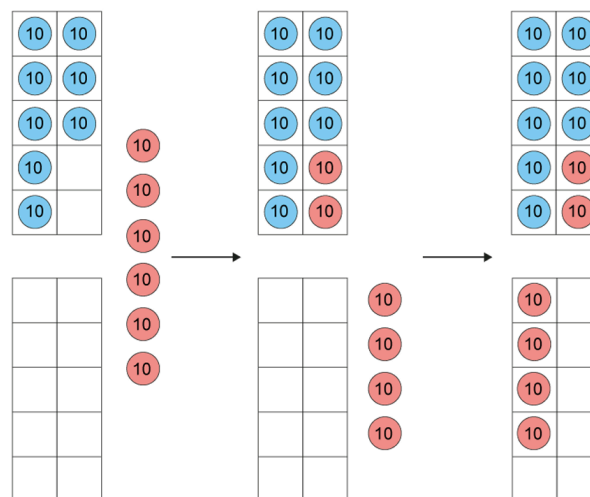


Figure 75: tens frames with 10-value counters showing $80 + 60 = 140$

Array of 10 value counters

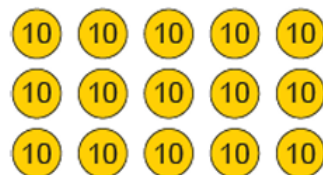
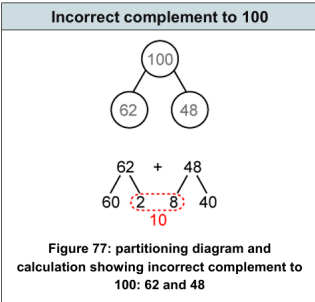
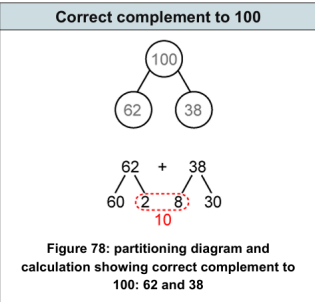
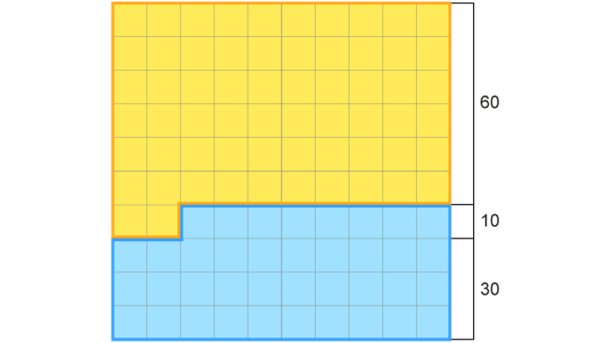
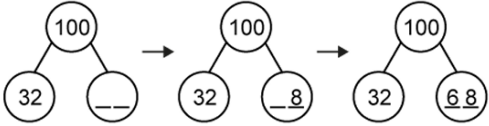


Figure 76: 3-by-5 array of 10-value place-value counters

“3 times 5 is equal to 15.” “3 times 5 tens is equal to 15 tens.” “15 tens is equal to 150.”
“3 times 5 is equal to 15.” “3 tens times 5 is equal to 15 tens.” “15 tens is equal to 150.”

3AS-1 Calculate complement s to 100	Partitioning Incorrect complement to 100  Figure 77: partitioning diagram and calculation showing incorrect complement to 100: 62 and 48 Correct complement to 100  Figure 78: partitioning diagram and calculation showing correct complement to 100: 62 and 38 100 square grid  Figure 79: a 100 grid shaded in 2 colours to represent 62 and 38 as a complement to 100 Partitioning - calculating a complement  Figure 80: calculating a complement to 100	“First we make 10 ones. The ones digits add up to 1 ten, so we need 9 more tens.”
3AS-2 Columnar addition and subtraction	Column subtraction (dienes)	“3 ones plus 5 ones is equal to 8 ones.” “4 tens plus 2 tens is equal to 6 tens.” “5 ones minus 3 ones is equal to 2 ones.” “6 tens minus 2 tens is equal to 4 tens.”

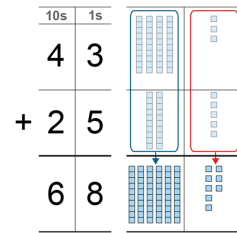


Figure 81: columnar addition with no regrouping: calculation and Dienes representation

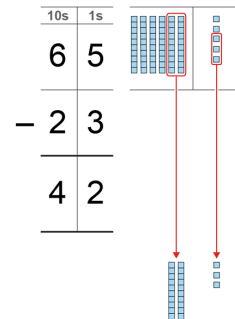


Figure 82: columnar subtraction with no exchange: calculation and Dienes representation

Modelling making efficient choices



Figure 83: making efficient choices for within-column calculations when adding 3 addends

Bar modelling

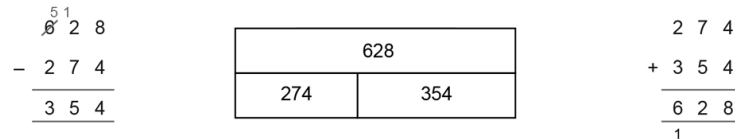


Figure 84: using addition to check a subtraction calculation

3AS-3
Manipulate
the additive
relationship

Partitioning diagrams - Bar model/part whole “cherry”

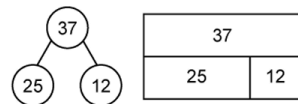


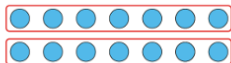
Figure 85: partitioning diagrams showing the additive relationship between 25, 12 and 37

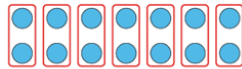
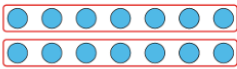
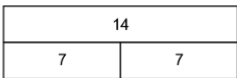
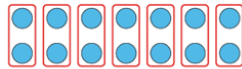
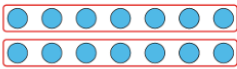
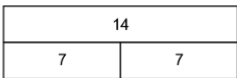
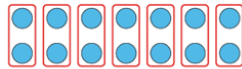
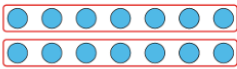
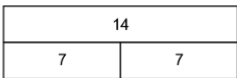
$$\begin{array}{ll} 25 + 12 = 37 & 37 - 12 = 25 \\ 12 + 25 = 37 & 37 - 25 = 12 \\ \\ 37 = 25 + 12 & 25 = 37 - 12 \\ 37 = 12 + 25 & 12 = 37 - 25 \end{array}$$

sum		minuend		minuend	
addend	addend	subtrahend	difference	difference	subtrahend

Figure 86: connecting the terms in addition and subtraction equations to the part-part-whole structure

“If we swap the values of the subtrahend and difference, the minuend remains the same.”

	<table><tr><td>Missing-addend problems</td><td>Type of problem: missing part Rewrite the addition equation as a subtraction equation, for example: $329 + \square = 743 \rightarrow 743 - 329 = \square$ Language focus "There is a missing part. To find the missing part, we subtract the other part from the whole."</td></tr><tr><td>Missing-subtrahend problems</td><td>Type of problem: missing part Rewrite the subtraction equation by swapping the subtrahend and the difference, for example: $477 - \square = 285 \rightarrow 477 - 285 = \square$ Language focus "There is a missing part. To find the missing part, we subtract the other part from the whole."</td></tr><tr><td>Missing-minuend problems</td><td>Type of problem: missing whole Rewrite the subtraction equation as an addition equation, for example: $\square - 527 = 87 \rightarrow 527 + 87 = \square$ Language focus "There is a missing whole. To find the missing whole, we add the 2 parts."</td></tr></table>	Missing-addend problems	Type of problem: missing part Rewrite the addition equation as a subtraction equation, for example: $329 + \square = 743 \rightarrow 743 - 329 = \square$ Language focus "There is a missing part. To find the missing part, we subtract the other part from the whole."	Missing-subtrahend problems	Type of problem: missing part Rewrite the subtraction equation by swapping the subtrahend and the difference, for example: $477 - \square = 285 \rightarrow 477 - 285 = \square$ Language focus "There is a missing part. To find the missing part, we subtract the other part from the whole."	Missing-minuend problems	Type of problem: missing whole Rewrite the subtraction equation as an addition equation, for example: $\square - 527 = 87 \rightarrow 527 + 87 = \square$ Language focus "There is a missing whole. To find the missing whole, we add the 2 parts."	"There is a missing part. To find the missing part, we subtract the other part from the whole." "There is a missing part. To find the missing part, we subtract the other part from the whole." "There is a missing whole. To find the missing whole, we add the 2 parts."
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3MD–1 Multiplication and division structures	Commutitive property - Array  7 groups of 2 $7 \times 2 = 14$  2 groups of 7 $2 \times 7 = 14$ $7 \times 2 = 2 \times 7$ Figure 87: using an array to show that 7 groups of 2 and 2 groups of 7 both correspond to the same total quantity Bar model <td> "factor times factor is equal to product" "The order of the factors does not affect the product." </td>	"factor times factor is equal to product" "The order of the factors does not affect the product."						

	<table><tr><td> Quotitive division I need 14 ping-pong balls. There are 2 ping-pong balls in a pack. How many packs do I need?  $14 \div 2 = 7$  Figure 88: using an array and bar model to show that 14 divided into groups of 2 is equal to 7 Language focus "7 times 2 is 14, so 14 divided by 2 is 7." "14 divided into groups of 2 is equal to 7." I need 7 packs of ping-pong balls. </td><td> Partitive division £14 is shared between 2 children. How much money does each child get?  $14 \div 2 = 7$  Figure 89: using an array and bar model to show that 14 shared between 2 is equal to 7 Language focus "7 times 2 is 14, so 14 divided by 2 is 7." "£14 shared between 2 is equal to £7 each." Each child gets £7. </td></tr></table>	Quotitive division I need 14 ping-pong balls. There are 2 ping-pong balls in a pack. How many packs do I need?  $14 \div 2 = 7$  Figure 88: using an array and bar model to show that 14 divided into groups of 2 is equal to 7 Language focus "7 times 2 is 14, so 14 divided by 2 is 7." "14 divided into groups of 2 is equal to 7." I need 7 packs of ping-pong balls.	Partitive division £14 is shared between 2 children. How much money does each child get?  $14 \div 2 = 7$  Figure 89: using an array and bar model to show that 14 shared between 2 is equal to 7 Language focus "7 times 2 is 14, so 14 divided by 2 is 7." "£14 shared between 2 is equal to £7 each." Each child gets £7.	"7 times 2 is 14, so 14 divided by 2 is 7." "14 divided into groups of 2 is equal to 7." "7 times 2 is 14, so 14 divided by 2 is 7." "£14 shared between 2 is equal to £7 each."						
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3F–1 Use and understand fraction notation	Concrete examples and diagrams such as:  Figure 90: a circle divided into 3 equal parts, with one part shaded  Is $\frac{1}{2}$ shaded?  Is $\frac{1}{3}$ shaded?  Is $\frac{1}{2}$ shaded?  Is $\frac{1}{4}$ shaded?  Is $\frac{1}{3}$ shaded?  Is $\frac{1}{9}$ shaded?  Is $\frac{1}{8}$ shaded?  Is $\frac{1}{6}$ shaded?	"The whole is divided into 3 equal parts. 1 of these parts is shaded." "The whole is divided into 3 equal parts. Each part is one-third of the whole." <table><tr><th>Say</th><th>Write</th></tr><tr><td>"The whole has been divided..."</td><td>The fraction bar: –</td></tr><tr><td>"...into 3 equal parts."</td><td>The denominator: 3</td></tr><tr><td>"1 of these parts is shaded."</td><td>The numerator: 1</td></tr></table> "The whole is divided into 8 equal parts and 5 of those parts are shaded. $\frac{5}{8}$ of the shape is shaded. $\frac{5}{8}$ is 5 one-eighths."	Say	Write	"The whole has been divided..."	The fraction bar: –	"...into 3 equal parts."	The denominator: 3	"1 of these parts is shaded."	The numerator: 1
Say	Write									
"The whole has been divided..."	The fraction bar: –									
"...into 3 equal parts."	The denominator: 3									
"1 of these parts is shaded."	The numerator: 1									
3F–2 Find unit	Array into equal parts	"The whole is 12 oranges. The whole is divided into 4 equal parts." "Each part is of the whole. of 12 oranges is 3 oranges."								

fractions of quantities	Figure 91: 12 oranges divided into 4 equal parts Bar model Figure 92: using a bar model to represent 15 divided into 5 equal parts Figure 93: using a bar model to represent one-fifth of 15	“To find $\frac{1}{5}$ of 15, we divide 15 into 5 equal parts.” “15 divided by 5 is equal to 3, so $\frac{1}{5}$ of 15 is equal to 3.”
3F–3 Fractions within 1 in the linear number system	Number lines	“One fifth, two fifths, three fifths...” “1 one-fifth, 2 one-fifths, 3 one-fifths...” “Each whole-number interval is divided into 4 equal parts, so we count in quarters.”

