📖 Introduction

Recommended level to read document: able to solve any Evil NG board without hints and under 180 seconds

Introduction

This document serves as a sequel to *Advanced Patterns* (Pattern Library) from 2021, and covers minesweeper's guessing patterns instead of logical patterns. This document has a tier-list type of format to list out all of the guessing patterns, with better patterns being placed higher up. This is to help with differentiating the quality of various guesses. (do note that one pattern ranking above another does not guarantee it will be a better guess in rare cases.) **There are also tabs containing other, related information.**

Credits

MSCoach - JSMinesweeper (primary analysis tool & board editor)

Sections & Areas

Sections are straightforward and coincide with the commonly used definition: groups of cells that can affect each other.

In this example there are three sections marked with different colors. Notice in the pink section that cells do not have to be connected to be part of the same section.

One property of sections not containing logic is that they may only be resolved by a guess in that section, or minecounting. This means that sections having a low probability of being solvable with minecounting could be guessed on early in the game to save time.

Seed: 114925261427464 (30x16/99 openingstart at 0,0) Analyse

All **exposed cells** (cells touching a number) can be divided into partitions known as **areas**. Cell C is part of an area A if it can interact with A. ~~"Interaction" precisely means that C can be determined safe or a mine without minecounting by assuming some other cells in A are mines/safe.~~ ~~(provided the assumption is not impossible)~~

In the example, pink, yellow and blue areas are shown. In addition, three other areas all colored purple are present on the board. In these areas, the exact number of mines is known. Areas with this property are known as **simple areas**. Notice that the pink and even the blue area are also simple.

There is a 7th "area" too, that being the group of cells that are not exposed cells (the uncolored cells in the example). These cells are called **floating cells** and all have the same probability of being mines (in the example this is 14.71%). This exact probability for a given board is important for locating optimal guesses and will be called D for density. Note that there is only a __% chance for a floating cell guess to be optimal in a random expert guessing situation, so try to avoid guessing them.

One convenient property of areas is that provided the density is similar, you can expect identical areas in two different boards to behave similarly in terms of the safety and progress chance for each cell in the area. This means that it is possible to apply a certain pattern recognition where you may recognize strong guesses because you have seen the particular area they are in before. In the example, the yellow area seems to contain cells worth guessing!

Ultra High Density

In this document, it is assumed that the density is reasonable, that is, less than 50% and ideally less than 30%.
Any probability estimations in this document should be accurate for any density provided minecount logic is not present.
>30% density scenarios may be addressed but they are not the focus of this document.

❓ Approximation

Approximation

D: Probability of floating cell being a mine (approximately $\frac{Remaining\ mines}{Remaining\ cells}$)

P_x: Probability of cell x being a mine

Expected safety is assuming $D = 20\%$ and range 16% to 25% on the pink cell

There are various ways to approximate safety of a cell, this document uses the following approximation:

It is possible to well-approximate most unknown cells on a minesweeper board with the form $P = \frac{1}{a+\frac{b}{D}+\frac{c}{D^2}}$ with additive error generally between $0.01\% - 0.1\%$, and such that two cells with the same approximation formula have identical probabilities.

The approximation also tells you whether the guess has affinity for low or high D:

$P \approx \frac{1}{2+3D} \cdot D$ grows slowly so such a cell works well with guessing on $D > 25\%$

$P \approx \frac{1}{\frac{2}{3}+D+\frac{1}{3D}} \cdot D$ grows quickly but starts slow, thus being great for $D < 20\%$

If you have trouble with judging approximation growth speed, try filling in $D = 1$, this returns the highest probability the cell will ever have of being a mine. If this value is small $(< 40\%)$, it is safe to say that the approximation must grow slowly. Alternatively, try using Desmos to graph using D as the x-axis and P as the y-axis.

Avoid using approximations in this document containing M and N, they are there for notekeeping.

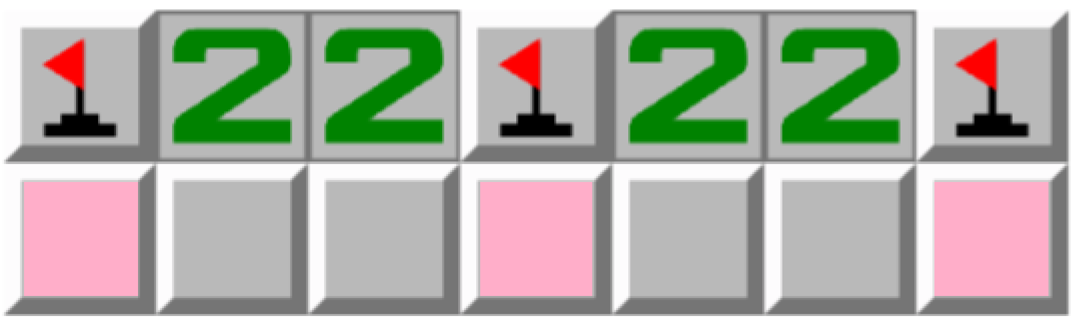
⬛ Guessing Patterns

The first number in front of the pattern name indicates the size of the area! ~~The second number indicates the region count.~~
Pink is the best guess in the area with $D \approx 20\%$ but read subsequent notes if there are any!
To search for a particular area, press ctrl+f then search "x/" where x is the size of the area. ~~You can also search "x/y" where y is region count.~~
For ranking, cells are assumed to have the number of neighbors they tend to have unless noted otherwise, this means that the cell may be a far stronger guess if it has fewer neighbors in game.

🟧 Tier 5 Guessing Patterns

these are effectively safe and/or provide a massive amount of progress

7/4 | Double 22 | 94% (92%-95%)

