Questions

Q1.

The curve C has parametric equations

$x = 3t - 4$, $y = 5 - \frac{6}{t}$, $t > 0$

(a) Find $\frac{dy}{dx}$ in terms of t

(2)

The point P lies on C where $t = \frac{1}{2}$

(b) Find the equation of the tangent to C at the point P. Give your answer in the form $y = px + q$, where p and q are integers to be determined.

(3)

(c) Show that the cartesian equation for C can be written in the form

$y = \frac{ax + b}{x + 4}$, $x > -4$

where a and b are integers to be determined.

(3)

(Total for question = 8 marks)

Q2.

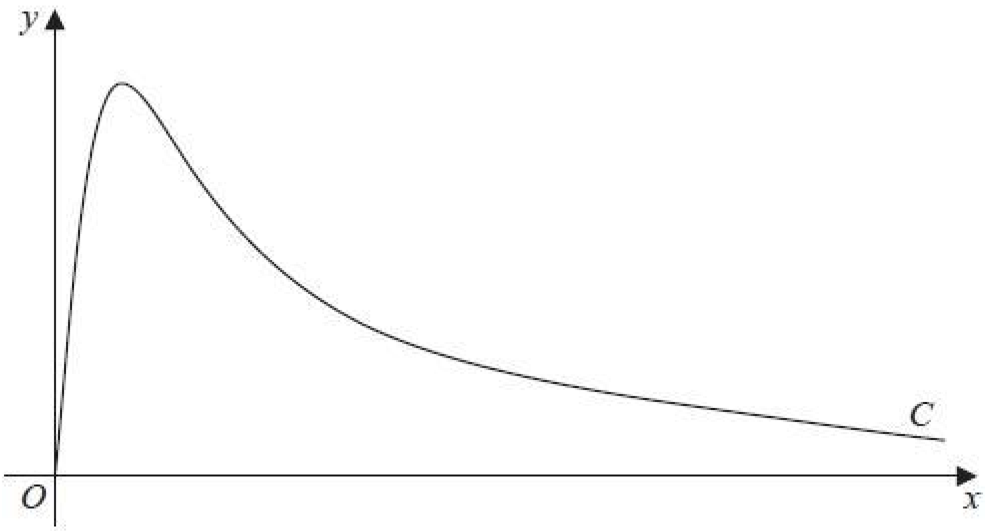


Figure 2

Figure 2 shows a sketch of the curve C with parametric equations

$$x = 4\tan t, \qquad y = 5\sqrt{3}\sin 2t, \qquad 0 \leqslant t < \frac{\pi}{2}$$

The point P lies on C and has coordinates $\left(4\sqrt{3}, \frac{15}{2}\right)$.

(a) Find the exact value of $\frac{\mathrm{d}y}{\mathrm{d}x}$ at the point P.

Give your answer as a simplified surd.

(4)

The point Q lies on the curve C, where $\frac{\mathrm{d}y}{\mathrm{d}x} = 0$

(b) Find the exact coordinates of the point Q.

(2)

(Total for question = 6 marks)

Q3.

The curve C has parametric equations

$$x = 2\cos t, \quad y = \sqrt{3}\cos 2t, \quad 0 \leqslant t \leqslant \pi$$

(a) Find an expression for $\frac{\mathrm{d}y}{\mathrm{d}x}$ in terms of t.

(2)

The point P lies on C where $t = \frac{2\pi}{3}$

The line l is the normal to C at P.

(b) Show that an equation for l is

$$2x - 2\sqrt{3}y - 1 = 0$$

(5)

The line l intersects the curve C again at the point Q.

(c) Find the exact coordinates of Q.

You must show clearly how you obtained your answers.

(6)

(Total for question = 13 marks)

Q4.

A curve C has parametric equations

$$x = 2\sin t, \quad y = 1 - \cos 2t, \quad -\frac{\pi}{2} \leqslant t \leqslant \frac{\pi}{2}$$

(a) Find $\frac{\mathrm{d}y}{\mathrm{d}x}$ at the point where $t = \frac{\pi}{6}$

(4)

(b) Find a cartesian equation for C in the form

$$y = \mathrm{f}(x), \; -k \leq x \leq k,$$

stating the value of the constant k.

(3)

(c) Write down the range of $\mathrm{f}(x)$.

(2)

(Total 9 marks)

Q5.

A curve has parametric equations

$$x = \tan^2 t, \qquad y = \sin t, \qquad 0 < t < \frac{\pi}{2}.$$

(a) Find an expression for $\frac{\mathrm{d}y}{\mathrm{d}x}$ in terms of t. You need not simplify your answer.

(3)

(b) Find an equation of the tangent to the curve at the point where $t = \frac{\pi}{4}$.

Give your answer in the form $y = ax + b$, where a and b are constants to be determined.

(5)

(c) Find a cartesian equation of the curve in the form $y^2 = \mathrm{f}(x)$.

(4)

(Total 12 marks)

Q6.

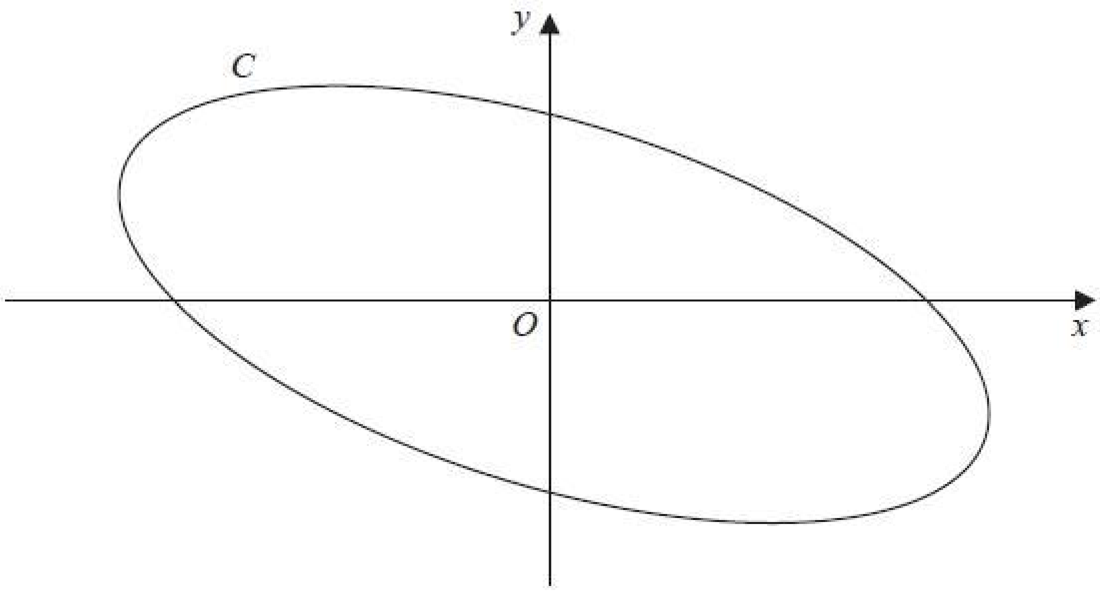


Figure 3

Figure 3 shows a sketch of the curve C with parametric equations

$$x = 4\cos\left(t + \frac{\pi}{6}\right), \quad y = 2\sin t, \quad 0 \leq t < 2\pi$$

(a) Show that

$$x + y = \sqrt{3}\cos t$$

(3)

(b) Show that a cartesian equation of C is

$$(x + y)^2 + ay^2 = b$$

where a and b are integers to be determined.

(2)

(Total 5 marks)

Q7.

A curve C has parametric equations

$$x = \frac{t^2 + 5}{t^2 + 1} \qquad y = \frac{4t}{t^2 + 1} \qquad t \in \mathbb{R}$$

Show that all points on C satisfy

$$(x - 3)^2 + y^2 = 4$$

(Total for question = 3 marks)

Q8.

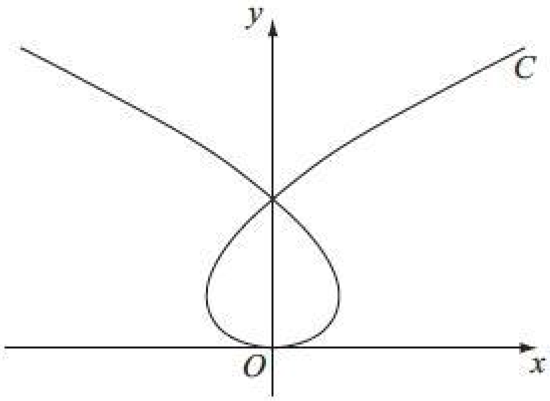


Figure 3

The curve *C* shown in Figure 3 has parametric equations

$$x = t^3 - 8t,\ y = t^2$$

where *t* is a parameter. Given that the point *A* has parameter $t = -1$,

(a) find the coordinates of *A*.

(1)

The line l is the tangent to *C* at *A*.

(b) Show that an equation for l is $2x - 5y - 9 = 0$.

(5)

The line l also intersects the curve at the point *B*.

(c) Find the coordinates of *B*.

(6)

(Total 12 marks)

Q9.

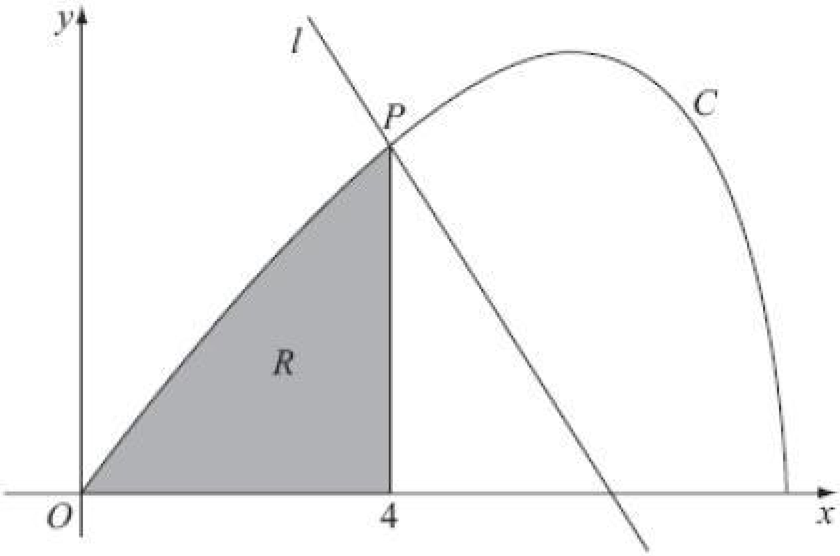


Figure 3

Figure 3 shows the curve *C* with parametric equations

$$x = 8\cos t, \qquad y = 4\sin 2t, \qquad 0 \leq t \leq \frac{\pi}{2}.$$

The point *P* lies on *C* and has coordinates $(4, 2\sqrt{3})$.