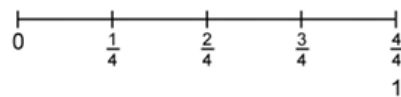


Figure 94: number line to support counting to 1 in multiples of one quarter

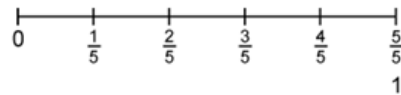


Figure 95: number line to support counting to 1 in multiples of one fifth

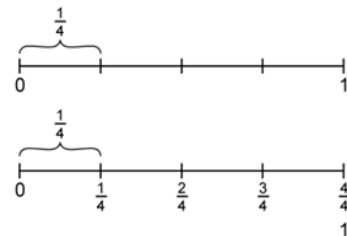


Figure 96: labelling a 0 to 1 number line marked in quarters

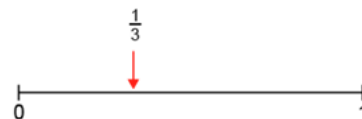


Figure 97: estimating the position of a fraction on a 0 to 1 number line

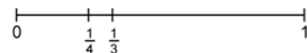


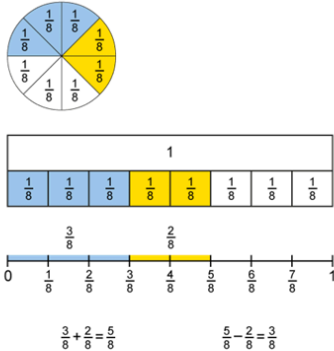
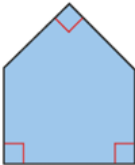
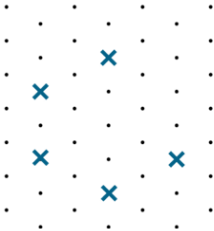
Figure 98: estimating the relative positions of one quarter and one third on a 0 to 1 number line

3F–4 Add and subtract fractions within 1

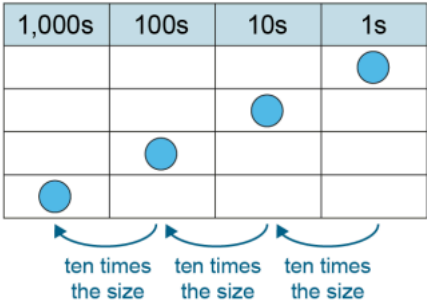
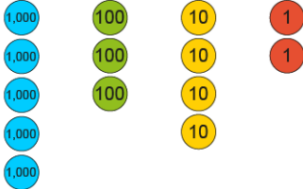
Pie model, Bar model, Number line

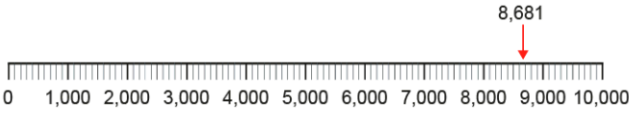
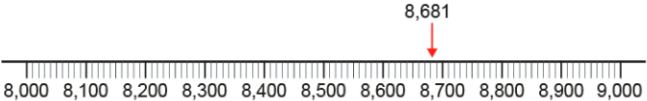
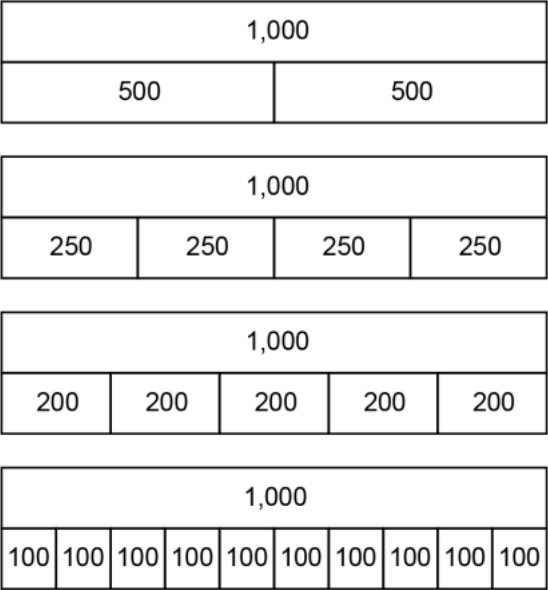
“3 one-eighths plus 2 one-eighths is equal to 5 one-eighths.” “5 one-eighths minus 2 one-eighths is equal to 3 one-eighths.”

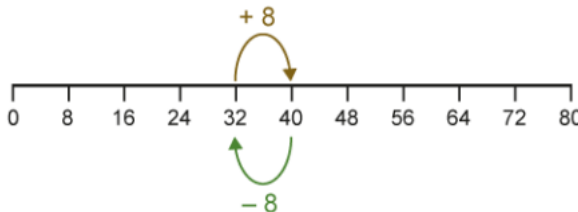
**“When adding fractions with the same denominators, just add the numerators.”
“When subtracting fractions with the**

	 Figure 99: adding and subtracting fractions with the same denominator: pie model, bar model and number line	same denominators, just subtract the numerators.”
3G–1 Recognise right angles	Shapes to demonstrate standard convention of right angle  Figure 100: irregular pentagon with 3 right angles marked using standard convention	
3G–2 Draw polygons and identify parallel and perpendicular sides	Spotted paper for accurate drawing 	“These 2 lines are parallel because they are always the same distance apart. They will never meet no matter how far we extend them.” “These 2 lines are perpendicular because they are at right angles to each other.”

Year 4		
	Representations	Language Focus

Ready to Progress		
4NPV-1 Equivalence of 10 hundreds and 1 thousand	Place value counters and tens frames:  Figure 104: eighteen 100-value place-value counters in 2 tens frames	“18 hundreds is equal to 10 hundreds and 8 more hundreds.” “10 hundreds is equal to 1,000.” “So 18 hundreds is equal to 1,000 and 8 more hundreds, which is 1,800.”
	Place value charts:  Figure 105: place-value chart illustrating the scaling relationship between ones, tens, hundreds and thousands	“1000 is 10 times the size of 100.” “1,800 is 10 times the size of 180.”
4NPV-2 Place value in four-digit numbers	Place value counters, part-whole models and number sentences:  $5,342 - 300 = \square$$5,342 - \square = 5,302$	

4NPV–3 Four-digit numbers in the linear number system	• Number lines:  Figure 109: using a number line to identify the previous and next multiple of 1,000  Figure 110: using a number line to identify the previous and next multiple of 100	“The previous multiple of 1,000 is 8,000. The next multiple of 1,000 is 9,000.” “The previous multiple of 100 is 8,600. The next multiple of 100 is 8,700.”
	• Inequality symbols:  Figure 111: identifying the nearest multiple of 1,000 with a number line for support	“The closest multiple of 1,000 is 9,000.” “8,681 rounded to the nearest thousand is 9,000.”
4NPV–4 Reading scales with 2, 4, 5 or 10 intervals	• Bar models:  Figure 112: bar models showing 1,000 partitioned into 2, 4, 5 and 10 equal parts	“Twenty-five, fifty, seventy-five, one hundred” needs to be a fluent spoken language pattern. Fluency in this language pattern provides the basis to count in multiples of 250.

4NF–1 Recall of multiplication tables	Expressions: <table><tr><td>2×2</td><td></td><td></td><td></td><td></td><td></td><td></td><td></td></tr><tr><td>3×2</td><td>3×3</td><td></td><td></td><td></td><td></td><td></td></tr><tr><td>4×2</td><td>4×3</td><td>4×4</td><td></td><td></td><td></td><td></td><td></td></tr><tr><td>5×2</td><td>5×3</td><td>5×4</td><td>5×5</td><td></td><td></td><td></td><td></td></tr><tr><td>6×2</td><td>6×3</td><td>6×4</td><td>6×5</td><td>6×6</td><td></td><td></td><td></td></tr><tr><td>7×2</td><td>7×3</td><td>7×4</td><td>7×5</td><td>7×6</td><td>7×7</td><td></td><td></td></tr><tr><td>8×2</td><td>8×3</td><td>8×4</td><td>8×5</td><td>8×6</td><td>8×7</td><td>8×8</td><td></td></tr><tr><td>9×2</td><td>9×3</td><td>9×4</td><td>9×5</td><td>9×6</td><td>9×7</td><td>9×8</td><td>9×9</td></tr></table>	2×2								3×2	3×3						4×2	4×3	4×4						5×2	5×3	5×4	5×5					6×2	6×3	6×4	6×5	6×6				7×2	7×3	7×4	7×5	7×6	7×7			8×2	8×3	8×4	8×5	8×6	8×7	8×8		9×2	9×3	9×4	9×5	9×6	9×7	9×8	9×9	When pupils commit multiplication table facts to memory, they do so using a verbal sound pattern to associate the 3 relevant numbers, for example, “nine sevens are sixtythree”. It is important to provide opportunities for pupils to verbalise each multiplication fact as part of the process of developing fluency.
	2×2																																																																
	3×2	3×3																																																															
4×2	4×3	4×4																																																															
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6×2	6×3	6×4	6×5	6×6																																																													
7×2	7×3	7×4	7×5	7×6	7×7																																																												
8×2	8×3	8×4	8×5	8×6	8×7	8×8																																																											
9×2	9×3	9×4	9×5	9×6	9×7	9×8	9×9																																																										
Number lines: 																																																																	
Arrays: <table><tr><td>×</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td><td>7</td><td>8</td></tr><tr><td>1</td><td>●</td><td>●</td><td>●</td><td>●</td><td>●</td><td>●</td><td>●</td><td>●</td></tr><tr><td>2</td><td>●</td><td>●</td><td>●</td><td>●</td><td>●</td><td>●</td><td>●</td><td>●</td></tr><tr><td>3</td><td>●</td><td>●</td><td>●</td><td>●</td><td>●</td><td>●</td><td>●</td><td>●</td></tr><tr><td>4</td><td>●</td><td>●</td><td>●</td><td>●</td><td>●</td><td>●</td><td>●</td><td>●</td></tr><tr><td>5</td><td>●</td><td>●</td><td>●</td><td>●</td><td>●</td><td>●</td><td>●</td><td>●</td></tr></table>	×	1	2	3	4	5	6	7	8	1	●	●	●	●	●	●	●	●	2	●	●	●	●	●	●	●	●	3	●	●	●	●	●	●	●	●	4	●	●	●	●	●	●	●	●	5	●	●	●	●	●	●	●	●											
×	1	2	3	4	5	6	7	8																																																									
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5	●	●	●	●	●	●	●	●																																																									

4NF–2 Division problems with remainders	Patterns:	“If the dividend is a multiple of the divisor there is no remainder.” “If the dividend is not a multiple of the divisor, there is a remainder.” “The remainder is always less than the divisor.”
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Total number of apples (dividend)	Number of apples in each tray (divisor)	Number of trays (quotient)	Number of apples left over (remainder)	Equation
12	4	3	0	$12 \div 4 = 3$
13	4	3	1	$13 \div 4 = 3 \text{ r } 1$
14	4	3	2	$14 \div 4 = 3 \text{ r } 2$
15	4	3	3	$15 \div 4 = 3 \text{ r } 3$
16	4	4	0	$16 \div 4 = 4$
17	4	4	1	$17 \div 4 = 4 \text{ r } 1$
18	4	4	2	$18 \div 4 = 4 \text{ r } 2$
19	4	4	3	$19 \div 4 = 4 \text{ r } 3$
20	4	5	0	$20 \div 4 = 5$

● **Unitised counters:**

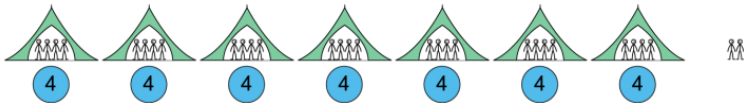


Figure 114: pictorial representation and counters: with 30 scouts and 4 per tent, 7 tents are insufficient

$$30 \div 4 = 7 \text{ r } 2$$

“The 30 represents the total number of scouts.”
 “The 4 represents the number of scouts in each tent.”
 “The 7 represents the number of full tents.”
 “The 2 represents the number of scouts left over.”
 “We need another tent for the 2 left-over scouts. 8 tents are needed.”