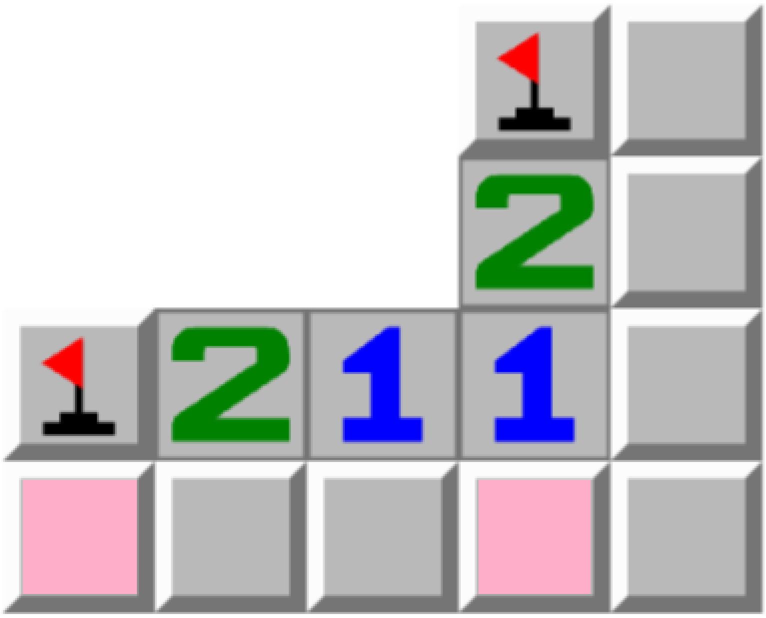


$P_{pink} \approx \frac{D}{4-3D}$

🟪 Tier 4 Guessing Patterns

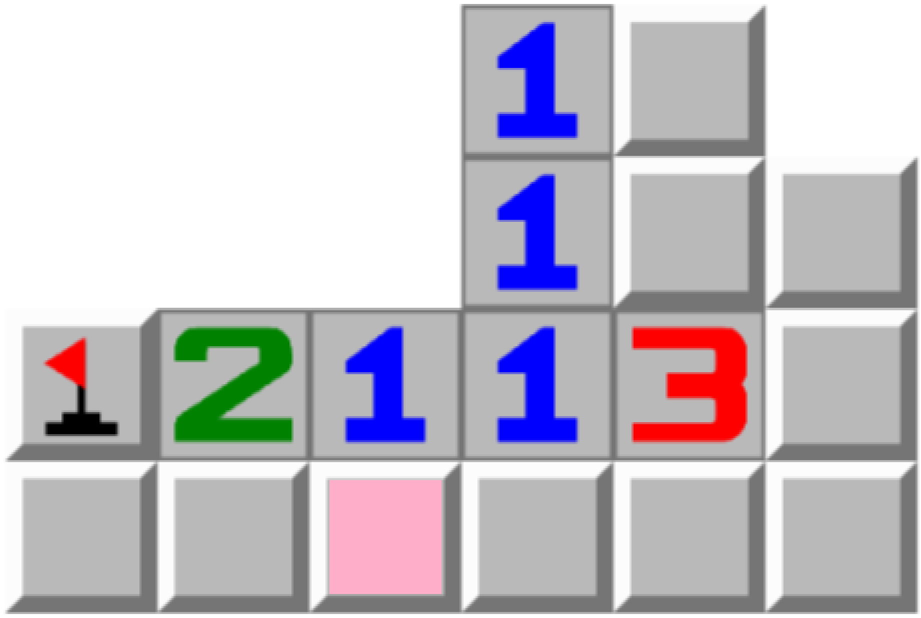
if these appear in-game you are pretty lucky

8/4 | ___ | 93% (91%-94%)



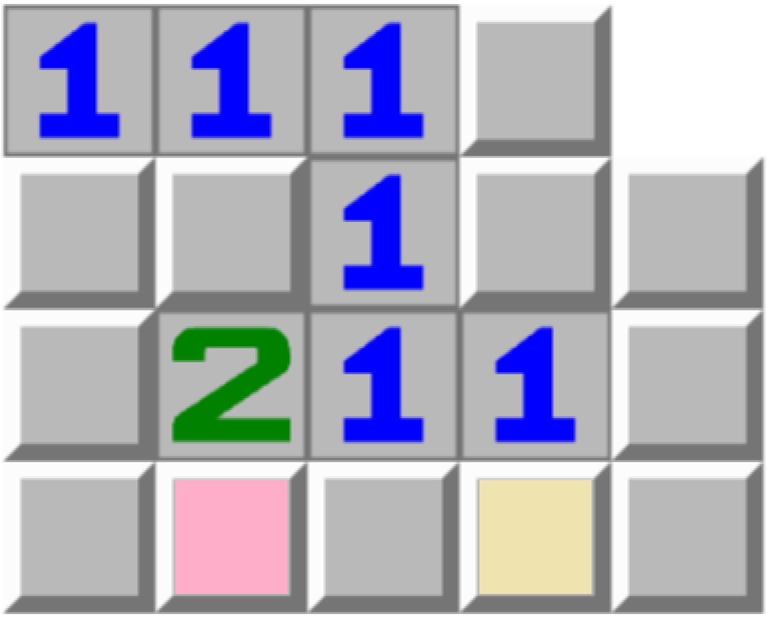
$$P_{pink} \approx \frac{D}{3-D}$$

10/5 | ___ | 95% (94%-96%)



$$P_{pink} \approx \frac{D}{3+4D}$$

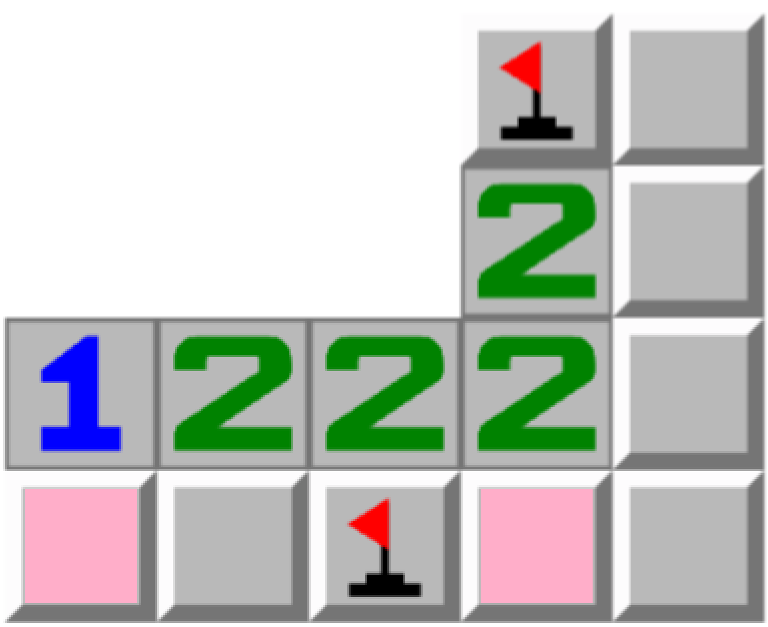
12/5 | ___ | 93% (91%-94%)



$P_{pink} \approx \frac{D}{3 - \frac{4}{3}D}$, $P_{yellow} \approx \frac{D}{\frac{9}{2} - 2D}$

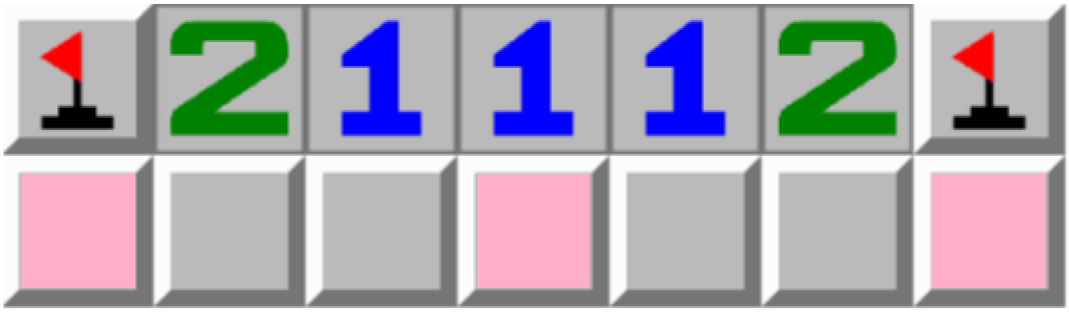
Instructive pattern: pink is 1.5x riskier (check this) than yellow but is much more likely to be useful. Yellow is better only for $D > 24\%$, and at $D > 34\%$ a different cell becomes optimal.