(a) Find the value of *t* at the point *P*.

(2)

The line l is a normal to C at P.

(b) Show that an equation for l is $y = -x\sqrt{3} + 6\sqrt{3}$.

(6)

The finite region R is enclosed by the curve C, the x-axis and the line $x = 4$, as shown shaded in Figure 3.

(c) Show that the area of R is given by the integral $\int_{\frac{\pi}{3}}^{\frac{\pi}{2}} 64 \sin^2 t \cos t \, dt$.

(4)

(d) Use this integral to find the area of R, giving your answer in the form $a + b\sqrt{3}$, where a and b are constants to be determined.

(4)

(Total 16 marks)

Q10.

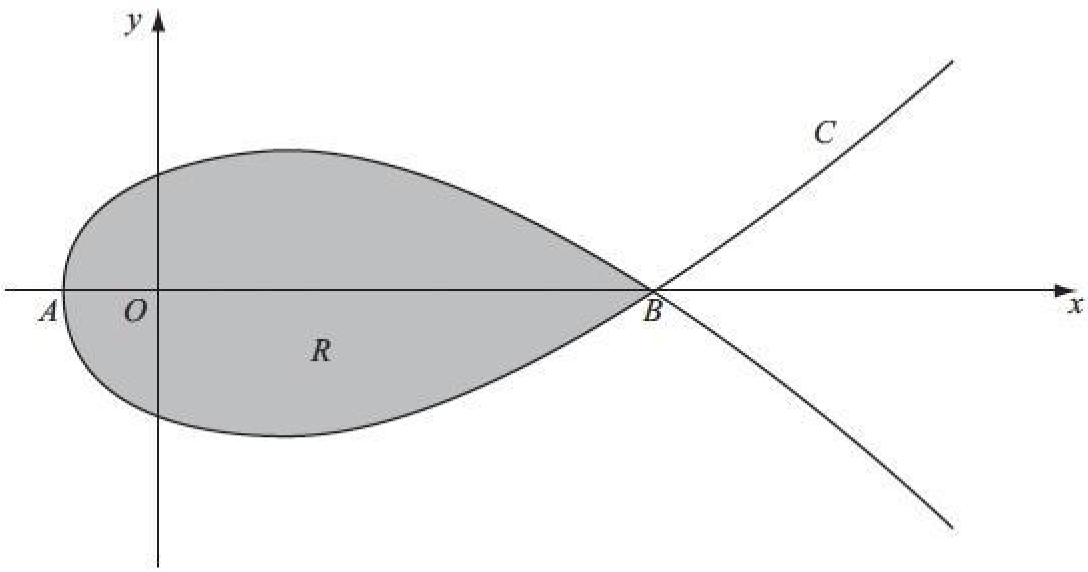


Figure 2

Figure 2 shows a sketch of the curve C with parametric equations

$$x = 5t^2 - 4, \qquad y = t(9 - t^2)$$

The curve C cuts the x-axis at the points A and B.

(a) Find the x-coordinate at the point A and the x-coordinate at the point B.

(3)

The region R, as shown shaded in Figure 2, is enclosed by the loop of the curve.

(b) Use integration to find the area of R.

(6)

(Total 9 marks)

Q11.

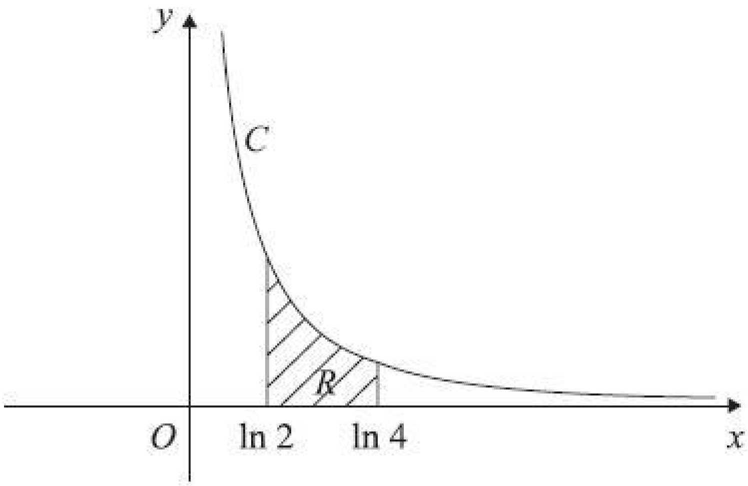


Figure 3

The curve C has parametric equations

$$x = \ln(t+2), \qquad y = \frac{1}{(t+1)}, \qquad t > -1.$$

The finite region R between the curve C and the x-axis, bounded by the lines with equations $x = \ln 2$ and $x = \ln 4$, is shown shaded in Figure 3.

(a) Show that the area of R is given by the integral

$$\int_0^2 \frac{1}{(t+1)(t+2)}\,\mathrm{d}t.$$

(4)

(b) Hence find an exact value for this area.

(6)

(c) Find a cartesian equation of the curve C, in the form $y = \mathrm{f}(x)$.

(4)

(d) State the domain of values for x for this curve.

(1)

(Total 15 marks)

Q12.

A curve C has parametric equations

$$x = 4t + 3, \quad y = 4t + 8 + \frac{5}{2t}, \quad t \neq 0$$

(a) Find the value of $\frac{\mathrm{d}y}{\mathrm{d}x}$ at the point on C where $t = 2$, giving your answer as a fraction in its simplest form.

(3)

(b) Show that the cartesian equation of the curve *C* can be written in the form

$$y = \frac{x^2 + ax + b}{x - 3}, \quad x \neq 3$$

where *a* and *b* are integers to be determined.

(3)

(Total for question = 6 marks)

Q13.

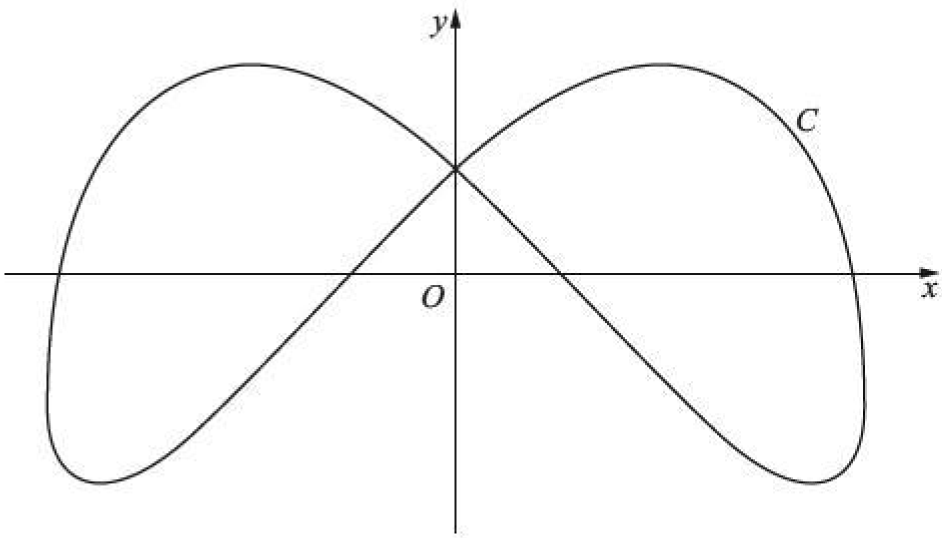


Figure 2

Figure 2 shows a sketch of the curve *C* with parametric equations

$$x = 4\sin\left(t + \frac{\pi}{6}\right), \quad y = 3\cos 2t, \quad 0 \leqslant t < 2\pi$$

(a) Find an expression for $\frac{\mathrm{d}y}{\mathrm{d}x}$ in terms of *t*.

(3)

Find the coordinates of all the points on *C* where $\frac{\mathrm{d}y}{\mathrm{d}x} = 0$

(5)

(Total 8 marks)

Q14.

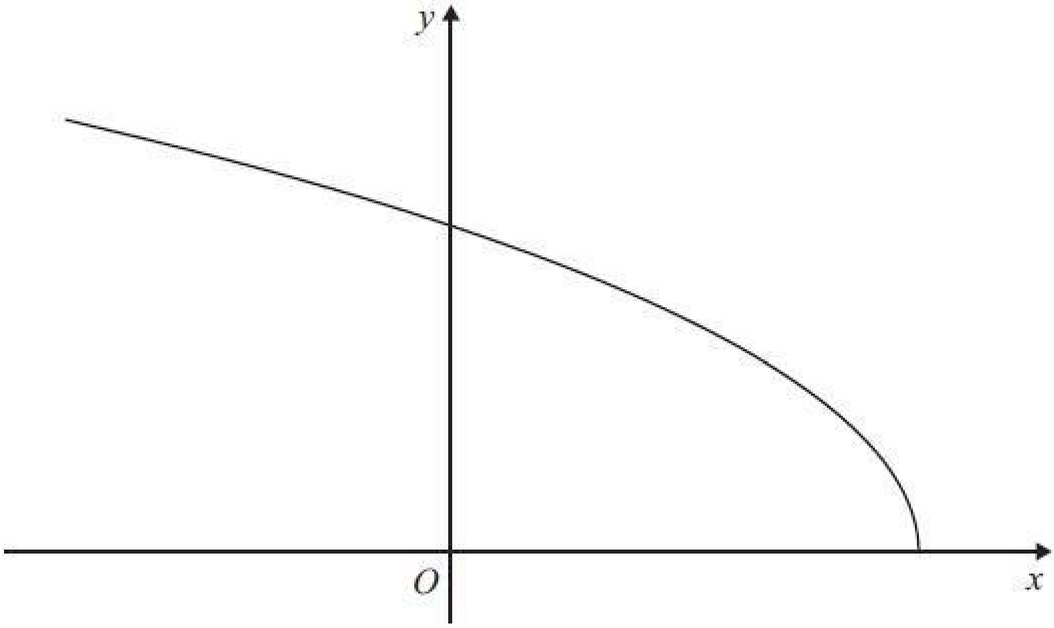


Figure 2

Figure 2 shows a sketch of the curve with parametric equations

$$x = 2\cos 2t, \quad y = 6\sin t, \quad 0 \leqslant t \leqslant \frac{\pi}{2}$$

(a) Find the gradient of the curve at the point where $t = \dfrac{\pi}{3}$.