**4NF–3 Scaling
number facts by
100**

● **Equations:**

$$8 + 6 = 14 \text{ and } 14 - 6 = 8$$

so

$$800 + 600 = 1,400 \text{ and } 1,400 - 600 = 800$$

$$3 \times 4 = 12 \text{ and } 12 \div 4 = 3$$

so

$$300 \times 4 = 1,200 \text{ and } 1,200 \div 4 = 300$$

● **Place value counters and tens frames:**

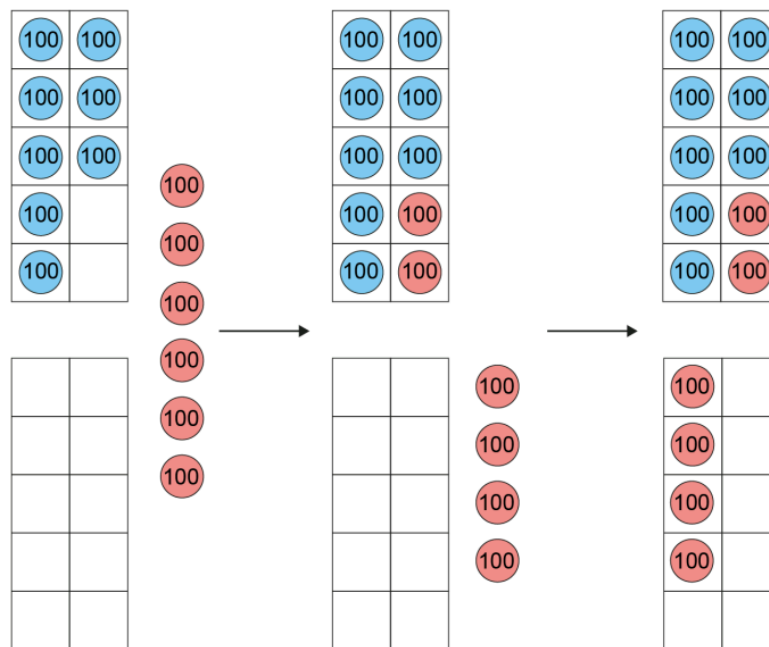


Figure 116: tens frames with 100-value counters showing $800 + 600 = 1,400$

$$8 + 6 = 14$$

$$800 + 600 = 1,400$$

$$14 - 6 = 8$$

$$1,400 - 600 = 800$$

$$14 - 8 = 6$$

$$1,400 - 800 = 600$$

• **Place value counters:**



Figure 117: 3-by-5 array of 100-value place-value counters

“3 times 5 is equal to 15.”
 “3 times 5 hundreds is equal to 15 hundreds.”
 “15 hundreds is equal to 1,500.”

“3 times 5 is equal to 15.”
 “3 hundreds times 5 is equal to 15 hundreds.”
 “15 hundreds is equal to 1,500.”

• **Equations:**

$$3 \times 5 = 15$$

$$3 \times 500 = 1,500$$

$$3 \times 5 = 15$$

$$300 \times 5 = 1,500$$

$$15 \div 3 = 5$$

$$1,500 \div 300 = 5$$

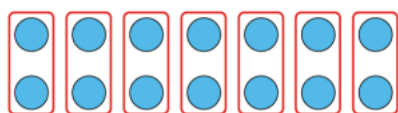
$$1,500 \div 3 = 500$$

“If I multiply one factor by 100, I must multiply the product by 100.”

“If I multiply the dividend by 100 and the divisor by 100, the quotient remains the same.”

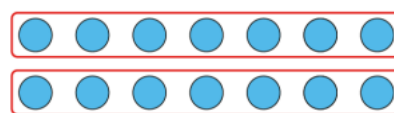
4MD–1 Multiplying and dividing by 10 and 100

		“If I multiply the dividend by 100 and keep the divisor the same, I must multiply the quotient by 100.”																																				
	Gattegno charts: <table><tr><td>1,000</td><td>2,000</td><td>3,000</td><td>4,000</td><td>5,000</td><td>6,000</td><td>7,000</td><td>8,000</td><td>9,000</td></tr><tr><td>100</td><td>200</td><td>300</td><td>400</td><td>500</td><td>600</td><td>700</td><td>800</td><td>900</td></tr><tr><td>10</td><td>20</td><td>30</td><td>40</td><td>50</td><td>60</td><td>70</td><td>80</td><td>90</td></tr><tr><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td><td>7</td><td>8</td><td>9</td></tr></table> × 10 ÷ 10 80 × 10 = 800 80 ÷ 10 = 8 Figure 118: using the Gattegno chart to multiply and divide by 10	1,000	2,000	3,000	4,000	5,000	6,000	7,000	8,000	9,000	100	200	300	400	500	600	700	800	900	10	20	30	40	50	60	70	80	90	1	2	3	4	5	6	7	8	9	“23, made 100 times the size, is 2,300.” “23 multiplied by 100 is equal to 2,300.” “First, we had 23 ones. Now we have 23 hundreds.” “1,450 is 10 times the size of 145.” “1,450 divided by 10 is equal to 145.”
	1,000	2,000	3,000	4,000	5,000	6,000	7,000	8,000	9,000																													
	100	200	300	400	500	600	700	800	900																													
10	20	30	40	50	60	70	80	90																														
1	2	3	4	5	6	7	8	9																														
Place value charts: <table><tr><td>1,000s</td><td>100s</td><td>10s</td><td>1s</td></tr><tr><td></td><td></td><td></td><td>8</td></tr><tr><td></td><td>8</td><td>0</td><td>0</td></tr></table> × 100 ÷ 100 8 × 100 = 800 800 ÷ 100 = 8 Figure 119: using a place-value chart to multiply and divide by 100	1,000s	100s	10s	1s				8		8	0	0	“First, we had 145 tens. Now we have 145 ones.”																									
1,000s	100s	10s	1s																																			
			8																																			
	8	0	0																																			
Equations: 13 × 10 = 130 130 ÷ 10 = 13 130 × 10 = 1,300 1,300 ÷ 10 = 130 13 × 100 = 1,300 1,300 ÷ 100 = 13																																						
4MD–2 Manipulating the multiplicative relationship	Arrays:	“2 groups of 3 is equal to 6.” “3, two times is equal to 6.” “2 groups of 3 is equal to 3, two times.” “2 groups of 7 is equal to 14.” “7 groups of 2 is equal to 14.”																																				



7 groups of 2

$$7 \times 2 = 14$$



2 groups of 7

$$2 \times 7 = 14$$

$$7 \times 2 = 2 \times 7$$

Figure 120: using an array to show that 7 groups of 2 and 2 groups of 7 both correspond to the same total quantity

“2 groups of 7 is equal to 7 groups of 2.”

“factor times factor is equal to product”

“The order of the factors does not alter the product.”

- Unitised counters:

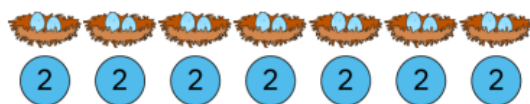


Figure 121: 7 groups of 2 – 7 nests of 2 eggs and seven 2-value counters

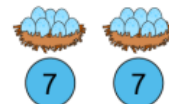


Figure 122: 2 groups of 7 – 2 nests of 7 eggs and two 7-value counters

“The 2 represents number of eggs in each nest/group”.

“The 7 represents the number of nests/groups.”

“The 14 represents the total number of eggs/product.”

“The 2 represents the number of nests/groups.”

“The 7 represents the number of eggs in each nest/group.”

“The 14 represents the total number of eggs/product.”

- Partitive division:

“14 shared between 7 is equal to 2.”

“The 14 represents the total number of eggs.”

“The 7 represents the number of nests/shares.”

“The 2 represents the number of eggs in each nest/share.”

Partitive division

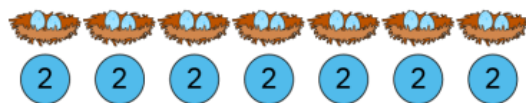


Figure 121: 7 groups of 2 – 7 nests of 2 eggs and seven 2-value counters

Quotitive division

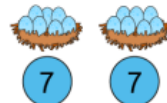
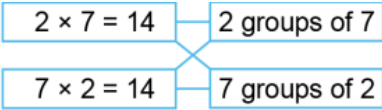
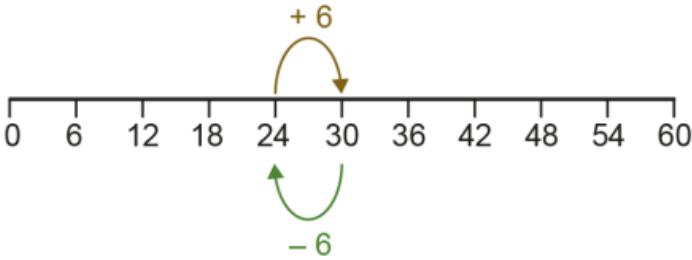


Figure 122: 2 groups of 7 – 2 nests of 7 eggs and two 7-value counters

- Quotitive division:

“14 divided into groups of 7 is equal to 2.

“The 14 represents the total number of eggs.”

		“The 7 represents the number of eggs in each nest/group.” “The 2 represents the number of nests/groups.”
	Schematic diagrams:  Figure 123: schematic diagram summarising the commutative property of multiplication and the different grouping interpretations	
	Equations: $2 \times 7 = 14$ $7 \times 2 = 14$ $14 \div 2 = 7$ $14 \div 7 = 2$	“If we swap the values of the divisor and quotient, the dividend remains the same.”
4MD–3 The distributive property of multiplication	Number lines: 	
	Arrays:	

×	1	2	3	4	5	6
1	●	●	●	●	●	●
2	●	●	●	●	●	●
3	●	●	●	●	●	●
4	●	●	●	●	●	●
5	●	●	●	●	●	●

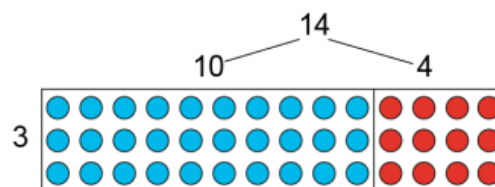


Figure 126: array showing the relationship between 14×3 , 10×3 and 4×3

• Equations:

$$5 \times 6 = 4 \times 6 + 6 \quad \text{or} \quad 5 \times 6 = 4 \times 6 + 1 \times 6$$

$$4 \times 6 = 5 \times 6 - 6 \quad \text{or} \quad 4 \times 6 = 5 \times 6 - 1 \times 6$$

“5 is equal to 3 plus 2, so 5 times 6 is equal to 3 times 6 plus 2 times 6.”