7/4 | ___ | 90% (88%-92%)



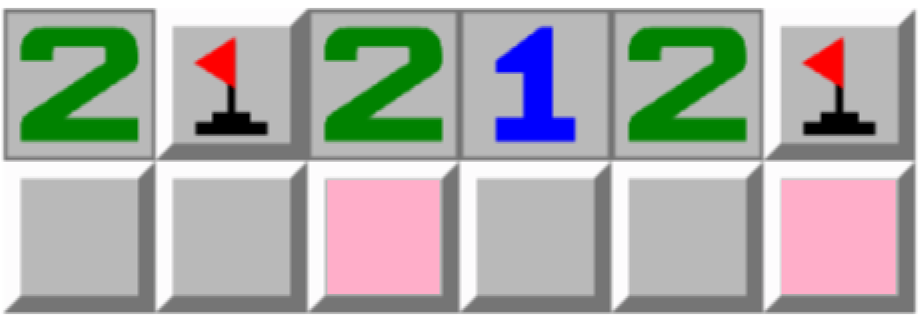
$P_{pink} = \frac{D}{2}$

7/5 | ___ | 88% (86%-91%)



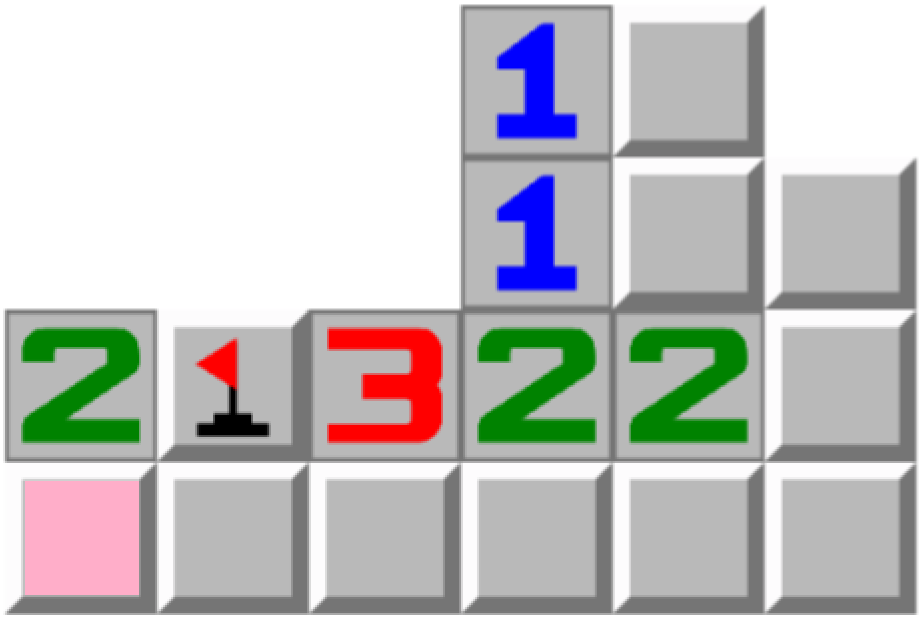
$P_{pink} \approx \frac{D}{2-D}$

6/4 | ___ | 88% (86%-91%)



$P_{pink} \approx \frac{D}{2-D}$

10/5 | ___ | 92% (89%-95%)



$P_{pink} \approx \frac{D}{\frac{2}{3}+D+\frac{1}{3D}}$

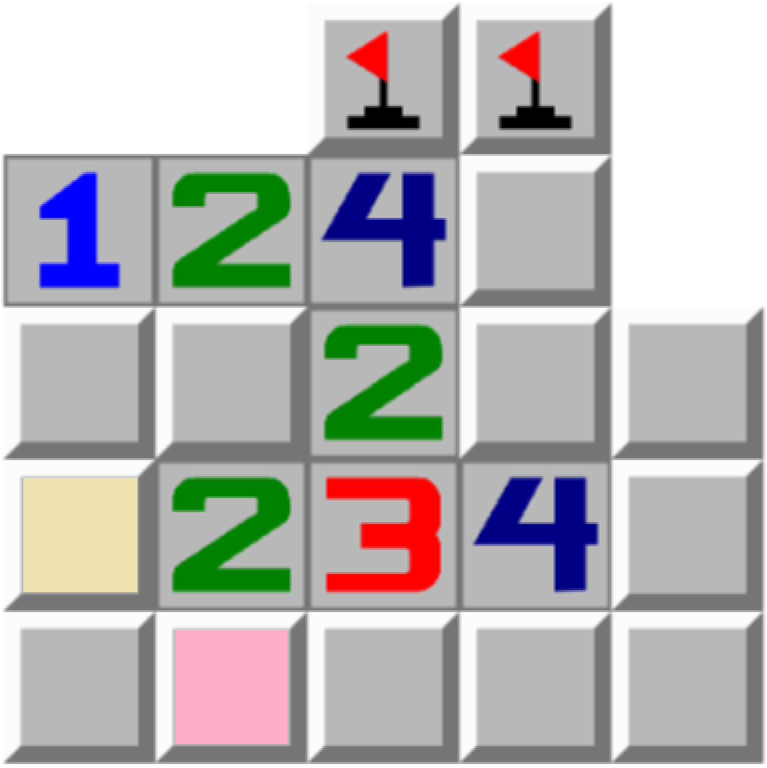
Provided pink on an edge. Better than [above] if $D < 21\%$, better than [2-above] if $D < 19\%$.

4/2 | 22 pattern | 88% (86%-91%)

$P_{pink} \approx \frac{D}{2-D}$

Better than [above] if $D > 22\%$.

12/5 | ___ | 92% (90%-94%)

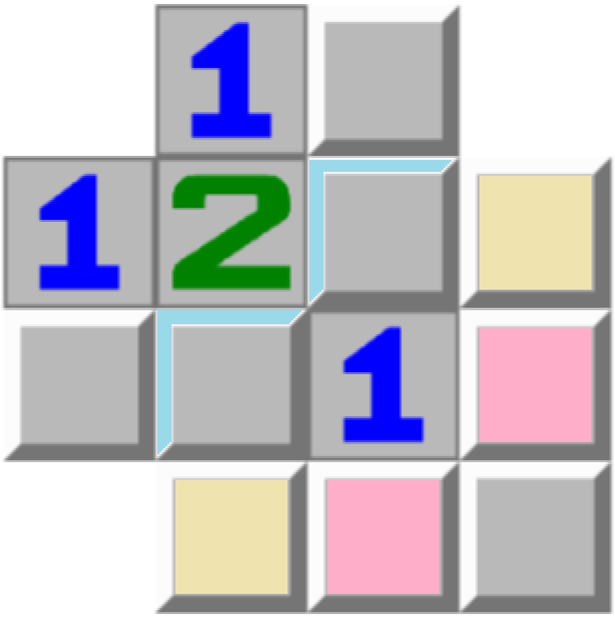


$P_{pink} \approx \frac{D}{3-2D}$

, $P_{yellow} \approx \frac{3+3D}{3+7D-6D^2} \cdot D$

Better than 22 pattern if $D < 18\%$. The yellow cell is stronger with few neighbors and if $D > 18\%$.