(4)

(b) Find a cartesian equation of the curve in the form

$$y = \mathrm{f}(x), \quad -k \leqslant x \leqslant k,$$

stating the value of the constant *k*.

(4)

(c) Write down the range of f(*x*).

(2)
(Total 10 marks)

Q15.

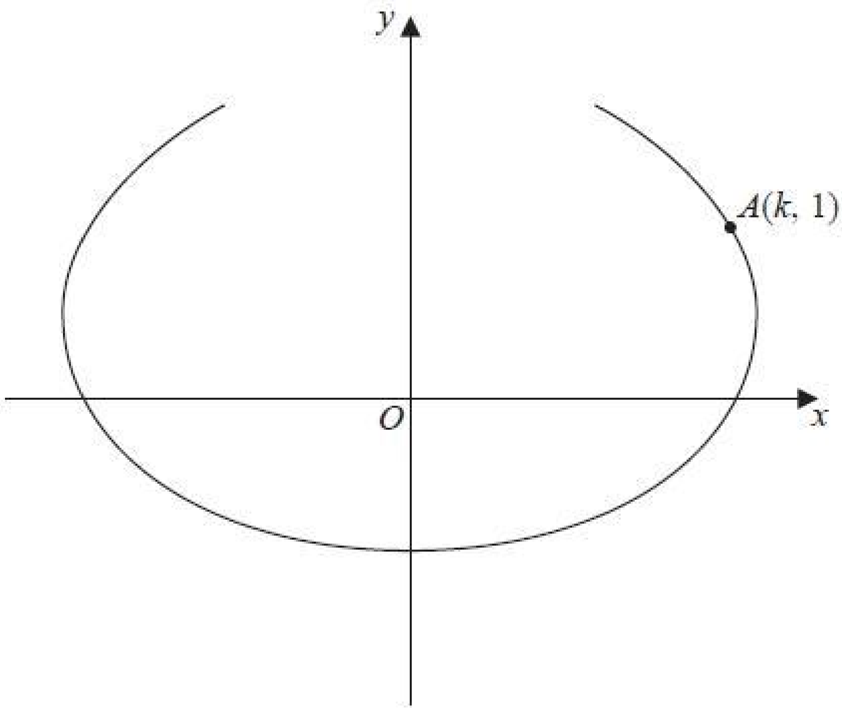


Figure 3

The curve shown in Figure 3 has parametric equations

$$x = t - 4\sin t,\ y = 1 - 2\cos t, \quad -\tfrac{2\pi}{3} \leq t \leq \tfrac{2\pi}{3}$$

The point A, with coordinates $(k, 1)$, lies on the curve.

Given that $k > 0$

(a) find the exact value of k,

(2)

(b) find the gradient of the curve at the point A.

(4)

There is one point on the curve where the gradient is equal to $-\tfrac{1}{2}$

(c) Find the value of t at this point, showing each step in your working and giving your answer to 4 decimal places.

[*Solutions based entirely on graphical or numerical methods are not acceptable.*]

(6)

(Total 12 marks)

Q16.

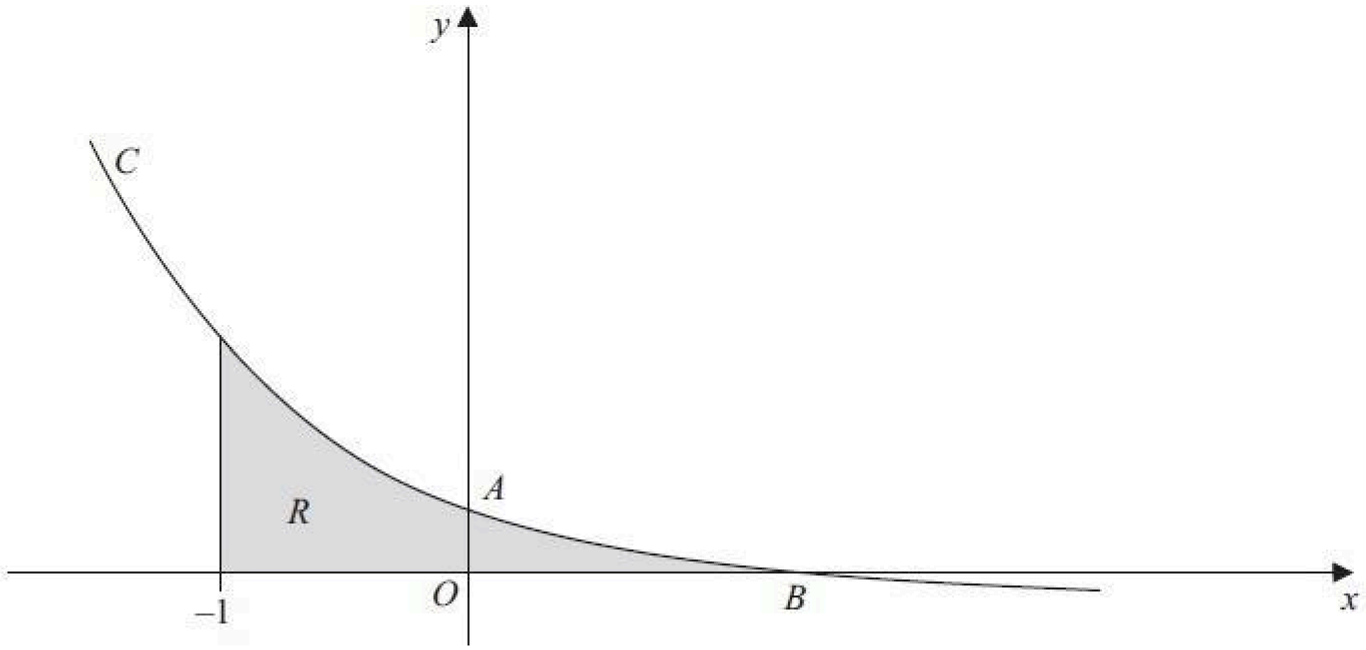


Figure 2

Figure 2 shows a sketch of part of the curve C with parametric equations

$$x = 1 - \frac{1}{2}t, \quad y = 2^t - 1$$

The curve crosses the y-axis at the point A and crosses the x-axis at the point B.

(a) Show that A has coordinates (0, 3).

(2)

(b) Find the x coordinate of the point B.

(2)

(c) Find an equation of the normal to C at the point A.

(5)

The region R, as shown shaded in Figure 2, is bounded by the curve C, the line $x = -1$ and the x-axis.

(d) Use integration to find the exact area of R.

(6)

(Total 15 marks)

Mark Scheme

Q1.

Question Number	Scheme	Notes	Marks
	$x = 3t - 4,\ y = 5 - \dfrac{6}{t},\quad t > 0$		
(a)	$\dfrac{dx}{dt} = 3,\quad \dfrac{dy}{dt} = 6t^{-2}$		
	$\dfrac{dy}{dx} = \dfrac{6t^{-2}}{3}\ \left\{= \dfrac{6}{3t^2} = 2t^{-2} = \dfrac{2}{t^2}\right\}$	their $\dfrac{dy}{dt}$ divided by their $\dfrac{dx}{dt}$ to give $\dfrac{dy}{dx}$ in terms of t **or** their $\dfrac{dy}{dt}$ multiplied by their $\dfrac{dt}{dx}$ to give $\dfrac{dy}{dx}$ in terms of t	M1
		$\dfrac{6t^{-2}}{3}$, simplified or un-simplified, in terms of t. **See note.**	A1 isw
	Award **Special Case 1st M1** if **both** $\dfrac{dx}{dt}$ **and** $\dfrac{dy}{dt}$ are stated **correctly** and **explicitly**.		[2]
	Note: You can recover the work for part (a) in part (b).		
(a) **Way 2**	$y = 5 - \dfrac{18}{x+4} \Rightarrow \dfrac{dy}{dx} = \dfrac{18}{(x+4)^2} = \dfrac{18}{(3t)^2}$	Writes $\dfrac{dy}{dx}$ in the form $\dfrac{\pm\lambda}{(x+4)^2}$, ***and*** writes $\dfrac{dy}{dx}$ as a function of t.	M1
		Correct un-simplified or simplified answer, in terms of t. **See note.**	A1 isw
			[2]
(b)	$\left\{t = \dfrac{1}{2} \Rightarrow\right\} P\left(-\dfrac{5}{2}, -7\right)$	$x = -\dfrac{5}{2},\ y = -7$ **or** $P\left(-\dfrac{5}{2}, -7\right)$ seen or implied.	B1
	$\dfrac{dy}{dx} = \dfrac{2}{\left(\frac{1}{2}\right)^2}$ **and** either • $y - "-7" = "8"\left(x - "-\frac{5}{2}"\right)$ • $"-7" = ("8")("-\frac{5}{2}") + c$ So, $y = (\text{their } m_T)x + "c"$	**Some** attempt to substitute $t = 0.5$ into their $\dfrac{dy}{dx}$ which contains t in order to find m_T **and either** applies $y - (\text{their } y_P) = (\text{their } m_T)(x - \text{their } x_P)$ **or** finds c from $(\text{their } y_P) = (\text{their } m_T)(\text{their } x_P) + c$ and uses their numerical c in $y = (\text{their } m_T)x + c$	M1
	$\mathbf{T}:\ y = 8x + 13$	$y = 8x + 13$ or $y = 13 + 8x$	A1 cso
	Note: their x_P, their y_P and their m_T must be numerical values in order to award M1		[3]
(c) **Way 1**	$\left\{t = \dfrac{x+4}{3} \Rightarrow\right\}\ y = 5 - \dfrac{6}{\left(\dfrac{x+4}{3}\right)}$	An attempt to eliminate t. **See notes.**	M1
		Achieves a correct equation in x and y only	A1 o.e.
	$\Rightarrow y = 5 - \dfrac{18}{x+4} \Rightarrow\ y = \dfrac{5(x+4) - 18}{x+4}$		
	So, $y = \dfrac{5x+2}{x+4},\ \{x > -4\}$	$y = \dfrac{5x+2}{x+4}$ (or implied equation)	A1 cso
			[3]
(c) **Way 2**	$\left\{t = \dfrac{6}{5-y} \Rightarrow\right\}\ x = \dfrac{18}{5-y} - 4$	An attempt to eliminate t. **See notes.**	M1
		Achieves a correct equation in x and y only	A1 o.e.
	$\Rightarrow (x+4)(5-y) = 18 \Rightarrow 5x - xy + 20 - 4y = 18$		
	$\{\Rightarrow 5x + 2 = y(x+4)\}$ So, $y = \dfrac{5x+2}{x+4},\ \{x > -4\}$	$y = \dfrac{5x+2}{x+4}$ (or implied equation)	A1 cso
			[3]
	Note: Some or all of the work for part (c) can be recovered in part (a) or part (b)		8