• Number lines:

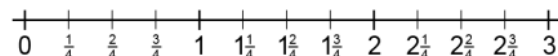


Figure 128: number line to support counting in multiples of one quarter

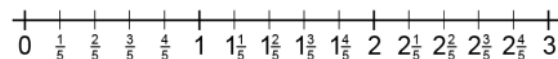


Figure 129: number line to support counting in multiples of one fifth

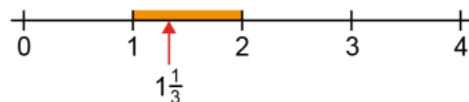


Figure 131: placing a mixed number on a number line with no fractional marks



$\frac{1}{4}$

4F–1 Mixed numbers in the linear number system

“Each interval is divided into 4 equal parts, so we count in quarters.”

“ is between 1 and 2.”

“The previous whole number is 1.”

“The next whole number is 2.”

**4F–2 Convert
between mixed
numbers and
improper fractions**

• **Number lines:**

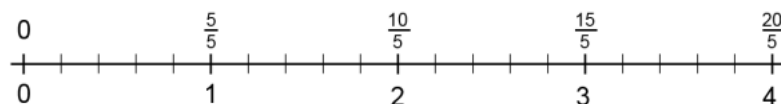


Figure 132: number line showing integers expressed as fractions

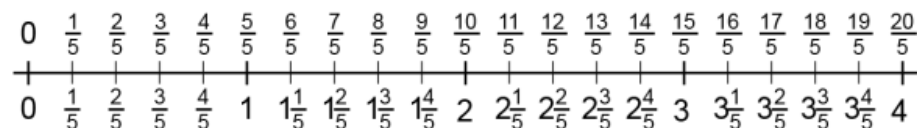
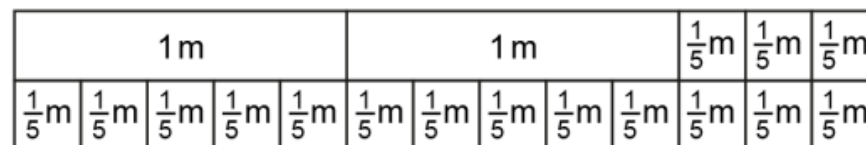


Figure 133: number line showing mixed number–improper fraction equivalence

“When the numerator is a multiple of the denominator, the fraction is equivalent to a whole number.”

• **Bar models:**



$$2\frac{3}{5}\text{m} = \frac{13}{5}\text{m}$$

Figure 134: bar model showing mixed number–improper fraction equivalence

“There are 2 groups of five-fifths, which is 10 fifths and 3 more fifths. This is 13 fifths.”

4F–3 Add and subtract improper and mixed fractions (same denominator)

• **Unitised counters:**

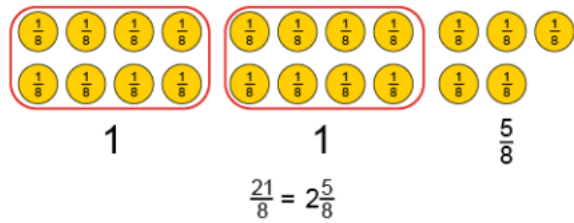


Figure 135: using unit fraction counters to support conversion from an improper fraction to a mixed number

“We have 21 eighths. 8 eighths is equal to 1 (whole).”

“21 eighths is equal to 2 groups of 8 eighths, and 5 more eighths. This is 2 and 5 eighths.”

• **Equations:**

$$\begin{aligned} \frac{7}{5} + \frac{4}{5} &= \frac{11}{5} \\ 3\frac{7}{8} - \frac{2}{8} &= 3\frac{5}{8} \\ 7\frac{2}{5} + \frac{4}{5} &= 8\frac{1}{5} \\ 8\frac{1}{5} - \frac{4}{5} &= 7\frac{2}{5} \end{aligned}$$

“7 one-fifths plus 4 one-fifths is equal to 11 one-fifths.”

• **Bar models:**

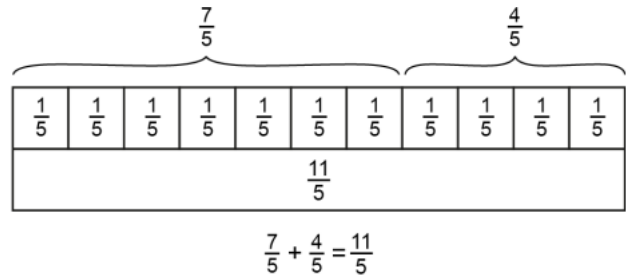


Figure 136: bar model showing addition of improper fractions with the same denominator

“7 one-fifths plus 4 one-fifths is equal to 11 one-fifths.”

• **Number lines and area models:**

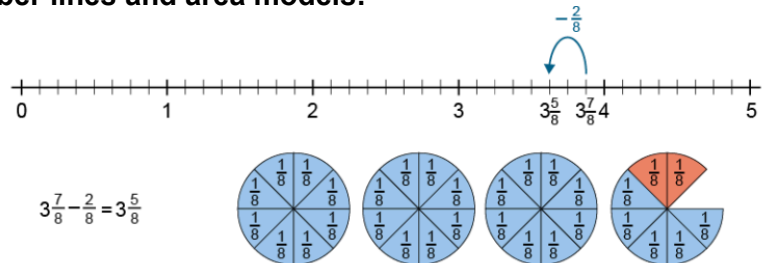
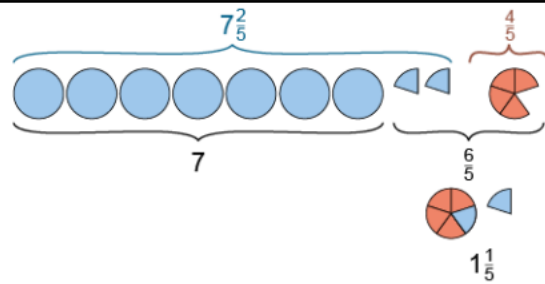


Figure 137: number line and pie model showing subtraction of a fraction from a mixed number (same denominator)

“When adding fractions with the same denominators, just add the numerators.”



“When subtracting fractions with the same denominators, just subtract the numerators.”

4G–1 Draw polygons specified by coordinates or by Translation

- Polygons and points on square grids:**

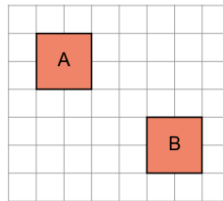


Figure 141: translation of a polygon on a square grid

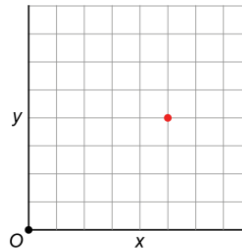


Figure 142: marking a point relative to the origin (O) on a grid with no number labels

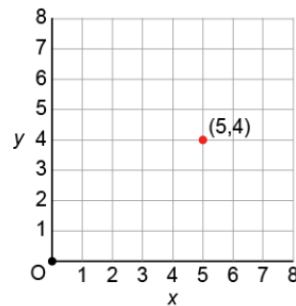


Figure 143: marking a point on a grid with number labels

“The polygon has been translated 4 squares to the right and 3 squares down.”

“First count along the x-axis, then count along the y-axis.”

“First count along the x-axis, then count along the y-axis.”

• **Polygons:**

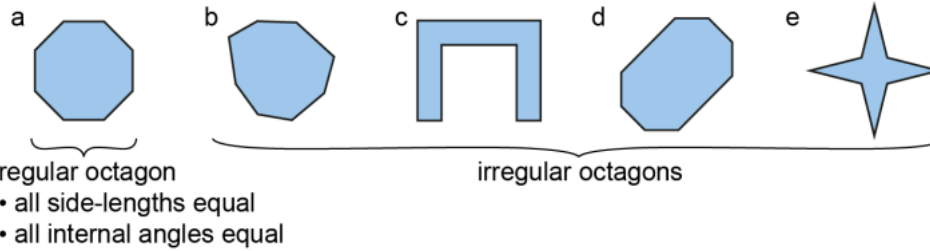
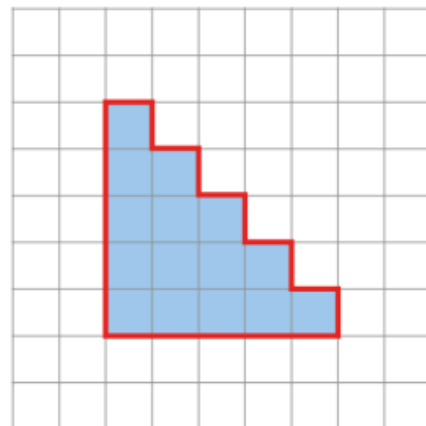


Figure 144: a regular octagon and 4 irregular octagons

“This is a regular polygon, because all of the sides are the same length, and all of the interior angles are equal.”

• **Rectilinear shapes and grids:**



perimeter = 20cm

1cm
1cm

• **Formulas and unitised counters:**

	use doubling and addition: $\text{perimeter} = 2 \times (\text{length} + \text{width})$ or $\text{perimeter} = (2 \times \text{length}) + (2 \times \text{width})$ Figure 146: 2 strategies for calculating the perimeter of a rectangle <i>Drawn to scale.</i>	
4G–3 Identify line symmetry in 2D shapes	• Polygons: no line symetry line symetry Figure 147: composing shapes from two identical isosceles triangles	“This is a line of symmetry because it splits the shape into two equal parts which are mirror images.”
	• Patterns and grids	

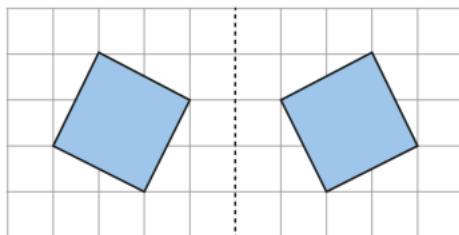


Figure 148: reflecting a shape in a line of symmetry

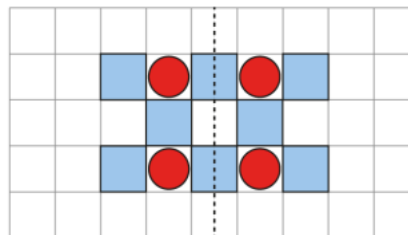


Figure 149: a symmetrical pattern

Year 5

Ready to Progress

Representation

Language Focus

5NPV–1 Tenths and hundredths

• Place value cha

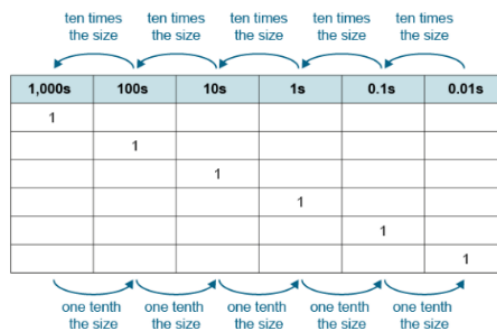


Figure 153: place-value chart illustrating the scaling relationship between adjacent positions, including tenths and hundredths

“1 is 10 times the size of one-tenth.”

“One-tenth is 10 times the size of one-hundredth.”

“1 is 100 times the size of one-hundredth.”

• Base-10:

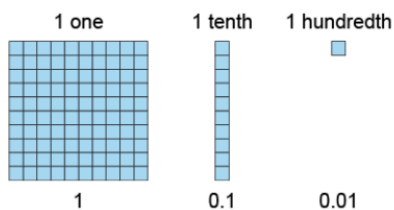


Figure 154: using Dienes to represent 1, one-tenth and one-hundredth

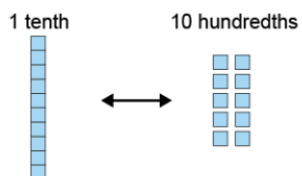


Figure 155: using Dienes to represent the equivalence of 1 tenth and 10 hundredths

“10 tenths is equal to 1 one.”

“10 hundredths is equal to 1 tenth.”

“100 hundredths is equal to 1 one.”