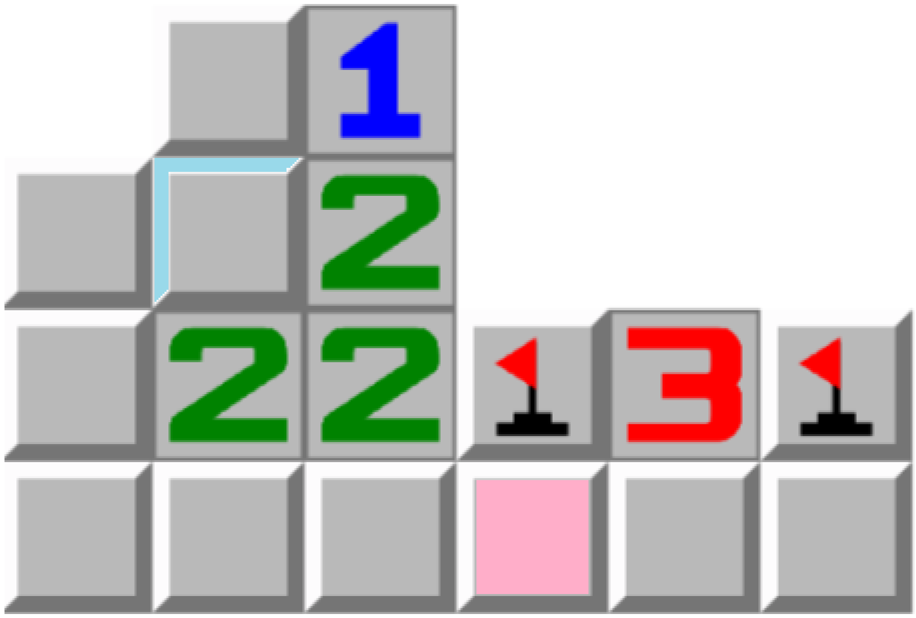
9/3 | 121-1 pattern | 92% (91%-94%)



$$P_{pink,yellow} \approx \frac{D}{2+3D}$$

Note: Pink is exclusively better than yellow for approximately $18.5\% < D < 20.5\%$, so if encountered as an expert opening it will be slightly better. Because expert has D extremely close to the tipover point, and yellow has easier continuations (beneficial for speed/endurance or if not very experienced in continuations), yellow is the recommended choice. Guess blue if $D > 45\%$.

10/4 | ___ | 92% (91%-94%)



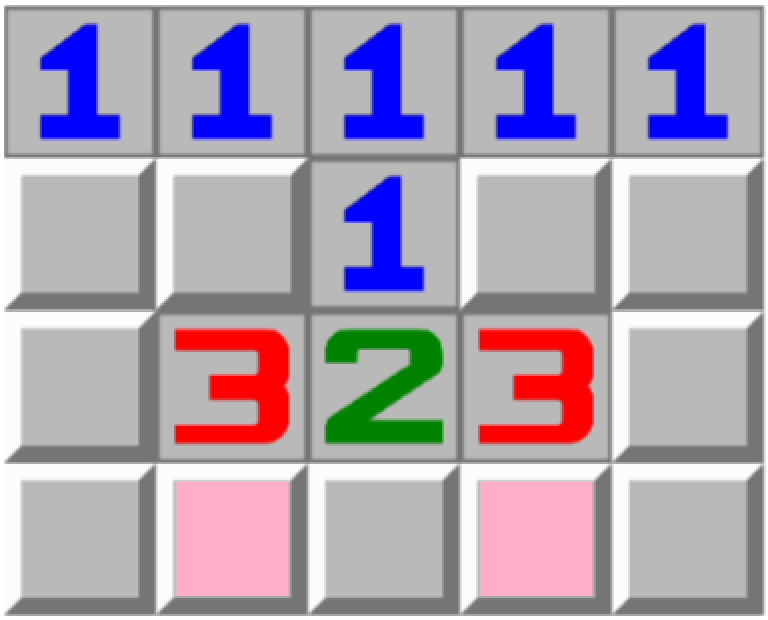
$P_{pink} \approx \frac{D}{2+3D}$

Same probability to pattern above but weaker progress. Guess blue if $D > 60\%$.

🟥 Tier 3 Guessing Patterns

these are often the best guess

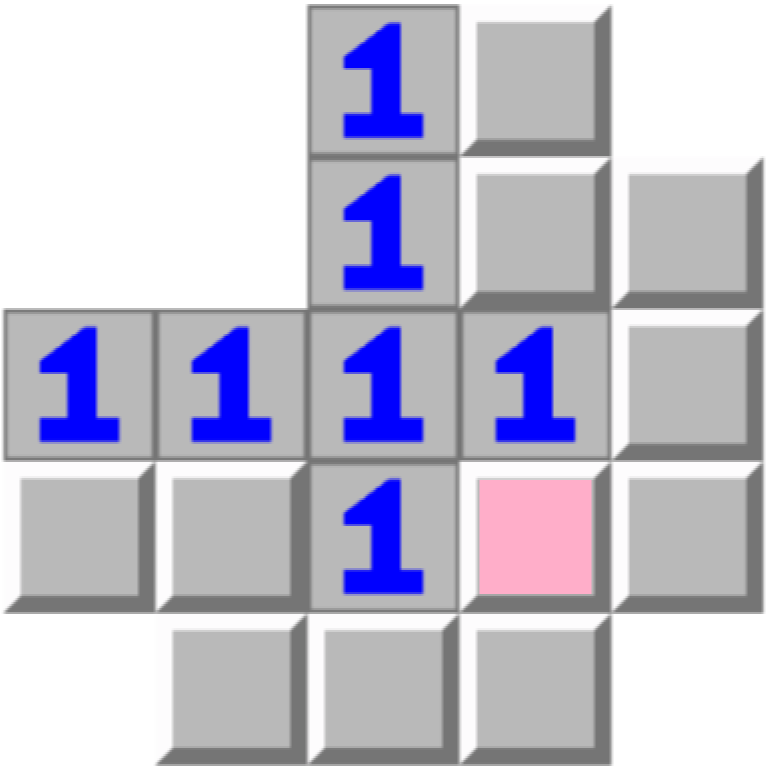
7/3 | 323-hole | 90% (88%-92%)



$P_{pink} = \frac{D}{2}$

(The image technically shows two areas but this makes it clearer how it looks in game)

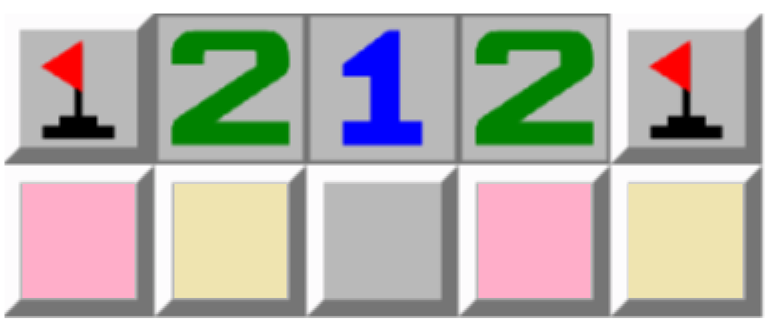
11/5 | Scissor pattern | 86%



$P_{pink} = \frac{1}{7} \approx 14\%$

It may not appear to be, but this area is a simple area since it must contain exactly 3 mines. This means that the probabilities of all cells in the area are constant. Better than 323-hole if $D > 23\%$.