Question Number	Scheme	Notes	Marks
(c) **Way 3**	$y=\dfrac{3at-4a+b}{3t-4+4}=\dfrac{3at}{3t}-\dfrac{4a-b}{3t}=a-\dfrac{4a-b}{3t}\Rightarrow a=5$	A full method leading to the value of a being found	M1
		$y=a-\dfrac{4a-b}{3t}$ and $a=5$	A1
	$\dfrac{4a-b}{3}=6\Rightarrow b=4(5)-6(3)=2$	**Both** $a=5$ **and** $b=2$	A1
			[3]

		Question Notes
(a)	**Note**	Condone $\dfrac{dy}{dx}=\dfrac{\left(\frac{6}{t^2}\right)}{3}$ for A1
	Note	You can ignore subsequent working following on from a correct expression for $\dfrac{dy}{dx}$ in terms of t.
(b)	**Note**	Using a changed gradient (i.e. applying $\dfrac{-1}{\text{their } \frac{dy}{dx}}$ or $\dfrac{1}{\text{their } \frac{dy}{dx}}$ or $-\left(\text{their } \frac{dy}{dx}\right)$) is M0.
	Note	**Final A1:** A correct solution is required from a correct $\dfrac{dy}{dx}$.
	Note	**Final A1:** You can ignore subsequent working following on from a correct solution.
(c)	**Note**	**1st M1:** A full attempt to eliminate t is defined as either • rearranging one of the parametric equations to make t the subject and substituting for t in the other parametric equation (only the RHS of the equation required for M mark) • rearranging both parametric equations to make t the subject and putting the results equal to each other.
	Note	Award M1A1 for $\dfrac{6}{5-y}=\dfrac{x+4}{3}$ or equivalent.

Q2.

Question Number	Scheme	Notes	Marks
	$x = 4\tan t, \quad y = 5\sqrt{3}\sin 2t, \quad 0 \leqslant t < \frac{\pi}{2}$		
(a) **Way 1**	$\frac{\mathrm{d}x}{\mathrm{d}t} = 4\sec^2 t,\ \frac{\mathrm{d}y}{\mathrm{d}t} = 10\sqrt{3}\cos 2t$ $\Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x} = \frac{10\sqrt{3}\cos 2t}{4\sec^2 t} \quad \left\{= \frac{5}{2}\sqrt{3}\cos 2t\cos^2 t\right\}$	**Either both** x and y are differentiated correctly with respect to t **or** their $\frac{\mathrm{d}y}{\mathrm{d}t}$ divided by their $\frac{\mathrm{d}x}{\mathrm{d}t}$ **or** applies $\frac{\mathrm{d}y}{\mathrm{d}t}$ multiplied by their $\frac{\mathrm{d}t}{\mathrm{d}x}$	M1
		Correct $\frac{\mathrm{d}y}{\mathrm{d}x}$ (Can be implied)	A1 oe
	$\left\{\text{At } P\left(4\sqrt{3}, \frac{15}{2}\right),\ t = \frac{\pi}{3}\right\}$		
	$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{10\sqrt{3}\cos\left(\frac{2\pi}{3}\right)}{4\sec^2\left(\frac{\pi}{3}\right)}$	**dependent on the previous M mark** *Some evidence* of substituting $t = \frac{\pi}{3}$ or $t = 60^\circ$ into their $\frac{\mathrm{d}y}{\mathrm{d}x}$	dM1
	$\frac{\mathrm{d}y}{\mathrm{d}x} = -\frac{5}{16}\sqrt{3}$ or $-\frac{15}{16\sqrt{3}}$	$-\frac{5}{16}\sqrt{3}$ or $-\frac{15}{16\sqrt{3}}$ **from a correct solution only**	A1 cso
			[4]
(b)	$\left\{10\sqrt{3}\cos 2t = 0 \Rightarrow t = \frac{\pi}{4}\right\}$		
	So $x = 4\tan\left(\frac{\pi}{4}\right),\ y = 5\sqrt{3}\sin\left(2\left(\frac{\pi}{4}\right)\right)$	At least one of either $x = 4\tan\left(\frac{\pi}{4}\right)$ or $y = 5\sqrt{3}\sin\left(2\left(\frac{\pi}{4}\right)\right)$ or $x = 4$ or $y = 5\sqrt{3}$ or $y =$ awrt 8.7	M1
	Coordinates are $(4, 5\sqrt{3})$	$(4, 5\sqrt{3})$ or $x = 4, y = 5\sqrt{3}$	A1
			[2]
			6

		Question Notes
(a)	**1st A1**	Correct $\frac{\mathrm{d}y}{\mathrm{d}x}$. E.g. $\frac{10\sqrt{3}\cos 2t}{4\sec^2 t}$ or $\frac{5}{2}\sqrt{3}\cos 2t\cos^2 t$ or $\frac{5}{2}\sqrt{3}\cos^2 t(\cos^2 t - \sin^2 t)$ or any equivalent form.
	Note	Give the final A0 for a final answer of $-\frac{10}{32}\sqrt{3}$ **without reference to** $-\frac{5}{16}\sqrt{3}$ or $-\frac{15}{16\sqrt{3}}$
	Note	Give the final A0 for more than one value stated for $\frac{\mathrm{d}y}{\mathrm{d}x}$
(b)	**Note**	Also allow M1 for either $x = 4\tan(45)$ or $y = 5\sqrt{3}\sin(2(45))$
	Note	M1 can be gained by ignoring previous working in part (a) and/or part (b)
	Note	Give A0 for stating more than one set of coordinates for Q.
	Note	Writing $x = 4, y = 5\sqrt{3}$ followed by $(5\sqrt{3}, 4)$ is A0.

Question Number	Scheme	Notes	Marks
	$x = 4\tan t,\quad y = 5\sqrt{3}\sin 2t,\quad 0 \leqslant t < \dfrac{\pi}{2}$		
(a) **Way 2**	$\tan t = \dfrac{x}{4} \Rightarrow \sin t = \dfrac{x}{\sqrt{(x^2+16)}},\ \cos t = \dfrac{4}{\sqrt{(x^2+16)}} \Rightarrow y = \dfrac{40\sqrt{3}x}{x^2+16}$		
	$\left\{\begin{matrix} u = 40\sqrt{3}x & v = x^2+16 \\ \dfrac{\mathrm{d}u}{\mathrm{d}x} = 40\sqrt{3} & \dfrac{\mathrm{d}v}{\mathrm{d}x} = 2x \end{matrix}\right\}$		
	$\dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{40\sqrt{3}(x^2+16) - 2x(40\sqrt{3}x)}{(x^2+16)^2}\ \left\{= \dfrac{40\sqrt{3}(16-x^2)}{(x^2+16)^2}\right\}$	$\dfrac{\pm A(x^2+16) \pm Bx^2}{(x^2+16)^2}$	M1
		Correct $\dfrac{\mathrm{d}y}{\mathrm{d}x}$; simplified or un-simplified	A1
	$\dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{40\sqrt{3}(48+16) - 80\sqrt{3}(48)}{(48+16)^2}$	**dependent on the previous M mark** *Some evidence* of substituting $x = 4\sqrt{3}$ into their $\dfrac{\mathrm{d}y}{\mathrm{d}x}$	dM1
	$\dfrac{\mathrm{d}y}{\mathrm{d}x} = -\dfrac{5}{16}\sqrt{3}$ or $-\dfrac{15}{16\sqrt{3}}$	$-\dfrac{5}{16}\sqrt{3}$ or $-\dfrac{15}{16\sqrt{3}}$ **from a correct solution only**	A1 cso
			[4]
(a) **Way 3**	$y = 5\sqrt{3}\sin\left(2\tan^{-1}\left(\dfrac{x}{4}\right)\right)$		
	$\dfrac{\mathrm{d}y}{\mathrm{d}x} = 5\sqrt{3}\cos\left(2\tan^{-1}\left(\dfrac{x}{4}\right)\right)\left(\dfrac{2}{1+\left(\frac{x}{4}\right)^2}\right)\left(\dfrac{1}{4}\right)$	$\dfrac{\mathrm{d}y}{\mathrm{d}x} = \pm A\cos\left(2\tan^{-1}\left(\dfrac{x}{4}\right)\right)\left(\dfrac{1}{1+x^2}\right)$	M1
		Correct $\dfrac{\mathrm{d}y}{\mathrm{d}x}$; simplified or un-simplified.	A1
	$\dfrac{\mathrm{d}y}{\mathrm{d}x} = 5\sqrt{3}\cos\left(2\tan^{-1}\left(\sqrt{3}\right)\right)\left(\dfrac{2}{1+3}\right)\left(\dfrac{1}{4}\right)\ \left\{= 5\sqrt{3}\left(-\dfrac{1}{2}\right)\left(\dfrac{1}{2}\right)\left(\dfrac{1}{4}\right)\right\}$	**dependent on the previous M mark** *Some evidence* of substituting $x = 4\sqrt{3}$ into their $\dfrac{\mathrm{d}y}{\mathrm{d}x}$	dM1
	$\dfrac{\mathrm{d}y}{\mathrm{d}x} = -\dfrac{5}{16}\sqrt{3}$ or $-\dfrac{15}{16\sqrt{3}}$	$-\dfrac{5}{16}\sqrt{3}$ or $-\dfrac{15}{16\sqrt{3}}$ **from a correct solution only**	A1 cso
			[4]

Q3.