	Tens frames and place-value counters: 0.01 0.01 0.01 0.01 0.01 0.01 0.01 0.01 0.01 0.01 0.01 0.01 0.01 0.01 0.01 0.01 0.01 0.01 Figure 156: eighteen 0.01-value place-value counters in 2 tens frames	“18 hundredths is equal to 10 hundredths and 8 more hundredths.” “10 hundredths is equal to 1 tenth.” “So 18 hundredths is equal to 1 tenth and 8 more hundredths, which is 0.18.”																																																						
5NPV–2 Place value in decimal fractions	Gattegno charts: <table><tr><td>1,000</td><td>2,000</td><td>3,000</td><td>4,000</td><td>5,000</td><td>6,000</td><td>7,000</td><td>8,000</td><td>9,000</td></tr><tr><td>100</td><td>200</td><td>300</td><td>400</td><td>500</td><td>600</td><td>700</td><td>800</td><td>900</td></tr><tr><td>10</td><td>20</td><td>30</td><td>40</td><td>50</td><td>60</td><td>70</td><td>80</td><td>90</td></tr><tr><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td><td>7</td><td>8</td><td>9</td></tr><tr><td>0.1</td><td>0.2</td><td>0.3</td><td>0.4</td><td>0.5</td><td>0.6</td><td>0.7</td><td>0.8</td><td>0.9</td></tr><tr><td>0.01</td><td>0.02</td><td>0.03</td><td>0.04</td><td>0.05</td><td>0.06</td><td>0.07</td><td>0.08</td><td>0.09</td></tr></table> Figure 157: Gattegno chart showing thousands, hundreds, tens, ones, tenths and hundredths	1,000	2,000	3,000	4,000	5,000	6,000	7,000	8,000	9,000	100	200	300	400	500	600	700	800	900	10	20	30	40	50	60	70	80	90	1	2	3	4	5	6	7	8	9	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	“Three hundredths is zero-point-zero-three.”
	1,000	2,000	3,000	4,000	5,000	6,000	7,000	8,000	9,000																																															
	100	200	300	400	500	600	700	800	900																																															
10	20	30	40	50	60	70	80	90																																																
1	2	3	4	5	6	7	8	9																																																
0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9																																																
0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09																																																
Place-value counters, part-whole models and number sentences. 10 10 10 10 10 1 1 1 0.1 0.1 0.1 0.1 0.01 0.01 53.42 0.4 3 0.02 50 53.42 – 3 = 53.42 – = 53.02 Figure 159: 2 representations of the place-value composition of 53.42																																																								
Digits and words: 0.03 is zero-point-zero-three																																																								

5NPV-3 Decimal fractions in the linear number system

• Number lines:

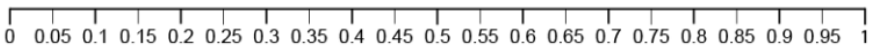


Figure 162: 0 to 1 number line marked and labelled in intervals of 5 hundredths

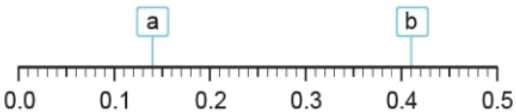


Figure 163: identifying 0.14 and 0.41 on a 0 to 0.5 number line marked with intervals of hundredths



Figure 164: placing 8.6 on an unmarked 8 to 9 number line

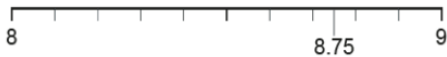


Figure 165: placing 8.75 on an 8 to 9 number line marked only in tenths

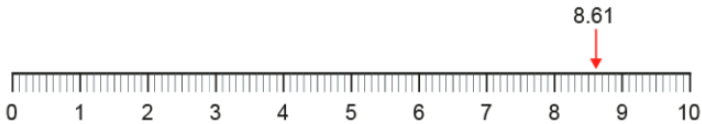


Figure 166: using a number line to identify the previous and next whole number

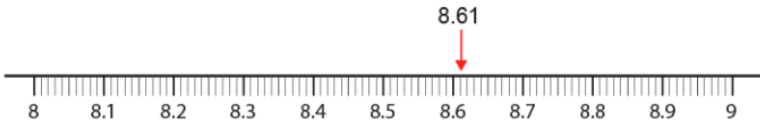


Figure 167: using a number line to identify the previous and next multiple of 0.1

“a is 0.14 because it is 1 hundredth less than the midpoint of 0.1 and 0.2, which is 0.15.”

“b is 0.41 because it is 1 hundredth more than 0.4.”

“The previous whole number is 8. The next whole number is 9.” “The previous multiple of 0.1 is 8.6. The next multiple of 0.1 is 8.7.”

• Inequality expressions:

“The closest whole number is 9.”
“8.61 rounded to the nearest whole number is 9.”

	previous whole number next whole number 8 < 8.61 < 9 Figure 168: Identifying the nearest whole number with a number line for support																						
5NPV–4 Reading scales with 2, 4, 5 or 10 intervals	• Bar models: Figure 169: bar models showing 1 partitioned into 2, 4, 5 and 10 equal parts	“Twenty-five, fifty, seventy-five, one hundred” needs to be a fluent spoken language pattern. Fluency in this language pattern provides the basis to count in multiples of 0.25.																					
5NPV–5 Convert between units of measure	• Conversion charts: <table border="1"> <thead> <tr> <th>Distance in km expressed as a fraction</th><th>Distance in km expressed as a decimal fraction</th><th>Distance in metres</th></tr> </thead> <tbody> <tr> <td>$\frac{1}{5}$ km</td><td>0.2km</td><td>200m</td></tr> <tr> <td>$\frac{1}{4}$ km</td><td>0.25km</td><td>250m</td></tr> <tr> <td>$\frac{1}{2}$ km</td><td>0.5km</td><td>500m</td></tr> <tr> <td>$\frac{3}{4}$ km</td><td>0.75m</td><td>750m</td></tr> <tr> <td>$\frac{1}{10}$ km</td><td>0.1km</td><td>100m</td></tr> <tr> <td>all other multiples of $\frac{1}{10}$ km, for example, $\frac{7}{10}$ km</td><td>0.7km</td><td>700m</td></tr> </tbody> </table>	Distance in km expressed as a fraction	Distance in km expressed as a decimal fraction	Distance in metres	$\frac{1}{5}$ km	0.2km	200m	$\frac{1}{4}$ km	0.25km	250m	$\frac{1}{2}$ km	0.5km	500m	$\frac{3}{4}$ km	0.75m	750m	$\frac{1}{10}$ km	0.1km	100m	all other multiples of $\frac{1}{10}$ km, for example, $\frac{7}{10}$ km	0.7km	700m	“1m is 100cm.” “So is 75cm.” “1,000ml is 1 litre.” “So, 3,000ml is 3 litres, and 3,700ml is 3.7 litres.” “100p is £1.” “So 50p is £0.50, and 52p is £0.52.”
Distance in km expressed as a fraction	Distance in km expressed as a decimal fraction	Distance in metres																					
$\frac{1}{5}$ km	0.2km	200m																					
$\frac{1}{4}$ km	0.25km	250m																					
$\frac{1}{2}$ km	0.5km	500m																					
$\frac{3}{4}$ km	0.75m	750m																					
$\frac{1}{10}$ km	0.1km	100m																					
all other multiples of $\frac{1}{10}$ km, for example, $\frac{7}{10}$ km	0.7km	700m																					

	Ratio tables:																
	<table><tr><td>1m</td><td>100cm</td></tr><tr><td>$\frac{3}{4}$m</td><td>75cm</td></tr></table>	1m	100cm	$\frac{3}{4}$ m	75cm	<table><tr><td>1,000ml</td><td>1 litre</td></tr><tr><td>3,700ml</td><td>3.7 litres</td></tr></table>	1,000ml	1 litre	3,700ml	3.7 litres	<table><tr><td>100p</td><td>£1</td></tr><tr><td>52p</td><td>£0.52</td></tr></table>	100p	£1	52p	£0.52		
1m	100cm																
$\frac{3}{4}$ m	75cm																
1,000ml	1 litre																
3,700ml	3.7 litres																
100p	£1																
52p	£0.52																

5NF–1 Secure fluency in multiplication and division facts

- Written short multiplication and division:

Identifying core number facts: short multiplication	Identifying core number facts: short division
3 4 2 x 7 2 , 3 9 4 2 1 Figure 170: short multiplication of 342 by 7 6 1 9 8 4 , 9 5 2 Figure 171: short division of 4,952 by 8	6 1 9 8 4 , 9 5 2 Figure 171: short division of 4,952 by 8
Within-column calculations: 7 × 2 = 14 7 × 4 + 1 = 28 + 1 7 × 3 + 2 = 21 + 2	Within-column calculations: 4 ÷ 8 = 0 r 4 49 ÷ 8 = 6 r 1 15 ÷ 8 = 1 r 7 72 ÷ 8 = 9

- Equations:

2×2=4	3×3=9						
3×2=6	4×3=12	4×4=16					
4×2=8	5×3=15	5×4=20	5×5=25				
5×2=10	6×3=18	6×4=24	6×5=30	6×6=36			
6×2=12	7×3=21	7×4=28	7×5=35	7×6=42	7×7=49		
7×2=14	8×3=24	8×4=32	8×5=40	8×6=48	8×7=56	8×8=64	
8×2=16	9×3=27	9×4=36	9×5=45	9×6=54	9×7=63	9×8=72	9×9=81
9×2=18							

5NF–2 Scaling
number facts by 0.1
or 0.01

• Equations:

$8 + 6 = 14$
 $0.8 + 0.6 = 1.4$
 $0.08 + 0.06 = 0.14$

$3 \times 4 = 12$
 $0.3 \times 4 = 1.2$
 $0.03 \times 4 = 0.12$

“8 plus 6 is equal to 14, so 8 tenths plus 6 tenths is equal to 14 tenths.” “14 tenths is equal to 1 one and 4 tenths.”

• Tens frames and place-value counters:

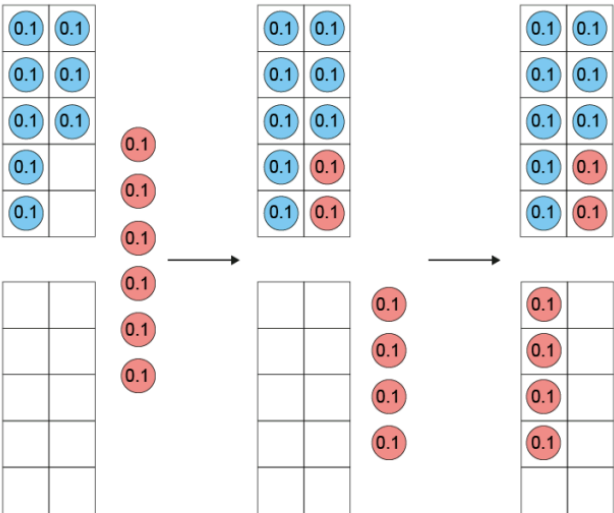


Figure 172: tens frames with 0.1-value counters showing $0.8 + 0.6 = 1.4$

$8 + 6 = 14$
 $0.8 + 0.6 = 1.4$

$14 - 6 = 8$
 $1.4 - 0.6 = 0.8$

$14 - 8 = 6$
 $1.4 - 0.8 = 0.6$

“8 plus 6 is equal to 14, so 8 tenths plus 6 tenths is equal to 14 tenths.” “14 tenths is equal to 1 one and 4 tenths.”