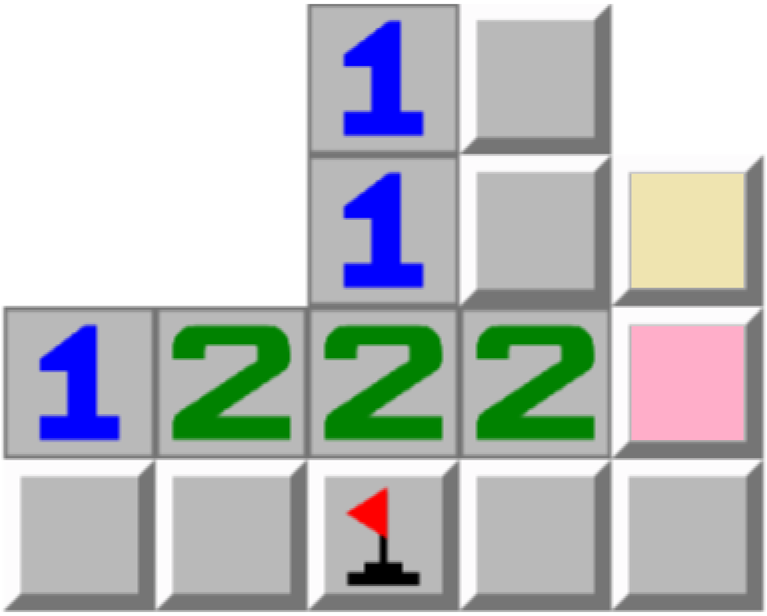
5/3 | ___ | 83% (80%-86%)



$P_{pink,yellow} \approx \frac{D}{1+D}$

Guess pink/yellow depending on the progress chance of the outer cells. Better than scissor if $D < 19\%$.

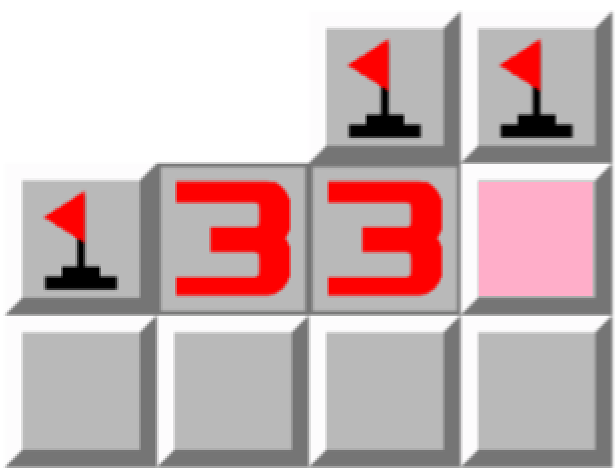
8/4 | ___ | 88% (86%-89%)



$P_{pink,yellow} \approx \frac{D}{1+3D}$

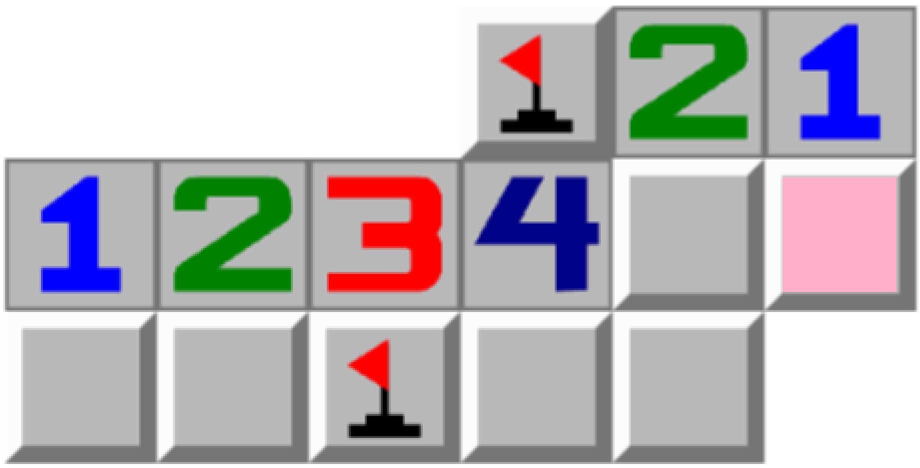
Better than scissor if $D < 18\%$.

5/2 | ___ | 90% (88%-92%)



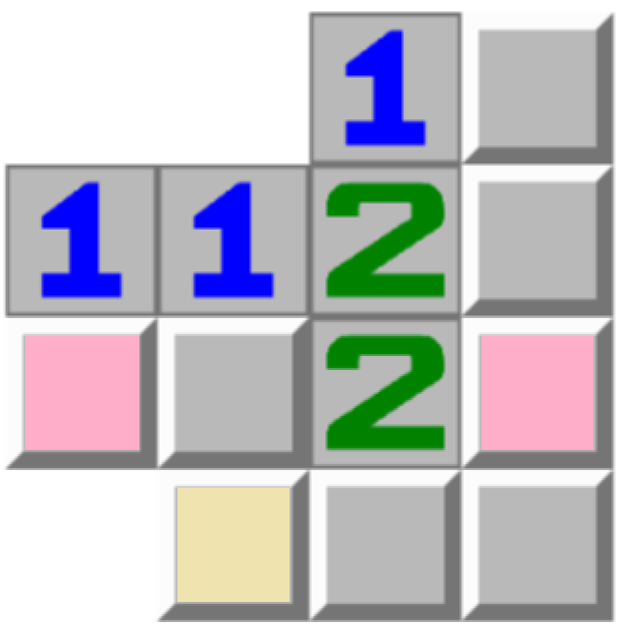
$P_{pink} = \frac{D}{2}$

6/4 | ___ | 88% (86%-91%)



$$P_{pink} \approx \frac{D}{2-D}$$

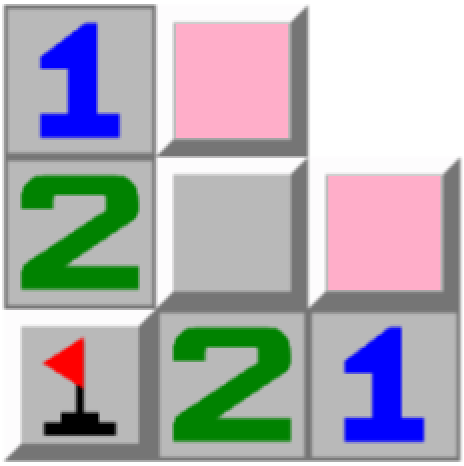
8/4 | Obelus2 | 80% (75%-84%)



$$P_{pink} = D,\ P_{yellow} \approx \frac{D}{1+2D}$$

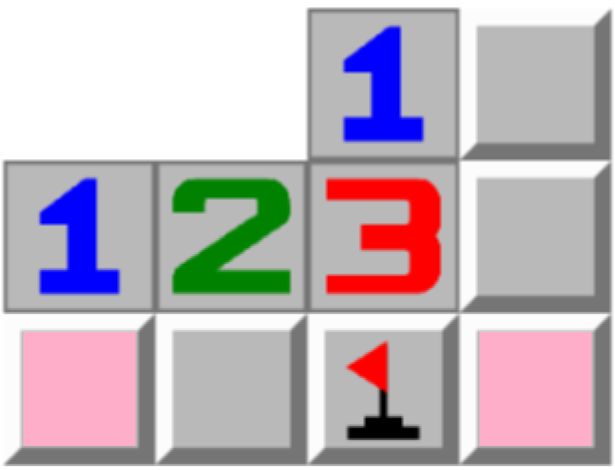
Yellow is better for $D > 25\%$.