Question	Scheme	Marks	AOs
(a)	Attempts $\frac{dy}{dx} = \frac{dy/dt}{dx/dt}$	M1	1.1b
	$\frac{dy}{dx} = \frac{\sqrt{3}\sin 2t}{\sin t} \quad \left(= 2\sqrt{3}\cos t\right)$	A1	1.1b
		(2)	
(b)	Substitutes $t = \frac{2\pi}{3}$ in $\frac{dy}{dx} = \frac{\sqrt{3}\sin 2t}{\sin t} = \left(-\sqrt{3}\right)$	M1	2.1
	Uses gradient of normal $= -\frac{1}{dy/dx} = \left(\frac{1}{\sqrt{3}}\right)$	M1	2.1
	Coordinates of $P = \left(-1, -\frac{\sqrt{3}}{2}\right)$	B1	1.1b
	Correct form of normal $y + \frac{\sqrt{3}}{2} = \frac{1}{\sqrt{3}}(x+1)$	M1	2.1
	Completes proof $\Rightarrow 2x - 2\sqrt{3}y - 1 = 0$ *	A1*	1.1b
		(5)	
(c)	Substitutes $x = 2\cos t$ and $y = \sqrt{3}\cos 2t$ into $2x - 2\sqrt{3}y - 1 = 0$	M1	3.1a
	Uses the identity $\cos 2t = 2\cos^2 t - 1$ to produce a quadratic in $\cos t$	M1	3.1a
	$\Rightarrow 12\cos^2 t - 4\cos t - 5 = 0$	A1	1.1b
	Finds $\cos t = \frac{5}{6}, \cancel{-\frac{1}{2}}$	M1	2.4
	Substitutes their $\cos t = \frac{5}{6}$ into $x = 2\cos t$, $y = \sqrt{3}\cos 2t$,	M1	1.1b
	$Q = \left(\frac{5}{3}, \frac{7}{18}\sqrt{3}\right)$	A1	1.1b
		(6)	
			(13 marks)

Notes:

(a)

M1: Attempts $\frac{dy}{dx} = \frac{dy/dt}{dx/dt}$ and achieves a form $k\frac{\sin 2t}{\sin t}$ Alternatively candidates may apply the double angle identity for $\cos 2t$ and achieve a form $k\frac{\sin t \cos t}{\sin t}$

A1: Scored for a correct answer, either $\frac{\sqrt{3}\sin 2t}{\sin t}$ or $2\sqrt{3}\cos t$

(b)

M1: For substituting $t = \frac{2\pi}{3}$ in their $\frac{dy}{dx}$ which must be in terms of t

M1: Uses the gradient of the normal is the negative reciprocal of the value of $\frac{dy}{dx}$. This may be seen in the equation of l.

B1: States or uses (in their tangent or normal) that $P = \left(-1, -\frac{\sqrt{3}}{2}\right)$

M1: Uses their numerical value of $-1\Big/\frac{dy}{dx}$ with their $\left(-1, -\frac{\sqrt{3}}{2}\right)$ to form an equation of the normal at P

A1*: This is a proof and all aspects need to be correct. Correct answer only $2x - 2\sqrt{3}y - 1 = 0$

(c)	
M1:	For substituting $x = 2\cos t$ and $y = \sqrt{3}\cos 2t$ into $2x - 2\sqrt{3}y - 1 = 0$ to produce an equation in t. Alternatively candidates could use $\cos 2t = 2\cos^2 t - 1$ to set up an equation of the form $y = Ax^2 + B$.
M1:	Uses the identity $\cos 2t = 2\cos^2 t - 1$ to produce a quadratic equation in $\cos t$ In the alternative method it is for combining their $y = Ax^2 + B$ with $2x - 2\sqrt{3}y - 1 = 0$ to get an equation in just one variable
A1:	For the correct quadratic equation $12\cos^2 t - 4\cos t - 5 = 0$ Alternatively the equations in x and y are $3x^2 - 2x - 5 = 0$ $12\sqrt{3}y^2 + 4y - 7\sqrt{3} = 0$
M1:	Solves the quadratic equation in $\cos t$ (or x or y) and rejects the value corresponding to P.
M1:	Substitutes their $\cos t = \frac{5}{6}$ or their $t = \arccos\left(\frac{5}{6}\right)$ in $x = 2\cos t$ and $y = \sqrt{3}\cos 2t$ If a value of x or y has been found it is for finding the other coordinate.
A1:	$Q = \left(\frac{5}{3}, \frac{7}{18}\sqrt{3}\right)$. Allow $x = \frac{5}{3}, y = \frac{7}{18}\sqrt{3}$ but do not allow decimal equivalents.

Q4.

Question Number	Scheme		Marks
	$x = 2\sin t, \quad y = 1 - \cos 2t \quad \left\{= 2\sin^2 t\right\}, \quad -\frac{\pi}{2} \leqslant t \leqslant \frac{\pi}{2}$		
(a)	$\frac{dx}{dt} = 2\cos t, \quad \frac{dy}{dt} = 2\sin 2t \;$ or $\; \frac{dy}{dt} = 4\sin t\cos t$	At least one of $\frac{dx}{dt}$ or $\frac{dy}{dt}$ correct. Both $\frac{dx}{dt}$ and $\frac{dy}{dt}$ are correct.	B1 B1
	So, $\frac{dy}{dx} = \frac{2\sin 2t}{2\cos t} \left\{= \frac{4\cos t\sin t}{2\cos t} = 2\sin t\right\}$	Applies their $\frac{dy}{dt}$ divided by their $\frac{dx}{dt}$	
	At $t = \frac{\pi}{6}$, $\frac{dy}{dx} = \frac{2\sin\left(\frac{2\pi}{6}\right)}{2\cos\left(\frac{\pi}{6}\right)}$; $= 1$	and substitutes $t = \frac{\pi}{6}$ into their $\frac{dy}{dx}$. Correct value for $\frac{dy}{dx}$ of 1	M1; A1 cao cso
			[4]
(b)	$y = 1 - \cos 2t = 1 - (1 - 2\sin^2 t)$ $= 2\sin^2 t$		M1
	So, $y = 2\left(\frac{x}{2}\right)^2$ or $y = \frac{x^2}{2}$ or $y = 2 - 2\left(1 - \left(\frac{x}{2}\right)^2\right)$	$y = \frac{x^2}{2}$ or equivalent.	A1 cso isw
	Either $k = 2$ or $-2 \leqslant x \leqslant 2$		B1
			[3]
(c)	Range: $0 \leqslant f(x) \leqslant 2$ or $0 \leqslant y \leqslant 2$ or $0 \leqslant f \leqslant 2$	See notes	B1 B1
			[2] 9

Notes for Question

(a)

B1: At least one of $\frac{dx}{dt}$ or $\frac{dy}{dt}$ correct. **Note:** that this mark can be implied from their working.

B1: Both $\frac{dx}{dt}$ and $\frac{dy}{dt}$ are correct. **Note:** that this mark can be implied from their working.

M1: Applies their $\frac{dy}{dt}$ divided by their $\frac{dx}{dt}$ **and** attempts to substitute $t = \frac{\pi}{6}$ into their expression for $\frac{dy}{dx}$.

This mark may be implied by their final answer.

Ie. $\frac{dy}{dx} = \frac{\sin 2t}{2\cos t}$ followed by an answer of $\frac{1}{2}$ would be M1 (implied).

A1: For an answer of 1 *by correct solution only.*

Note: Don't just look at the answer! A number of candidates are finding $\frac{dy}{dx} = 1$ from incorrect methods.

Note: Applying $\frac{dx}{dt}$ divided by their $\frac{dy}{dt}$ is M0, even if they state $\frac{dy}{dx} = \frac{dy}{dt} \div \frac{dx}{dt}$.

Special Case: Award SC: B0B0M1A1 for $\frac{dx}{dt} = -2\cos t$, $\frac{dy}{dt} = -2\sin 2t$ leading to $\frac{dy}{dx} = \frac{-2\sin 2t}{-2\cos t}$

which after substitution of $t = \frac{\pi}{6}$, yields $\frac{dy}{dx} = 1$

Note: It is possible for you to mark part(a), part (b) and part (c) together. Ignore labelling!