• 100 grids:

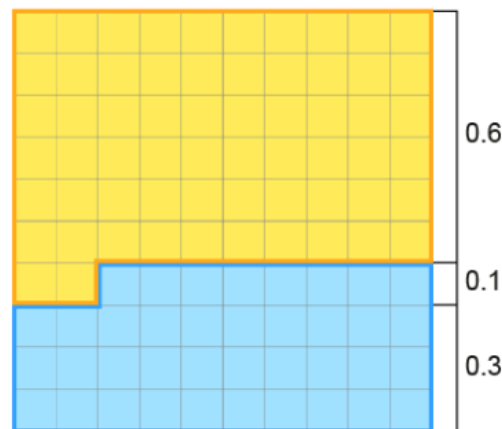


Figure 173: a 100 grid shaded in 2 colours to represent 0.62 and 0.38 as a complement to 1

- Place-value counters and equations:



Figure 174: 3-by-5 array of 0.01-value place-value counters

$$3 \times 5 = 15$$

$$3 \times 0.05 = 0.15$$

$$3 \times 5 = 15$$

$$0.03 \times 5 = 0.15$$

$$15 \div 3 = 5$$

$$0.15 \div 0.03 = 5$$

$$0.15 \div 3 = 0.05$$

“3 times 5 is equal to 15.”

“3 times 5 hundredths is equal to 15 hundredths.”

“15 hundredths is equal to 0.15.”

“3 times 5 is equal to 15.”

“3 hundredths times 5 is equal to 15 hundredths.”

“15 hundredths is equal to 0.15.”

If I make one factor one-hundredth times the size, I must make the product one hundredth times the size.”

“If I make the dividend one-hundredth times the size and the divisor one-hundredth times the size, the quotient remains the same.”

“If I make the dividend one-hundredth times the size and keep the divisor the same, I must make the quotient one-hundredth times the size.”

5MD–1 Multiplying and dividing by 10 and 100

Multiplicative and division chains:

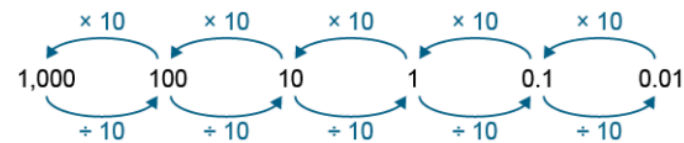


Figure 175: multiplicative relationships between powers of 10: 10 times the size and one-tenth times the size

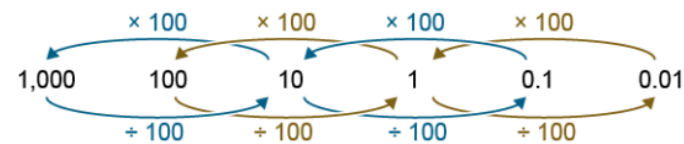


Figure 176: multiplicative relationships between powers of 10: 100 times the size and one-hundredth times the size

Gattegno charts:

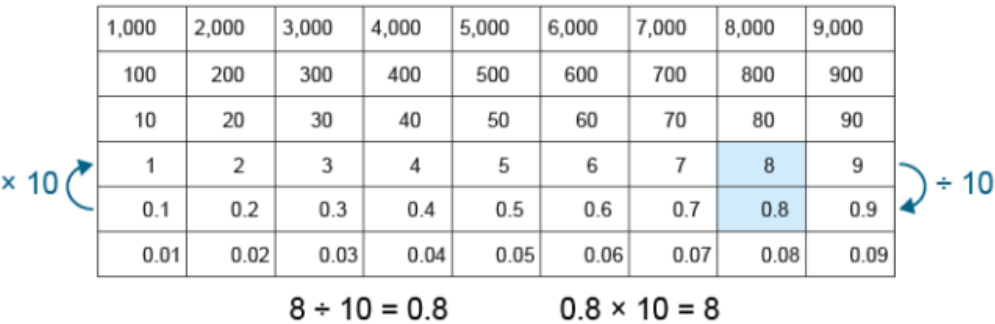


Figure 177: using the Gattegno chart to multiply and divide by 10, crossing 1

“8, made one-tenth of the size, is 0.8.”

“8 divided by 10 is equal to 0.8.”

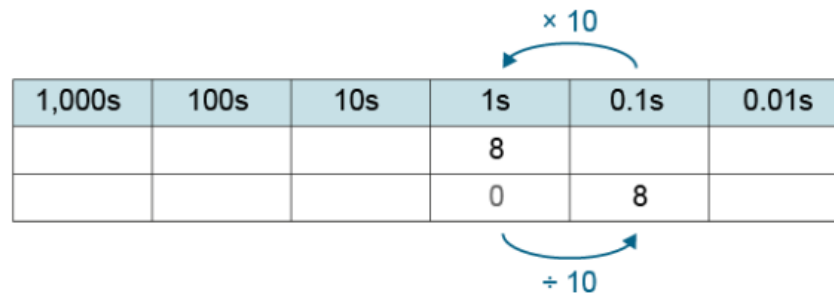
“First, we had 8 ones. Now we have 8 tenths.”

“0.8, made 10 times the size, is 8.”

“0.8 multiplied by 10 is equal to 8.”

“First, we had 8 tenths. Now we have 8 ones.”

- Place-value charts:



$$8 \div 10 = 0.8$$

$$0.8 \times 10 = 8$$

Figure 178: using a place-value chart to multiply and divide by 10, crossing 1

“8, made one-tenth of the size, is 0.8.”

“8 divided by 10 is equal to 0.8.”

“First we had 8 ones. Now we have 8 tenths.”

“0.8, made 10 times the size, is 8.”

“0.8 multiplied by 10 is equal to 8.”

“First we had 8 tenths. Now we have 8 ones.”

- Equations:

$$13 \div 100 = 0.13$$

$$0.13 \times 100 = 13$$

$$26.5 \div 10 = 2.65$$

$$2.65 \times 10 = 26.5$$

$$4,710 \div 100 = 47.1$$

$$47.1 \times 100 = 4,710$$

“8cm is 10 times the length of 0.8cm.”

“0.25kg is one-hundredth times the mass of 25kg.”

“150 pencils is 10 times as many as 15 pencils.”

5MD–2 Find factors and multiples

- Equations:

$$7 \times 3 = 21$$

$$7 \times 300 = 2,100$$

$$700 \times 3 = 2,100$$

Applying commutativity		Applying associativity (example)
$3 \times 7 \times 10 = 210$	$3 \times 10 \times 7 = 210$	$3 \times 7 \times 10 = 210$
$7 \times 3 \times 10 = 210$	$7 \times 10 \times 3 = 210$	$(3 \times 7) \times 10 = 21 \times 10 = 210$
$10 \times 3 \times 7 = 210$	$10 \times 7 \times 3 = 210$	$3 \times (7 \times 10) = 3 \times 70 = 210$

“A multiple of a given number is the product of the given number and any whole number.”

“A factor of a given number is a whole number that the given number can be divided by without giving a remainder.”

“21 is a multiple of 3. 3 is a factor of 21.”

“21 is a multiple of 3, so...

“2,100 is a multiple of 300”

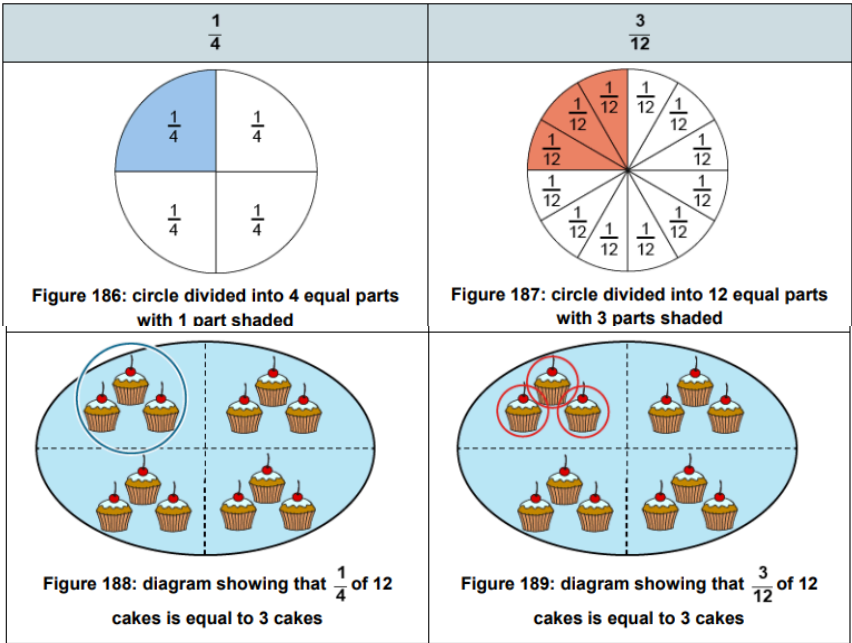
“2,100 is a multiple of 3”

5MD–3 Multiply using a formal written method	Place-value counters: 10 10 10 10 10 10 1 1 1 1 1 1 1 1 Figure 179: place-value counters showing 34×2 248							
	Written methods: <table><tr><th>Informal written method</th><th>Expanded multiplication algorithm</th><th>Short multiplication</th></tr><tr><td> 34×2 = 30×2 + 4×2 = 60 + 8 = 68 </td><td> 10s 1s 3 4 × 2 8 6 0 6 8 2 × 4 ones = 8 ones 2 × 3 tens = 6 tens </td><td> 10s 1s 3 4 × 2 8 6 </td></tr></table> 367 × 4 1 4 6 8 2 2 Figure 180: multiplying 367 by 4 using short multiplication 4×7 ones = 28 ones 4×6 tens = 24 tens plus 2 more tens = 2 hundreds + 6 tens 4×3 hundreds = 12 hundreds plus 2 more hundreds = 1 thousand + 4 hundreds = 2 tens + 8 ones = 2 hundreds + 4 tens = 2 hundreds + 6 tens = 1 thousand + 2 hundreds = 1 thousand + 4 hundreds	Informal written method	Expanded multiplication algorithm	Short multiplication	34×2 = 30×2 + 4×2 = 60 + 8 = 68	10s 1s 3 4 × 2 8 6 0 6 8 2 × 4 ones = 8 ones 2 × 3 tens = 6 tens	10s 1s 3 4 × 2 8 6	“2 times 4 ones is equal to 8 ones: write 8 in the ones column.” “2 times 3 tens = 6 tens: write 6 in the tens column.”
Informal written method	Expanded multiplication algorithm	Short multiplication						
34×2 = 30×2 + 4×2 = 60 + 8 = 68	10s 1s 3 4 × 2 8 6 0 6 8 2 × 4 ones = 8 ones 2 × 3 tens = 6 tens	10s 1s 3 4 × 2 8 6						
5MD–4 Divide using a formal written method	Place-value counters and written methods:	“8 tens divided by 4 is equal to 2 tens: write 2 in the tens column.” “4 ones divided by 4 is equal to 1 one: write 1 in the ones column.”						

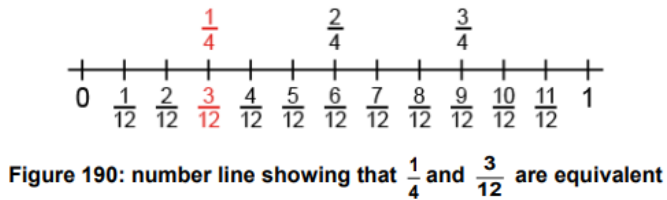
	<table><thead><tr><th>Short division with place-value counters</th><th>Short division</th></tr></thead><tbody><tr><td> 21 4 1010 1010 1010 1010 1 1 1 1 8 tens ÷ 4 = 2 tens 4 ones ÷ 4 = 1 one Figure 181: dividing 84 by 4 using short division with place-value counters </td><td> 10s1s 21 4 8 4 Figure 182: dividing 84 by 4 using short division with place-value headings </td></tr></tbody></table> 153 4 6 2 1 2 Figure 183: dividing 612 by 4 using short division 6 hundreds ÷ 4 = 1 hundred remainder 2 hundreds 2 hundreds = 20 tens plus 1 more ten = 21 tens 21 tens ÷ 4 = 5 tens remainder 1 ten 1 ten = 10 ones plus 2 more ones = 12 ones 12 ones ÷ 4 = 3 ones	Short division with place-value counters	Short division	21 4 1010 1010 1010 1010 1 1 1 1 8 tens ÷ 4 = 2 tens 4 ones ÷ 4 = 1 one Figure 181: dividing 84 by 4 using short division with place-value counters	10s1s 21 4 8 4 Figure 182: dividing 84 by 4 using short division with place-value headings	
Short division with place-value counters	Short division					
21 4 1010 1010 1010 1010 1 1 1 1 8 tens ÷ 4 = 2 tens 4 ones ÷ 4 = 1 one Figure 181: dividing 84 by 4 using short division with place-value counters	10s1s 21 4 8 4 Figure 182: dividing 84 by 4 using short division with place-value headings					
5F–1 Find non-unit fractions of quantities	● Bar models and equations: 40 8 8 8 8 8 15 15 15 15 15 35 Figure 184: bar model to support finding three-fifths of 40 40 ÷ 5 = 8 So, 15 of 40 = 8 8 × 3 = 24 So, 35 of 40 = 24 30 56 → 180 30 30 30 30 30 30 56 Figure 185: using a bar model to solve more complex problems related to fractions of quantities	“To find of 15, we divide 15 into 5 equal parts.” “15 divided by 5 is equal to 3, so of 15 is equal to 3.” “Three-fifths is equal to 3 one-fifths.” “To find 3 one-fifths of 40, first find one-fifth of 40 by dividing by 5, and then multiply by 3.”				