3/2 | Box | 80% (75%-84%)



$P_{pink} = D$

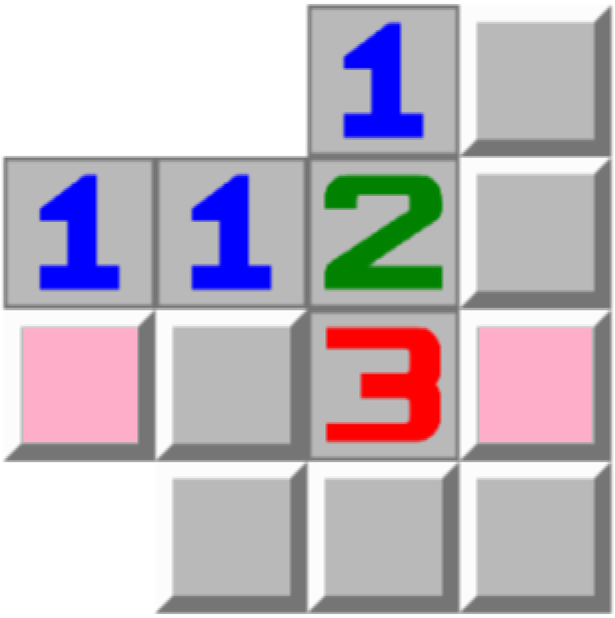
5/3 | ObelusB | 80% (75%-84%)



$P_{pink} = D$

Generally has weaker progress than the pink cells of box.

8/4 | Obelus3 | 80% (75%-84%)

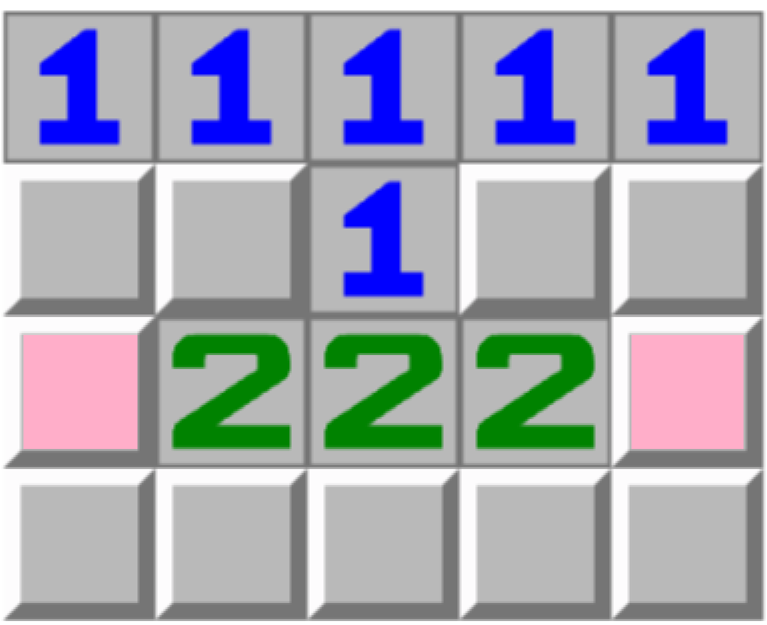


$P_{pink} = D$

🟦 Tier 2 Guessing Patterns

these can be expected to be better than guessing a corner

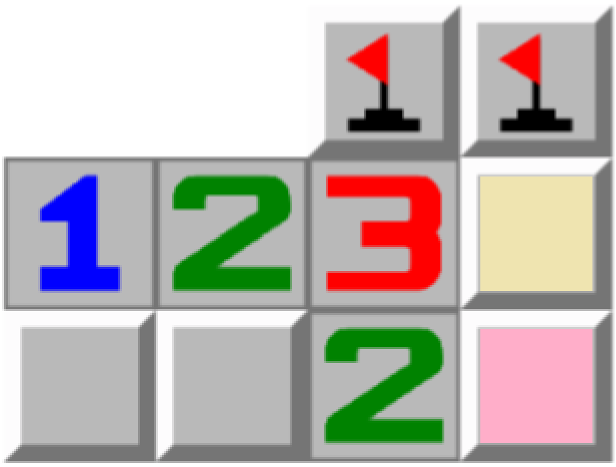
7/3 | 222-hole | 88% (86%-89%)



$P_{pink} \approx \frac{D}{1+3D}$

Look out for 222-holes with a pink cell on the board edge, those are far stronger cells to guess on. (The image technically shows two areas but this makes it clearer how it looks in game)

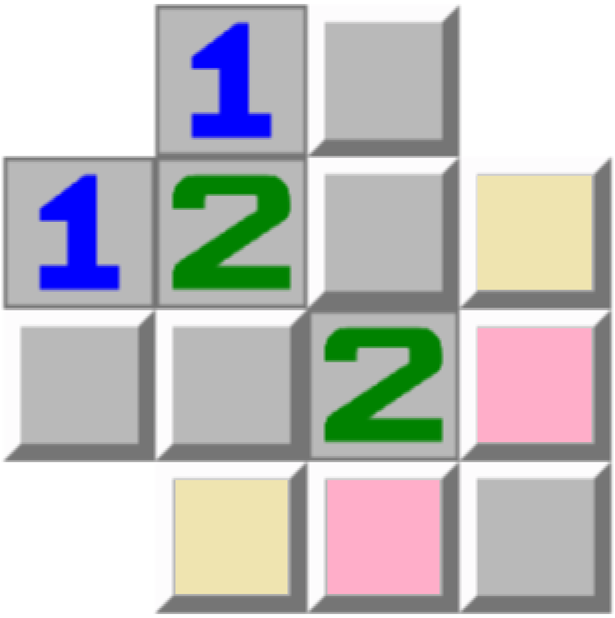
4/2 | ___ | 83% (80%-86%)



$P_{pink,yellow} \approx \frac{D}{1+D}$

If $D < 19\%$ guess a corner instead. Pay attention to the yellow cell as it might have fewer neighbors,_

9/3 | 121-2 pattern | 82% (80%-84%)



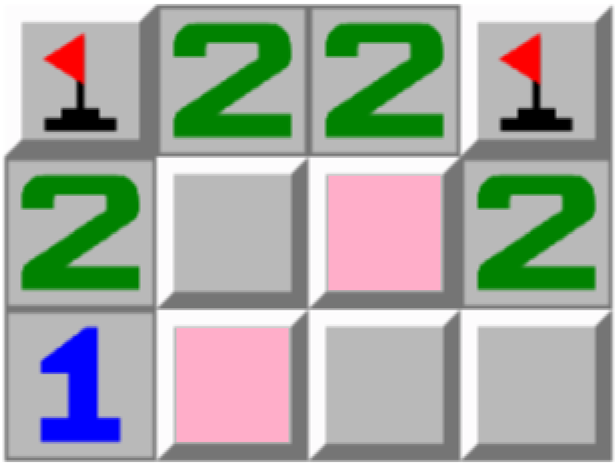
$$P_{pink,yellow} \approx \frac{2+2D}{1+8D+D^2} \cdot D$$

If $D < 21\%$ guess a corner instead (preferably the pseudo in the corner where the 121-2 is). If $30\% < D < 35\%$, guess yellow.