Notes for Question Continued		
(b)	**M1**: Uses the **correct** double angle formula $\cos 2t = 1 - 2\sin^2 t$ or $\cos 2t = 2\cos^2 t - 1$ or $\cos 2t = \cos^2 t - \sin^2 t$ in an attempt to get y in terms of $\sin^2 t$ or get y in terms of $\cos^2 t$ or get y in terms of $\sin^2 t$ and $\cos^2 t$. Writing down $y = 2\sin^2 t$ is fine for M1. **A1**: Achieves $y = \frac{x^2}{2}$ or un-simplified equivalents **in the form** $y = \mathrm{f}(x)$. For example: $y = \frac{2x^2}{4}$ or $y = 2\left(\frac{x}{2}\right)^2$ or $y = 2 - 2\left(1 - \left(\frac{x}{2}\right)^2\right)$ or $y = 1 - \frac{4 - x^2}{4} + \frac{x^2}{4}$ and you can ignore subsequent working if a candidate states a correct version of the Cartesian equation. **IMPORTANT: Please check working as this result can be fluked from an incorrect method.** **Award A0** if there is a $+c$ added to their answer. **B1**: Either $k = 2$ or a candidate writes down $-2 \leqslant x \leqslant 2$. Note: $-2 \leqslant k \leqslant 2$ unless k stated as 2 is B0.	
(c)	**Note: The values of 0 and/or 2 need to be evaluated in this part** **B1**: Achieves an inclusive upper **or** lower limit, using acceptable notation. Eg: $\mathrm{f}(x) \geqslant 0$ or $\mathrm{f}(x) \leqslant 2$ **B1**: $0 \leqslant \mathrm{f}(x) \leqslant 2$ or $0 \leqslant y \leqslant 2$ or $0 \leqslant \mathrm{f} \leqslant 2$ **Special Case: SC: B1B0** for either $0 < \mathrm{f}(x) < 2$ or $0 < \mathrm{f} < 2$ or $0 < y < 2$ or $(0, 2)$ **Special Case: SC: B1B0** for $0 \leqslant x \leqslant 2$. **IMPORTANT: Note that:** Therefore candidates can use either y or f in place of $\mathrm{f}(x)$ **Examples:** $0 \leqslant x \leqslant 2$ is SC: B1B0 — $0 < x < 2$ is B0B0 $x \geqslant 0$ is B0B0 — $x \leqslant 2$ is B0B0 $\mathrm{f}(x) > 0$ is B0B0 — $\mathrm{f}(x) < 2$ is B0B0 $x > 0$ is B0B0 — $x < 2$ is B0B0 $0 \geqslant \mathrm{f}(x) \geqslant 2$ is B0B0 — $0 < \mathrm{f}(x) \leqslant 2$ is B1B0 $0 \leqslant \mathrm{f}(x) < 2$ is B1B0. — $\mathrm{f}(x) \geqslant 0$ is B1B0 $\mathrm{f}(x) \leqslant 2$ is B1B0 — $\mathrm{f}(x) \geqslant 0$ and $\mathrm{f}(x) \leqslant 2$ is B1B1. Must state AND {or} $\cap$ $2 \leqslant \mathrm{f}(x) \leqslant 2$ is B0B0 — $\mathrm{f}(x) \geqslant 0$ or $\mathrm{f}(x) \leqslant 2$ is B1B0. $\lvert \mathrm{f}(x) \rvert \leqslant 2$ is B1B0 — $\lvert \mathrm{f}(x) \rvert \geqslant 2$ is B0B0 $1 \leqslant \mathrm{f}(x) \leqslant 2$ is B1B0 — $1 < \mathrm{f}(x) < 2$ is B0B0 $0 \leqslant \mathrm{f}(x) \leqslant 4$ is B1B0 — $0 < \mathrm{f}(x) < 4$ is B0B0 $0 \leqslant$ Range $\leqslant 2$ is B1B0 — Range is in between 0 and 2 is B1B0 $0 <$ Range < 2 is B0B0. — Range $\geqslant 0$ is B1B0 Range $\leqslant 2$ is B1B0 — Range $\geqslant 0$ and Range $\leqslant 2$ is B1B0. $[0, 2]$ is B1B1 — $(0, 2)$ is SC B1B0	
Aliter (a) Way 2	$\frac{dx}{dt} = 2\cos t$, $\frac{dy}{dt} = 2\sin 2t$, At $t = \frac{\pi}{6}$, $\frac{dx}{dt} = 2\cos\left(\frac{\pi}{6}\right) = \sqrt{3}$, $\frac{dy}{dt} = 2\sin\left(\frac{2\pi}{6}\right) = \sqrt{3}$ Hence $\frac{dy}{dx} = 1$	So B1, B1. So implied M1, A1.

Notes for Question Continued			
Aliter (a) Way 3	$y=\frac{1}{2}x^2 \Rightarrow \frac{dy}{dx}=x$	Correct differentiation of their Cartesian equation.	B1ft
		Finds $\frac{dy}{dx}=x$, using the correct Cartesian equation only.	B1
	At $t=\frac{\pi}{6}$, $\frac{dy}{dx}=2\sin\left(\frac{\pi}{6}\right)$	Finds the value of "x" when $t=\frac{\pi}{6}$ and substitutes this into their $\frac{dy}{dx}$	M1
	$=1$	Correct value for $\frac{dy}{dx}$ of 1	A1
Aliter (b) Way 2	$y=1-\cos 2t=1-(2\cos^2 t-1)$		M1
	$y=2-2\cos^2 t \Rightarrow \cos^2 t=\frac{2-y}{2} \Rightarrow 1-\sin^2 t=\frac{2-y}{2}$		
	$1-\left(\frac{x}{2}\right)^2=\frac{2-y}{2}$		(Must be in the form $y=\mathrm{f}(x)$).
	$y=2-2\left(1-\left(\frac{x}{2}\right)^2\right)$		A1
Aliter (b) Way 3	$x=2\sin t \Rightarrow t=\sin^{-1}\left(\frac{x}{2}\right)$		
	So, $y=1-\cos\left(2\sin^{-1}\left(\frac{x}{2}\right)\right)$	Rearranges to make t the subject and substitutes the result into y.	M1
		$y=1-\cos\left(2\sin^{-1}\left(\frac{x}{2}\right)\right)$	A1 oe
Aliter (b) Way 4	$y=1-\cos 2t \Rightarrow \cos 2t=1-y \Rightarrow t=\frac{1}{2}\cos^{-1}(1-y)$		
	So, $x=\pm 2\sin\left(\frac{1}{2}\cos^{-1}(1-y)\right)$	Rearranges to make t the subject and substitutes the result into y.	M1
	So, $y=1-\cos\left(2\sin^{-1}\left(\frac{x}{2}\right)\right)$	$y=1-\cos\left(2\sin^{-1}\left(\frac{x}{2}\right)\right)$	A1 oe
Aliter (b) Way 5	$\frac{dy}{dx}=2\sin t=x \Rightarrow y=\frac{1}{2}x^2+c$	$\frac{dy}{dx}=x \Rightarrow y=\frac{1}{2}x^2+c$	M1
	Eg: when eg: $t=0$ (nb: $-\frac{\pi}{2}\leqslant t\leqslant\frac{\pi}{2}$), $x=0,\ y=1-1=0 \Rightarrow c=0 \Rightarrow y=\frac{1}{2}x^2$	Full method of finding $y=\frac{1}{2}x^2$ using a value of t: $-\frac{\pi}{2}\leqslant t\leqslant\frac{\pi}{2}$	A1
	Note: $\frac{dy}{dx}=2\sin t=x \Rightarrow y=\frac{1}{2}x^2$, with no attempt to find c is M1A0.		

Q5.

Question Number	Scheme		Marks
(a)	$x = \tan^2 t\,, \qquad y = \sin t$		
	$\frac{dx}{dt} = 2(\tan t)\sec^2 t\,,\ \frac{dy}{dt} = \cos t$	Correct $\frac{dx}{dt}$ and $\frac{dy}{dt}$	B1
	$\therefore \frac{dy}{dx} = \frac{\cos t}{2\tan t \sec^2 t} \quad \left(= \frac{\cos^4 t}{2\sin t}\right)$	$\frac{\pm\cos t}{\text{their } \frac{dx}{dt}}$	M1
		$\frac{+\cos t}{\text{their } \frac{dx}{dt}}$	A1 $\sqrt{}$
			[3]
(b)	When $t = \frac{\pi}{4}$, $\quad x = 1,\ y = \frac{1}{\sqrt{2}}$ (need values)	The point $\underline{\left(1, \frac{1}{\sqrt{2}}\right)}$ or $\underline{(1, \text{awrt } 0.71)}$ **These coordinates can be implied.** ($y = \sin\left(\frac{\pi}{4}\right)$ is not sufficient for B1)	B1, B1
	When $t = \frac{\pi}{4}$, m(**T**) $= \frac{dy}{dx} = \frac{\cos\frac{\pi}{4}}{2\tan\frac{\pi}{4}\sec^2\frac{\pi}{4}}$		
	$= \underline{\frac{\frac{1}{\sqrt{2}}}{2.(1)\left(\frac{1}{\frac{1}{\sqrt{2}}}\right)^2}} = \underline{\frac{\frac{1}{\sqrt{2}}}{2.(1)\left(\frac{1}{\frac{1}{2}}\right)}} = \underline{\frac{\frac{1}{\sqrt{2}}}{2.(1)(2)}} = \underline{\frac{1}{4\sqrt{2}}} = \underline{\frac{\sqrt{2}}{8}}$	any of the five underlined expressions or awrt 0.18	B1 aef
	T: $y - \frac{1}{\sqrt{2}} = \frac{1}{4\sqrt{2}}(x - 1)$	Finding an equation of a tangent with ***their point*** and ***their tangent gradient*** or finds c by using $y = \underline{(\text{their gradient})}x + "\underline{c}"$.	M1 $\sqrt{}$ aef
	T: $\underline{y = \frac{1}{4\sqrt{2}}x + \frac{3}{4\sqrt{2}}}$ or $\underline{y = \frac{\sqrt{2}}{8}x + \frac{3\sqrt{2}}{8}}$	Correct simplified EXACT equation of <u>tangent</u>	<u>A1</u> aef **cso**
	or $\frac{1}{\sqrt{2}} = \frac{1}{4\sqrt{2}}(1) + c \ \Rightarrow\ c = \frac{1}{\sqrt{2}} - \frac{1}{4\sqrt{2}} = \frac{3}{4\sqrt{2}}$		
	Hence **T:** $\underline{y = \frac{1}{4\sqrt{2}}x + \frac{3}{4\sqrt{2}}}$ or $\underline{y = \frac{\sqrt{2}}{8}x + \frac{3\sqrt{2}}{8}}$		
			[5]

Note: The x and y coordinates must be the right way round.