5F–2 Find equivalent fractions

Area models:



Number lines:



“The whole is divided into 4 equal parts and 1 of those parts is shaded.”

“The whole is divided into 12 equal parts and 3 of those parts is shaded.”

“ and are equivalent because 1 is the same portion of 4 as 3 is of 12.

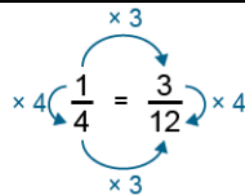


Figure 191: diagram showing the multiplicative relationships between the numerators and denominators in $\frac{1}{4}$ and $\frac{3}{12}$

5F-3 Recall decimal equivalents for common fractions

- Conversions charts:

Unit fraction	Decimal fraction
$\frac{1}{2}$	0.5
$\frac{1}{4}$	0.25
$\frac{1}{5}$	0.2
$\frac{1}{10}$	0.1

- Hundred grids:

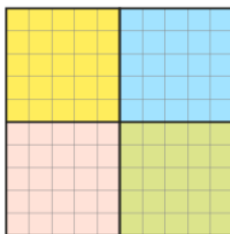


Figure 192: hundred grid divided into 4 equal parts: $\frac{1}{4}$ is equal to 25 hundredths

$$\frac{1}{4} = \frac{25}{100} = 0.25$$

- Number lines:

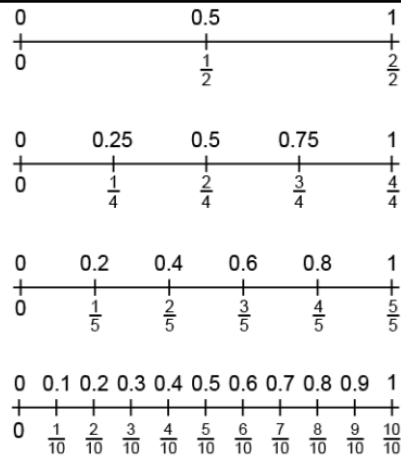


Figure 193: 0 to 1 number lines illustrating common proper fraction – decimal fraction equivalents

5G–1 Compare, estimate, measure and draw angles

- Shapes:

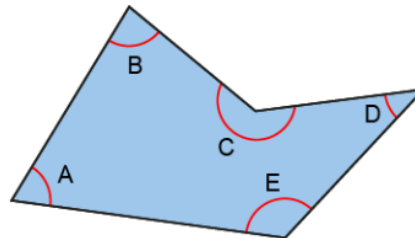


Figure 194: irregular pentagon with 3 acute internal angles, 1 obtuse internal angle and 1 reflex internal angle

- Angles:

“An acute angle is smaller than a right angle.”

“An obtuse angle is larger than a right angle but less than the angle on a straight line.”

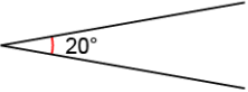
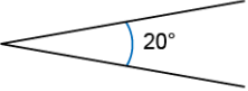
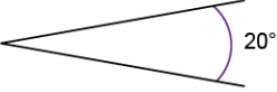
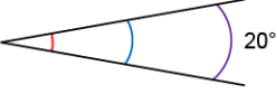
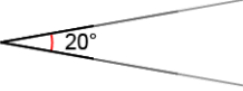
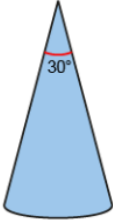
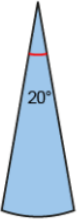
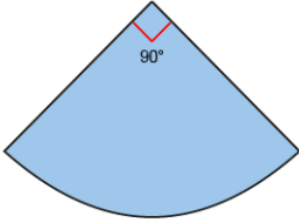
“A reflex angle is larger than the angle on a straight line, but less than the angle for a full turn.”

“D is the smallest angle. It is an acute angle.”

“C is the largest angle. It is a reflex angle.”

“An acute angle is less than 90° .”

“An obtuse angle is greater than 90° but less than 180° .”

	    Figure 195: the position of the arc indicating an angle does not affect the size of the angle     Figure 196: the length of the lines does not affect the size of the angle between them     Figure 197: a selection of 'standard angle' measuring tools	“A reflex angle is greater than 180° but less than 360°.”
5G–2 Compare and calculate areas	• Shapes:	“To find the area of a rectangle, multiply the length by the width.”

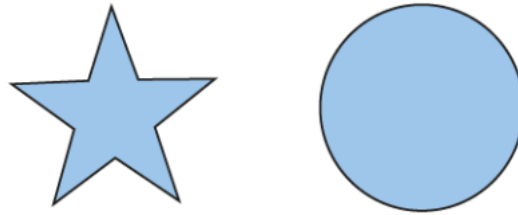


Figure 198: a decagon and a circle with a clear visual difference in area

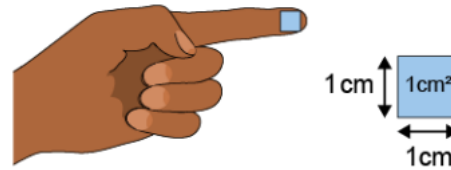


Figure 199: a square centimetre

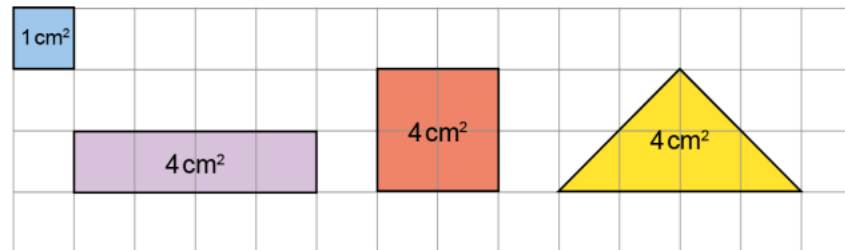


Figure 200: a rectangle, square and triangle with equal areas

Drawn to actual size.

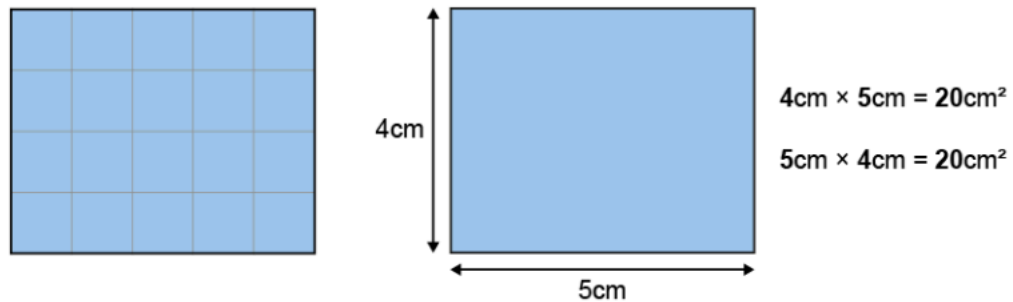


Figure 201: the area of a rectangle can be calculated by multiplying the length by the width

Drawn to actual size.

Year 6

Ready to Progress

Representations

Language Focus

6NPV-1 Powers of 10

- **Digits and words:**

0 . 0 1	one hundredth
0 . 1	one tenth
1	one
1 0	ten
1 0 0	one hundred
1 , 0 0 0	one thousand
1 0 , 0 0 0	ten thousand
1 0 0 , 0 0 0	one hundred thousand
1 , 0 0 0 , 0 0 0	one million
1 0 , 0 0 0 , 0 0 0	ten million

- **Gattegno charts:**

10,000,000	20,000,000	30,000,000	40,000,000	50,000,000	60,000,000	70,000,000	80,000,000	90,000,000
1,000,000	2,000,000	3,000,000	4,000,000	5,000,000	6,000,000	7,000,000	8,000,000	9,000,000
100,000	200,000	300,000	400,000	500,000	600,000	700,000	800,000	900,000
10,000	20,000	30,000	40,000	50,000	60,000	70,000	80,000	90,000
1,000	2,000	3,000	4,000	5,000	6,000	7,000	8,000	9,000
100	200	300	400	500	600	700	800	900
10	20	30	40	50	60	70	80	90
1	2	3	4	5	6	7	8	9
0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09

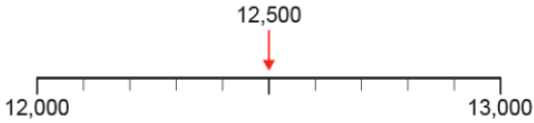
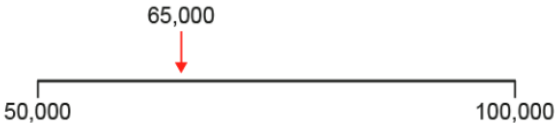
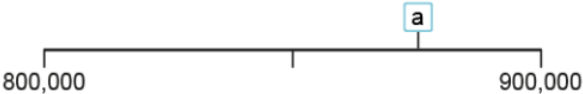
Figure 207: using the Gattegno chart to multiply and divide by 100

“10 hundred-thousands is equal to 1 million.”
“1,000,000 is 10 times the size of 100,000.”
“100,000 is one-tenth times the size of 1,000,000.”
“10 thousands is equal to 10,000.”
“10,000 is 10 times the size of 1,000.”
“1,000 is one-tenth times the size of 10,000.”
“50,000 is 100 times the size of 500.”
“500 multiplied by 100 is equal to 50,000.”
“500 is one-hundredth times the size of 50,000.”
“50,000 divided by 100 is equal to 500.”

- **Place value counters and tens frames:**



Figure 206: ten 100,000-value place-value counters in a tens frame

6NPV–2 Place value in numbers up to 10,000,000	• Equations: $518.32 + 30 = 548.32$ $381,920 - 60,000 = 321,920$	“In 67,000.4... - the 6 represents 6 ten-thousands; the value of the 6 is 60,000 - the 7 represents 7 thousands; the value of the 7 is 7,000 - the 4 represents 4 tenths; the value of the 4 is 0.4”
6NPV–3 Numbers up to 10 million in the linear number system	• Number lines:  Figure 208: placing 12,500 on a 12,000 to 13,000 number line marked, but not labelled, in multiples of 100  Figure 209: placing 12,500 on a 10,000 to 20,000 number line marked, but not labelled, in multiples of 1,000  Figure 210: estimating the position of 65,000 on an unmarked 50,000 to 100,00 number line  Figure 211: identifying 875,000 on a 800,00 to 900,000 number line marked only with a midpoint	“The previous multiple of 1 million is 5 million. The next multiple of 1 million is 6 million.” “The previous multiple of 100,000 is 5,100,000. The next multiple of 100,000 is 5,200,000.” “The closest multiple of 1 million is 5 million.” “5,192,012 rounded to the nearest million is 5 million.” “The closest multiple of 100,000 is 5,200,000.” “5,192,012 rounded to the nearest 100,000 is 5,200,000.”