🟩 Tier 1 Guessing Patterns

dont guess these

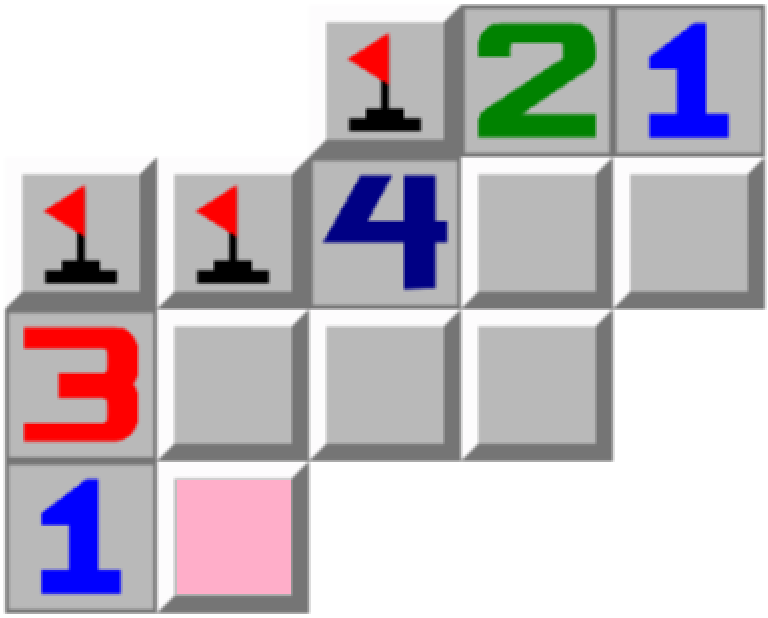
5/3 | ___ | 67%



$P_{pink} = \frac{1}{3} \approx 33\%$

Better than a corner if $D > 30\%$. If it exists, the floating cell two below the rightmost 2 is possibly a better guess than a corner.

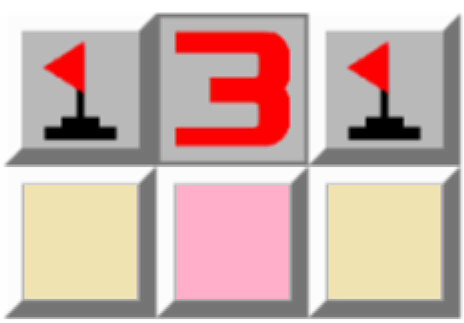
6/3 | ___ | 73% (68%-77%)



$$P_{pink} \approx \frac{2-D}{1+2D-2D^2} \cdot D$$

If $D > 35\%$, better than a corner. May also be viable for $D > 28\%$ if no corners reveal 1

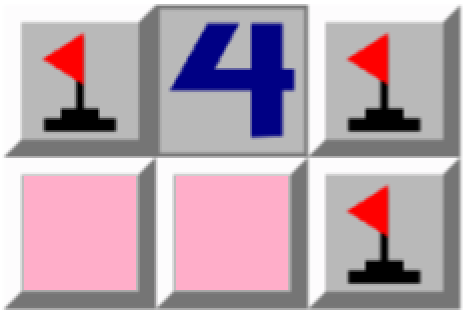
3/1 | 1 in 3 | 67%



$$P_{pink,yellow} = \frac{1}{3} \approx 33\%$$

Avoid this and go for a floating cell instead unless $D > 35\%$, then go for pink. If $D > 40\%$ go for a yellow cell. If one of the yellow cells has fewer neighbors (e.g. along an edge) guess there.

2/1 | 1 in 2 | 50%



$$P_{pink} = \frac{1}{2} = 50\%$$

Whichever cell has fewer neighbors is better, avoid this pattern unless D is close to 60% or one of the pink cells has few neighbors.

Tab 5

$c \in E(R) \Rightarrow$

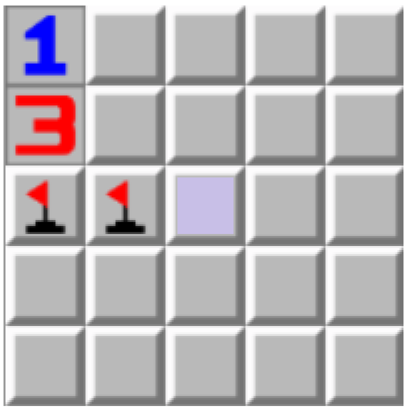
(1) $P_0(c) = 0$

(2) $P_1(c) = 1$

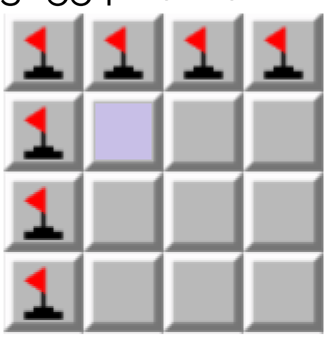
(3) $\exists x \in (0, 1)\colon |P_x(c) - x| \leq$...

(4) $\forall a, b \in (0, 1)\colon |\frac{P_a(c)}{P_b(c)} - \frac{a}{b}| \leq$...

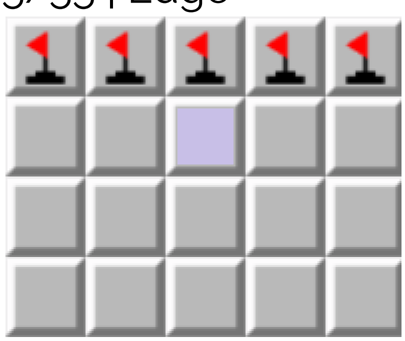
7/567b | 50/50 Breaker



3/55 | Corner

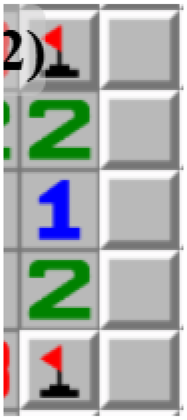


5/55 | Edge

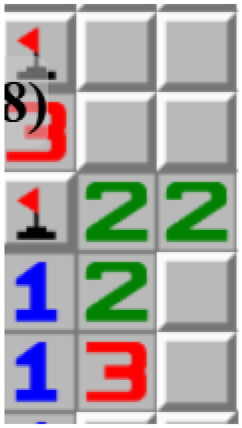


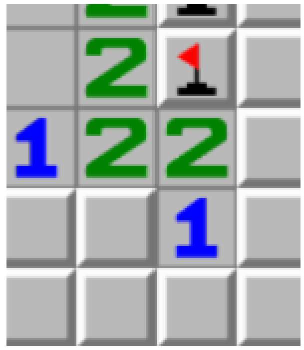
queue

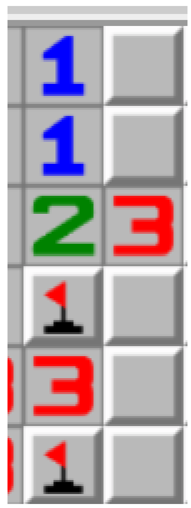
2)