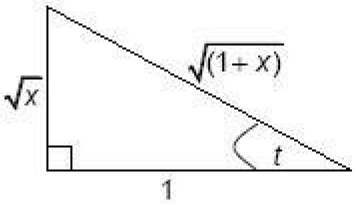
A candidate who incorrectly differentiates $\tan^2 t$ to give $\frac{dx}{dt} = 2\sec^2 t$ or $\frac{dx}{dt} = \sec^4 t$ is then able to fluke the correct answer in part (b). Such candidates can potentially get: (a) B0M1A1 $\sqrt{}$ (b) B1B1B1M1A0 **cso**.
Note: cso means "correct solution only".
Note: part (a) not fully correct implies candidate can achieve a maximum of 4 out of 5 marks in part (b).

(c) **Way 1**	$x = \tan^2 t = \dfrac{\sin^2 t}{\cos^2 t}$ $\quad y = \sin t$		
	$x = \dfrac{\sin^2 t}{1-\sin^2 t}$	Uses $\cos^2 t = 1-\sin^2 t$	M1
	$x = \dfrac{y^2}{1-y^2}$	Eliminates *'t' to* write an equation involving x and y.	M1
	$x(1-y^2) = y^2 \Rightarrow x - xy^2 = y^2$		
	$x = y^2 + xy^2 \quad \Rightarrow \quad x = y^2(1+x)$	Rearranging and factorising with an attempt to make y^2 the subject.	ddM1
	$y^2 = \dfrac{x}{1+x}$	$\dfrac{x}{1+x}$	A1
			[4]
Aliter (c) **Way 2**	$1+\cot^2 t = co\sec^2 t$	Uses $1+\cot^2 t = co\sec^2 t$	M1
	$= \dfrac{1}{\sin^2 t}$	Uses $\cos ec^2 t = \dfrac{1}{\sin^2 t}$	M1 implied
	Hence, $\quad 1+\dfrac{1}{x} = \dfrac{1}{y^2}$	Eliminates *'t' to* write an equation involving x and y.	ddM1
	Hence, $y^2 = 1 - \dfrac{1}{(1+x)}$ or $\dfrac{x}{1+x}$	$1 - \dfrac{1}{(1+x)}$ or $\dfrac{x}{1+x}$	A1
			[4]

$\dfrac{1}{1+\frac{1}{x}}$ is an acceptable response for the final accuracy A1 mark.

Aliter (c) **Way 3**	$x = \tan^2 t \qquad y = \sin t$		
	$1 + \tan^2 t = \sec^2 t$	Uses $1 + \tan^2 t = \sec^2 t$	M1
	$= \dfrac{1}{\cos^2 t}$	Uses $\sec^2 t = \dfrac{1}{\cos^2 t}$	M1
	$= \dfrac{1}{1 - \sin^2 t}$		
	Hence, $\quad 1 + x = \dfrac{1}{1 - y^2}$	Eliminates '*t*' *to* write an equation involving *x* and *y*.	ddM1
	Hence, $y^2 = 1 - \dfrac{1}{(1 + x)}$ or $\dfrac{x}{1 + x}$	$1 - \dfrac{1}{(1 + x)}$ or $\dfrac{x}{1 + x}$	A1
			[4]
Aliter (c) **Way 4**	$y^2 = \sin^2 t = 1 - \cos^2 t$	Uses $\sin^2 t = 1 - \cos^2 t$	M1
	$= 1 - \dfrac{1}{\sec^2 t}$	Uses $\cos^2 t = \dfrac{1}{\sec^2 t}$	M1
	$= 1 - \dfrac{1}{(1 + \tan^2 t)}$	then uses $\sec^2 t = 1 + \tan^2 t$	ddM1
	Hence, $y^2 = 1 - \dfrac{1}{(1 + x)}$ or $\dfrac{x}{1 + x}$	$1 - \dfrac{1}{(1 + x)}$ or $\dfrac{x}{1 + x}$	A1
			[4]

$\dfrac{1}{1 + \frac{1}{x}}$ is an acceptable response for the final accuracy A1 mark.

Aliter (c) **Way 5**	$x=\tan^2 t \qquad y=\sin t$		
	$x=\tan^2 t \;\Rightarrow\; \tan t=\sqrt{x}$		
		Draws a right-angled triangle and places both $\sqrt{x}$ and 1 on the triangle	M1
		Uses Pythagoras to deduce the hypotenuse	M1
	Hence, $y=\sin t=\dfrac{\sqrt{x}}{\sqrt{1+x}}$	Eliminates '*t*' *to* write an equation involving *x* and *y*.	ddM1
	Hence, $y^2=\dfrac{x}{1+x}$	$\dfrac{x}{1+x}$	A1
			[4]
			12 marks

$\dfrac{1}{1+\frac{1}{x}}$ is an acceptable response for the final accuracy A1 mark.

There are so many ways that a candidate can proceed with part (c). If a candidate produces a correct solution then please award all four marks. If they use a method commensurate with the five ways as detailed on the mark scheme then award the marks appropriately. If you are unsure of how to apply the scheme please escalate your response up to your team leader.

Q6.

Question Number	Scheme		Marks
	$x = 4\cos\left(t + \frac{\pi}{6}\right), \quad y = 2\sin t$		
	Main Scheme		
(a)	$x = 4\left(\cos t\cos\left(\frac{\pi}{6}\right) - \sin t\sin\left(\frac{\pi}{6}\right)\right)$	$\cos\left(t + \frac{\pi}{6}\right) \to \cos t\cos\left(\frac{\pi}{6}\right) \pm \sin t\sin\left(\frac{\pi}{6}\right)$	M1 oe
	So, $\{x + y\} = 4\left(\cos t\cos\left(\frac{\pi}{6}\right) - \sin t\sin\left(\frac{\pi}{6}\right)\right) + 2\sin t$	Adds their expanded x (which is in terms of t) to $2\sin t$	dM1
	$= 4\left(\left(\frac{\sqrt{3}}{2}\right)\cos t - \left(\frac{1}{2}\right)\sin t\right) + 2\sin t$		
	$= 2\sqrt{3}\cos t$ *	Correct proof	A1 *
			[3]
(a)	**Alternative Method 1**		
	$x = 4\left(\cos t\cos\left(\frac{\pi}{6}\right) - \sin t\sin\left(\frac{\pi}{6}\right)\right)$	$\cos\left(t + \frac{\pi}{6}\right) \to \cos t\cos\left(\frac{\pi}{6}\right) \pm \sin t\sin\left(\frac{\pi}{6}\right)$	M1 oe
	$= 4\left(\left(\frac{\sqrt{3}}{2}\right)\cos t - \left(\frac{1}{2}\right)\sin t\right) = 2\sqrt{3}\cos t - 2\sin t$		
	So, $x = 2\sqrt{3}\cos t - y$	Forms an equation in x, y and t.	dM1
	$x + y = 2\sqrt{3}\cos t$ *	Correct proof	A1 *
			[3]
	Main Scheme		
(b)	$\left(\frac{x + y}{2\sqrt{3}}\right)^2 + \left(\frac{y}{2}\right)^2 = 1$	Applies $\cos^2 t + \sin^2 t = 1$ to achieve an equation containing **only** x's and y's.	M1
	$\Rightarrow \frac{(x + y)^2}{12} + \frac{y^2}{4} = 1$		
	$\Rightarrow (x + y)^2 + 3y^2 = 12$	$(x + y)^2 + 3y^2 = 12$	A1
		$\{a = 3, b = 12\}$	[2]
(b)	**Alternative Method 1**		
	$(x + y)^2 = 12\cos^2 t = 12(1 - \sin^2 t) = 12 - 12\sin^2 t$		
	So, $(x + y)^2 = 12 - 3y^2$	Applies $\cos^2 t + \sin^2 t = 1$ to achieve an equation containing **only** x's and y's.	M1
	$\Rightarrow (x + y)^2 + 3y^2 = 12$	$(x + y)^2 + 3y^2 = 12$	A1
			[2]
(b)	**Alternative Method 2**		
	$(x + y)^2 = 12\cos^2 t$		
	As $12\cos^2 t + 12\sin^2 t = 12$		
	then $(x + y)^2 + 3y^2 = 12$		M1, A1
			[2]
			5

	Question Notes	
(a)	M1	$\cos\left(t+\frac{\pi}{6}\right) \to \cos t\cos\left(\frac{\pi}{6}\right) \pm \sin t\sin\left(\frac{\pi}{6}\right)$ or $\cos\left(t+\frac{\pi}{6}\right) \to \left(\frac{\sqrt{3}}{2}\right)\cos t \pm \left(\frac{1}{2}\right)\sin t$
	Note	**If a candidate states** $\cos(A+B) = \cos A\cos B \pm \sin A\sin B$, but there is an error ***in its application*** then give M1. **<u>Awarding the dM1 mark which is dependent on the first method mark</u>**
Main	dM1 **Note**	Adds their expanded x (which is in terms of t) to $2\sin t$ Writing $x + y = ...$ is not needed in the **Main Scheme** method.
Alt 1	dM1	Forms an equation in x, y and t.
	A1*	Evidence of $\cos\left(\frac{\pi}{6}\right)$ and $\sin\left(\frac{\pi}{6}\right)$ evaluated and the proof is correct with no errors.
	Note	$\{x+y\} = 4\cos\left(t+\frac{\pi}{6}\right) + 2\sin t$, by itself is M0M0A0.
(b)	M1 A1	Applies $\cos^2 t + \sin^2 t = 1$ to achieve an equation containing **only** x's and y's. leading $(x+y)^2 + 3y^2 = 12$
	SC	Award **Special Case B1B0** for a candidate who writes down **either** • $(x+y)^2 + 3y^2 = 12$ from no working • $a = 3$, $b = 12$, but **<u>does not provide a correct proof</u>**.
	Note Note	Alternative method 2 is fine for M1 A1 Writing $(x+y)^2 = 12\cos^2 t$ followed by $12\cos^2 t + a(4\sin^2 t) = b \Rightarrow a = 3, b = 12$ is SC: B1B0
	Note	Writing $(x+y)^2 = 12\cos^2 t$ followed by $12\cos^2 t + a(4\sin^2 t) = b$ • states $a = 3$, $b = 12$ • and refers to either $\cos^2 t + \sin^2 t = 1$ or $12\cos^2 t + 12\sin^2 t = 12$ • and there is no incorrect working would get M1A1

Q7.