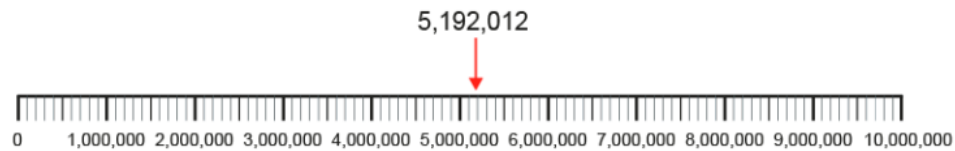


Figure 212: using a number line to identify the previous and next multiple of 1 million

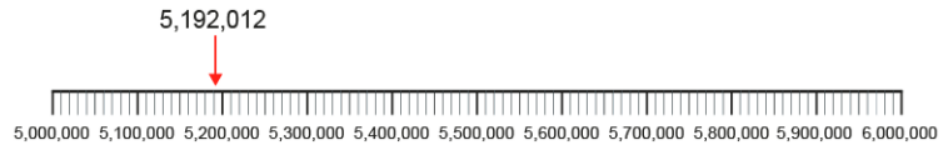


Figure 213: using a number line to identify the previous and next multiple of 100,000

- Inequality expressions:

previous multiple of 1,000,000		next multiple of 1,000,000
5,000,000	$< 5,192,012 <$	6,000,000

previous multiple of 100,000		next multiple of 100,000
5,100,000	$< 5,192,012 <$	5,200,000

Figure 214: identifying the nearest multiple of 1 million and the nearest multiple of 100,000

- Bar models:

6NPV–4 Reading
scales with 2, 4, 5
or 10 intervals

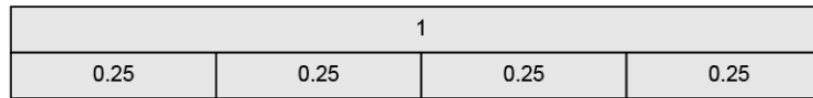
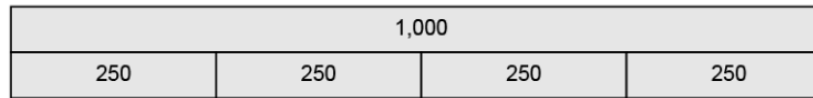
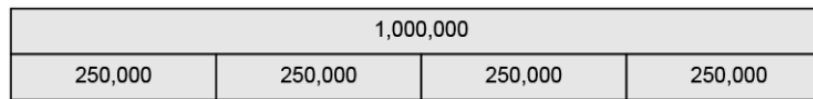


Figure 215: bar models showing 1 million, 1,000 and 1 partitioned into 4 equal parts

- Equations:**

$$750,000 + 250,000 = 1,000,000$$

$$1,000,000 - 250,000 = 750,000$$

$$1,000,000 - 750,000 = 250,000$$

$$1,000,000 \div 4 = 250,000$$

$$1,000,000 \div 250,000 = 4$$

$$4 \times 250,000 = 1,000,000$$

$$250,000 \times 4 = 1,000,000$$

$$\frac{1}{4} \text{ of } 1,000,000 = 250,000$$

- Diagrams:**



Figure 216: additive relationship between 20 and 60



Figure 217: multiplicative relationship between 20 and 60

6AS/MD–1 Quantify additive and multiplicative relationships

“The relationship between 2 numbers can be expressed additively or multiplicatively.”

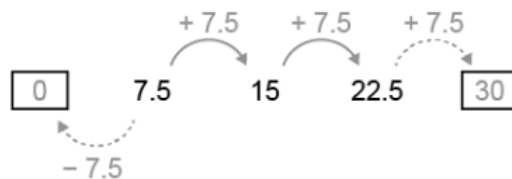


Figure 218: completing a sequence where the difference between adjacent terms is 7.5

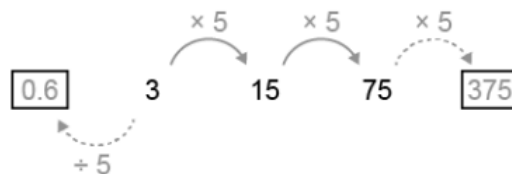


Figure 219: completing a sequence where each term is 5 times the previous

6AS/MD–2 Derive related calculations

- Equations:

$$70 + 2 = 72 \qquad 60 + 12 = 72$$

$$5 \times 8 = 10 \times 4$$

“If one addend is increased and the other is decreased by the same amount, the sum stays the same.”

“If I multiply one factor by a number, I must divide the other factor by the same number for the product to stay the same”

“If I multiply one factor by a number, and keep the other factor the same, I must multiply the product by the same number.”

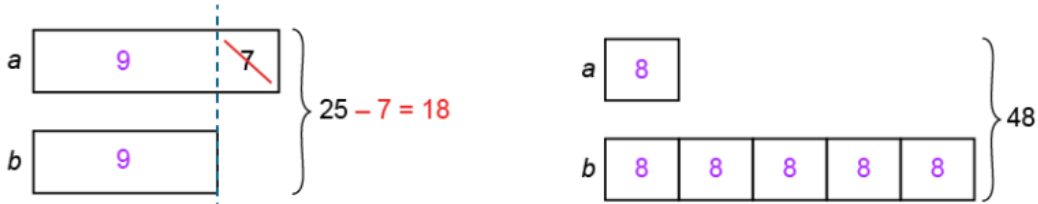
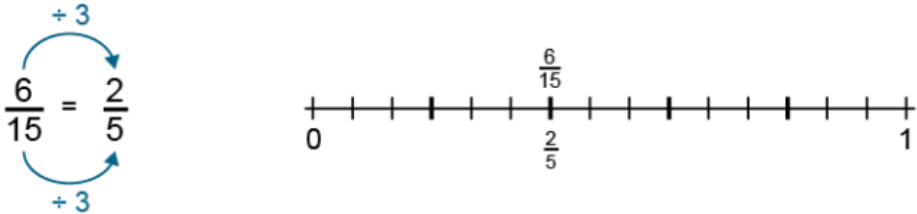
6AS/MD–3 Solve problems involving ratio relationships

- Ratio tables:

“For every 1 cup of rice you cook, you need 2 cups of water.”

“For every 10 children on the school trip, there must be 1 adult.”

	<table><tr><td>cups of rice</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td></tr><tr><td>cups of water</td><td>2</td><td>4</td><td>6</td><td>8</td><td>10</td><td>12</td></tr></table> <table><tr><td>number of children</td><td>10</td><td>20</td><td>30</td><td>40</td><td>50</td><td>60</td></tr><tr><td>number of adults</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td></tr></table>	cups of rice	1	2	3	4	5	6	cups of water	2	4	6	8	10	12	number of children	10	20	30	40	50	60	number of adults	1	2	3	4	5	6	“For every 2 yellow beads there are 3 green beads”.
cups of rice	1	2	3	4	5	6																								
cups of water	2	4	6	8	10	12																								
number of children	10	20	30	40	50	60																								
number of adults	1	2	3	4	5	6																								
	Bead strings:  Key ● red ● blue Figure 220: bead strings, each with the structure ‘for every 1 red bead, there are 3 blue beads’	“For every 3 blue beads there is 1 red bead”																												
6AS/MD–4 Solve problems with 2 unknowns	Patterns: <table><tr><td>Number of coins</td><td>Quantity in 50p coins</td><td>Quantity in 20p coins</td></tr><tr><td>1</td><td>50p</td><td>20p</td></tr><tr><td>2</td><td>£1</td><td>40p</td></tr><tr><td>3</td><td>£1.50</td><td>60p</td></tr><tr><td>4</td><td></td><td>80p</td></tr><tr><td>5</td><td></td><td>£1</td></tr><tr><td>6</td><td></td><td>£1.20</td></tr><tr><td>7</td><td></td><td>£1.40</td></tr><tr><td>8</td><td></td><td>£1.60</td></tr></table> Figure 221: finding the 2 solutions to the coin problem: one 50p coin and six 20p coins, or three 50p coins and one 20p coin	Number of coins	Quantity in 50p coins	Quantity in 20p coins	1	50p	20p	2	£1	40p	3	£1.50	60p	4		80p	5		£1	6		£1.20	7		£1.40	8		£1.60		
Number of coins	Quantity in 50p coins	Quantity in 20p coins																												
1	50p	20p																												
2	£1	40p																												
3	£1.50	60p																												
4		80p																												
5		£1																												
6		£1.20																												
7		£1.40																												
8		£1.60																												

	Bar models:  Figure 225: simplifying $\frac{6}{15}$ by dividing the numerator and denominator by the common factor of 3	
6F–1 Simplify fractions	Number lines and schematic diagrams:  Figure 225: simplifying $\frac{6}{15}$ by dividing the numerator and denominator by the common factor of 3	“A fraction can be simplified when the numerator and denominator have a common factor other than 1.” “To convert a fraction to its simplest form, divide both the numerator and the denominator by their highest common factor.”
6F–2 Express fractions in a common denomination	Diagrams:	“We need to compare the denominators of and .” “15 is a multiple of 5.” “We can use 15 as the common denominator.” “We need to express both fractions in fifteenths.” “We need to compare the denominators of and .” “8 is not a multiple of 3.” “24 is a multiple of both 3 and 8.” “We can use 24 as the common denominator.” “We need to express both fractions in twenty-fourths.” “If one denominator is

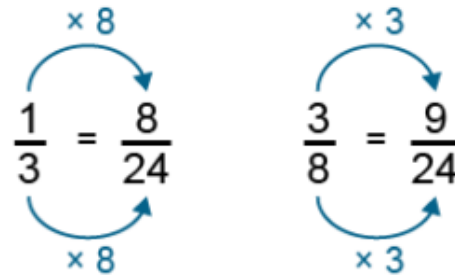


Figure 231: expressing $\frac{1}{3}$ and $\frac{3}{8}$ in twenty-fourths

not a multiple of the other, we can multiply the two denominators to find a common denominator.” “If the denominators are the same, then the larger the numerator, the larger the fraction.”

6F–3 Compare fractions with different denominators

- Bar models and inequality expressions:

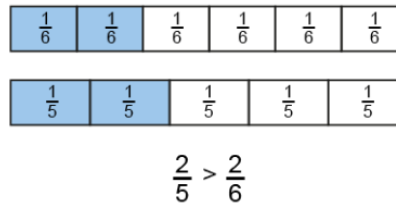


Figure 232: bar models to compare $\frac{2}{5}$ and $\frac{2}{6}$

“If the numerators are both 1, then the larger the denominator, the smaller the fraction.”
“If the numerators are the same, then the larger the denominator, the smaller the fraction.”

6G–1 Draw, compose and decompose shapes

- Polygons and grids

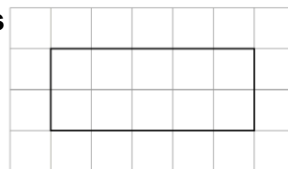


Figure 233: a 5cm by 2cm rectangle on a squared-centimetre grid

Drawn to actual size.

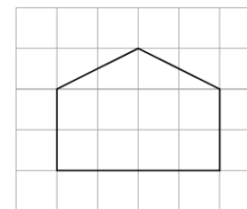


Figure 234: an irregular pentagon with an area of 10cm²

Drawn to actual size.

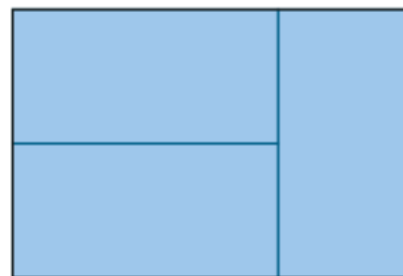
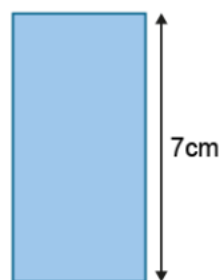


Figure 235: problem involving a compound shape made from 3 identical rectangles