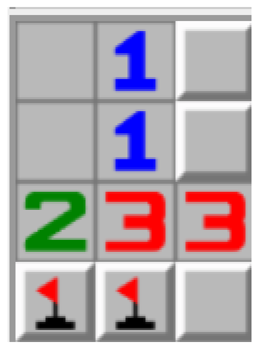


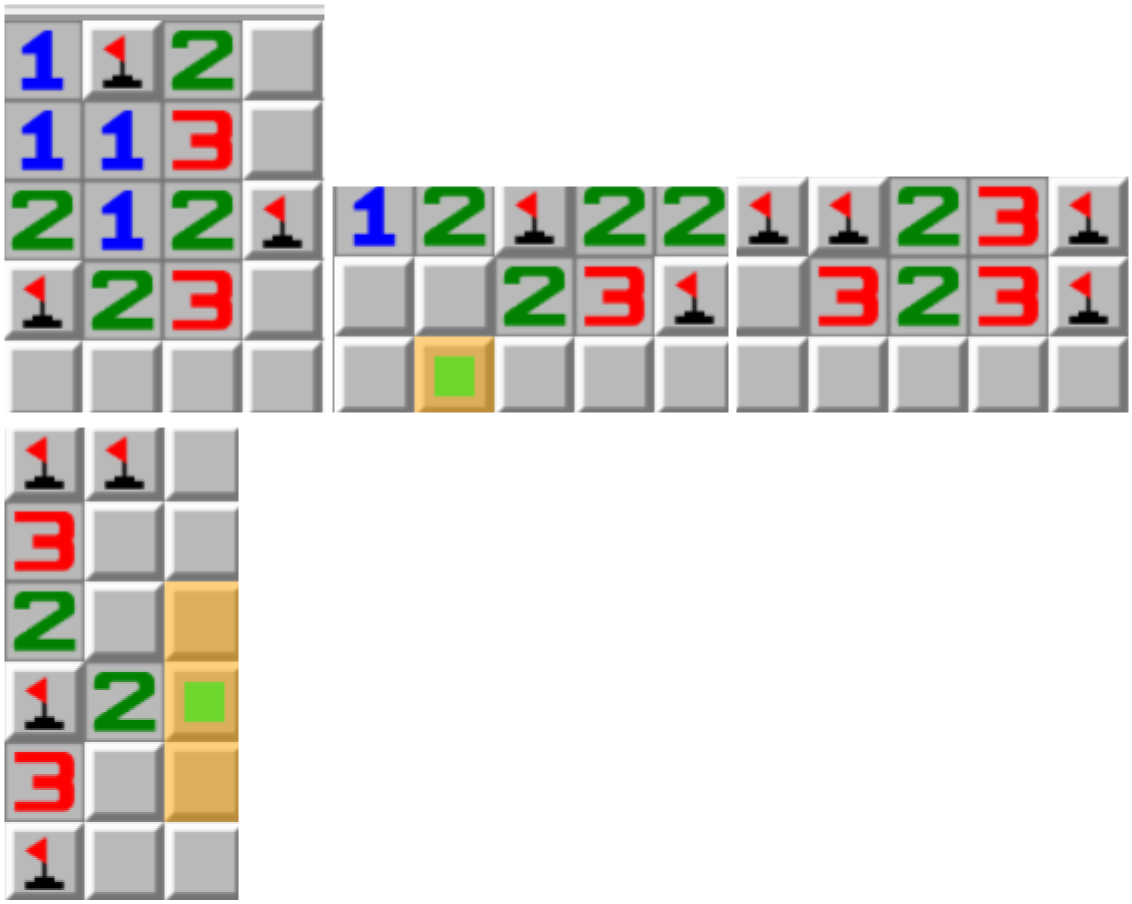
8)











↩️ U-Turn Algorithm

This algorithm is a simple method to find the strongest guess in low/medium-complexity sections with decent accuracy.
Below are the three steps for the algorithm described with an example:

Scanning

Consider the orange tiles, there are exactly two cells in between them. If you spot something like this in a section, it may function as a good starting point for the U-Turn algorithm.

Forward Step (coloring)

The colored and gray cells indicate the entire section. Taking the two previously mentioned tiles, the parts of the section adjacent to them can be colored.

It is now important to mark as much of the remaining gray cells as possible. This can be done best by trying to color the gray cell that is touching an already colored cell.

The blue 3 is the only number that can do this, because candidate tiles already adjacent to a colored cell cannot be used (because then a cell would have to be colored twice)

There is only one gray cell left now. This means that the coloring is done.

Backward Step (propagation)

Let's remove all of the coloring. For this final step, assume the gray cell is safe, and calculate the implied safe cells.

This implies that the red cell is a mine...

...and other safe cells / mines can be found to be implied!

Now, indicate the safe cell that was found last (the cell furthest removed from the gray cell)

This cell should then be a good guess!

Notes

- If doing the **backward step** on the gray cell produces no implied safe cells, the gray cell itself will qualify as the final safe cell.
- Notice in the example how all 3 tiles used for coloring neighbors are almost in a grid formation with tiles in between them, this is no coincidence and such formations should be watched out for.
- You do not have to color every cell excluding one, but it will cause the algorithm to lose more and more accuracy if no adjustments are made.

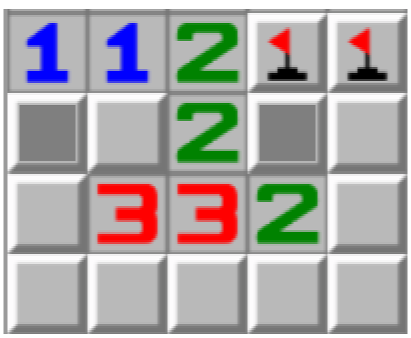
    - Maybe coloring will cause gray cells to appear in multiple locations (such as on both ends of the section), in that case, try the **backward step** on each of them and pick the cell that is safe in the most amount of cases
        - If there are no cells that are implied safe in more than one case, pick the final safe cell produced by the
- This algorithm only partially accounts for progress with equivalent cells, but not progress in general (progress = probability of the guess revealing a number that provides safe cells).

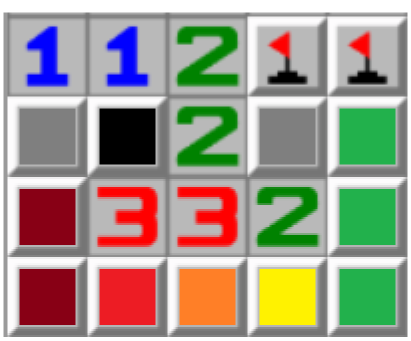
↔ Relative Probability

Approximation for finding the safest guess in a section that reduces the amount of combinations that need to be checked

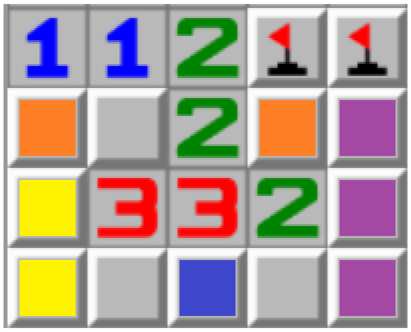
Label all cells

Reduction A: equivalent states the same color





Reduction B: equivalent clues the same color



Reduction C: uncolor cells where if one of that color is safe, some other color is implied safe

- **(a) Cells with the same color have the same probability of being safe**
- **(b) For any minecount, the safest guess in the section must be one of the remaining colors**

Enumerate all valid mine combinations of the colors :

1. Orange + Yellow (5 mines)
2. Orange + Blue (4 mines)
3. Yellow + Blue (4 mines)
4. Yellow + Purple (5 mines)
5. Blue + Purple (4 mines)

Each color is part of combinations with _ mines:

- Orange: **4m** + **5m**
- Yellow: **4m** + 2***5m**
- Blue : 3***4m**
- Purple : **4m** + **5m**

Relative comparisons that can be found now (for all minecounts):

- **Prob(Orange) = Prob(Purple)** (identical combination count)
- **Prob(Orange or Purple) ≤ Prob(Yellow)** (yellow has an additional 5-mine combination)

Notes about Blue:

- It has three low-mine combinations, which weigh heavy and therefore increase blue's mine probability to a high chance for normal boards
- However, blue's probability is not guaranteed to be less than or greater than any of the other colors, because it does not have the same combinations as any other color with some extra. (Alternatively, subtracting the mine combination counts for blue and any other color yields an equation with both positive and negative coefficients, this makes blue incomparable)

The safest color:

- Orange and Purple are the safest
- Blue is not dominated by another color, therefore it is the safest for mine density over 72%
- Yellow is dominated by Orange / Purple and is never the safest (except for the lowest valid minecount, but even then it only ties Orange / Purple)