Question	Scheme	Marks	AOs
	$(x-3)^2 + y^2 = \left(\dfrac{t^2+5}{t^2+1} - 3\right)^2 + \left(\dfrac{4t}{t^2+1}\right)^2$	M1	3.1a
	$= \dfrac{(2-2t^2)^2 + 16t^2}{(t^2+1)^2} = \dfrac{4+8t^2+4t^4}{(t^2+1)^2}$	dM1	1.1b
	$\dfrac{4(t^4+2t^2+1)}{(t^2+1)^2} = \dfrac{4(t^2+1)^2}{(t^2+1)^2} = 4*$	A1*	2.1
		(3)	

M1: Attempts to substitute the given parametric forms into the Cartesian equation or the lhs of the Cartesian equation. There may have been an (incorrect) attempt to multiply out the $(x-3)^2$ term.

dM1: Attempts to combine (at least the lhs) using correct processing into a single fraction, multiplies out and collects terms on the numerator.

A1*: Fully correct proof showing all key steps

Question	Scheme	Marks	AOs
Alt	$x=\dfrac{t^2+5}{t^2+1}\Rightarrow xt^2+x=t^2+5\Rightarrow t^2=\dfrac{5-x}{x-1}$ $y=\dfrac{4t}{t^2+1}\Rightarrow y^2=\dfrac{16t^2}{(t^2+1)^2}=\dfrac{16\left(\dfrac{5-x}{x-1}\right)}{\left(\dfrac{5-x}{x-1}+1\right)^2}$	M1	3.1a
	$y^2=\dfrac{16\left(\dfrac{5-x}{x-1}\right)}{\left(\dfrac{5-x}{x-1}+1\right)^2}=16\left(\dfrac{5-x}{x-1}\right)\times\left(\dfrac{(x-1)}{5-x+x-1}\right)^2\Rightarrow y^2=(5-x)(x-1)$	dM1	1.1b
	$y^2=(5-x)(x-1)\Rightarrow y^2=6x-x^2-5$ $\Rightarrow y^2=4-(x-3)^2$ or other intermediate step $\Rightarrow (x-3)^2+y^2=4$*	A1*	2.1
		(3)	
		(3 marks)	
Notes			

M1: Adopts a correct strategy for eliminating t to obtain an equation in terms of x and y only. See scheme.

Other methods exist which also lead to an appropriate equation. E.g using $t=\dfrac{y}{x-1}$

dM1: Uses correct processing to eliminate the fractions and start to simplify

A1*: Fully correct proof showing all key steps

Q8.

Question Number	Scheme		Marks	
(a)	At A, $x=-1+8=7$ & $y=(-1)^2=1 \Rightarrow A(7,1)$	$A(7,1)$	B1	
				(1)
(b)	$x=t^3-8t$, $y=t^2$,			
	$\frac{dx}{dt}=3t^2-8$, $\frac{dy}{dt}=2t$			
	$\therefore \frac{dy}{dx}=\frac{2t}{3t^2-8}$	Their $\frac{dy}{dt}$ divided by their $\frac{dx}{dt}$	M1	
		Correct $\frac{dy}{dx}$	A1	
	At A, $m(\mathbf{T})=\underline{\frac{2(-1)}{3(-1)^2-8}}=\underline{\frac{-2}{3-8}}=\underline{\frac{-2}{-5}}=\underline{\frac{2}{5}}$	Substitutes for t to give any of the four underlined oe:		
	$\mathbf{T}$: $y-(\text{their } 1)=m_T(x-(\text{their } 7))$ or $1=\frac{2}{5}(7)+c \Rightarrow c=1-\frac{14}{5}=-\frac{9}{5}$	Finding an equation of a tangent with their point and their tangent gradient or finds c and uses $y=(\text{their gradient})x+"c"$.	dM1	
	Hence $\mathbf{T}$: $y=\frac{2}{5}x-\frac{9}{5}$			
	gives $\mathbf{T}$: $\underline{2x-5y-9=0}$ **AG**	$\underline{2x-5y-9=0}$	A1	cso
				(5)
(c)	$2(t^3-8t)-5t^2-9=0$	Substitution of both $x=t^3-8t$ and $y=t^2$ into $\mathbf{T}$	M1	
	$2t^3-5t^2-16t-9=0$			
	$(t+1)\{(2t^2-7t-9)=0\}$ $(t+1)\{(t+1)(2t-9)=0\}$	A realisation that $(t+1)$ is a factor.	dM1	
	$\{t=-1 \text{ (at } A)\}$ $t=\frac{9}{2}$ at B	$t=\frac{9}{2}$	A1	
	$x=\left(\frac{9}{2}\right)^3-8\left(\frac{9}{2}\right)=\frac{729}{8}-36=\frac{441}{8}=55.125$ or awrt 55.1	Candidate uses their value of t to find either the x or y coordinate	ddM1	
	$y=\left(\frac{9}{2}\right)^2=\frac{81}{4}=20.25$ or awrt 20.3	One of either x or y correct.	A1	
		Both x and y correct.	A1	
	Hence $B\left(\frac{441}{8},\frac{81}{4}\right)$	awrt		(6)
				[12]

Q9.

Question Number	Scheme	Marks
(a)	At $P(4,2\sqrt{3})$ either $\underline{4=8\cos t}$ or $\underline{2\sqrt{3}=4\sin 2t}$	M1
	$\Rightarrow$ only solution is $\underline{t=\frac{\pi}{3}}$ where $0\leqslant t\leqslant \frac{\pi}{2}$	A1
(b)	$x=8\cos t, \quad y=4\sin 2t$	
	$\frac{dx}{dt}=-8\sin t, \quad \frac{dy}{dt}=8\cos 2t$	M1 A1
	At P, $\frac{dy}{dx}=\underline{\frac{8\cos\left(\frac{2\pi}{3}\right)}{-8\sin\left(\frac{\pi}{3}\right)}}$	M1
	$\left\{=\frac{8\left(-\frac{1}{2}\right)}{(-8)\left(\frac{\sqrt{3}}{2}\right)}=\frac{1}{\sqrt{3}}=\text{awrt } 0.58\right\}$	
	Hence $\text{m}(\mathbf{N})=-\sqrt{3}$ or $\frac{-1}{\frac{1}{\sqrt{3}}}$	M1
	N: $y-2\sqrt{3}=-\sqrt{3}(x-4)$	M1
	N: $\underline{y=-\sqrt{3}x+6\sqrt{3}}$ (*)	A1 cso (6)
(c)	$A=\int_0^4 y\,dx=\int_{\frac{\pi}{2}}^{\frac{\pi}{3}} 4\sin 2t.(-8\sin t)\,dt$	M1 A1
	$A=\int_{\frac{\pi}{2}}^{\frac{\pi}{3}} -32\sin 2t.\sin t\,dt = \int_{\frac{\pi}{2}}^{\frac{\pi}{3}} -32(2\sin t\cos t).\sin t\,dt$	M1
	$A=\int_{\frac{\pi}{2}}^{\frac{\pi}{3}} -64.\sin^2 t\cos t\,dt$	
	$A=\int_{\frac{\pi}{3}}^{\frac{\pi}{2}} 64.\sin^2 t\cos t\,dt$ (*)	A1 (4)
(d)	$A=64\left[\frac{\sin^3 t}{3}\right]_{\frac{\pi}{3}}^{\frac{\pi}{2}}$ or $A=64\left[\frac{u^3}{3}\right]_{\frac{\sqrt{3}}{2}}^{1}$	M1 A1
	$A=64\left[\frac{1}{3}-\left(\frac{1}{3}.\frac{\sqrt{3}}{2}.\frac{\sqrt{3}}{2}.\frac{\sqrt{3}}{2}\right)\right]$	M1
	$A=64\left(\frac{1}{3}-\frac{1}{8}\sqrt{3}\right)=\underline{\frac{64}{3}-8\sqrt{3}}$	A1 (4)
		(16 marks)

Question Number	Scheme		Marks	
	(a) $y=0 \Rightarrow t(9-t^2)=t(3-t)(3+t)=0$			
	$t=0,3,-3$	Any one correct value	B1	
	At $t=0$, $x=5(0)^2-4=-4$	Method for finding one value of x	M1	
	At $t=3$, $x=5(3)^2-4=41$			
	$\left(\text{At } t=-3,\ x=5(-3)^2-4=41\right)$			
	At A, $x=-4$; at B, $x=41$	Both	A1	(3)
	(b) $\frac{dx}{dt}=10t$	Seen or implied	B1	
	$\int y\,dx=\int y\frac{dx}{dt}dt=\int t(9-t^2)10t\,dt$		M1 A1	
	$=\int(90t^2-10t^4)dt$			
	$=\frac{90t^3}{3}-\frac{10t^5}{5}\ (+C)\quad\left(=30t^3-2t^5\ (+C)\right)$		A1	
	$\left[\frac{90t^3}{3}-\frac{10t^5}{5}\right]_0^3=30\times3^3-2\times3^5\quad(=324)$		M1	
	$A=2\int y\,dx=648\quad(\text{units}^2)$		A1	(6)
				[9]

Q11.

Question Number	Scheme		Marks
(a)	$\left[x = \ln(t+2),\ \ y = \frac{1}{t+1}\right],\quad \Rightarrow \frac{dx}{dt} = \frac{1}{t+2}$	Must state $\frac{dx}{dt} = \frac{1}{t+2}$	B1
	$\text{Area}(R) = \int_{\ln 2}^{\ln 4} \frac{1}{t+1}\,dx; = \int_0^2 \left(\frac{1}{t+1}\right)\left(\frac{1}{t+2}\right)dt$	Area $= \int \frac{1}{t+1}dx$. Ignore limits.	M1;
		$\int \left(\frac{1}{t+1}\right) \times \left(\frac{1}{t+2}\right)dt$. Ignore limits.	A1 **AG**
	Changing limits, when: $x = \ln 2 \ \Rightarrow \ln 2 = \ln(t+2) \ \Rightarrow\ 2 = t+2 \ \Rightarrow t = 0$ $x = \ln 4 \ \Rightarrow \ln 4 = \ln(t+2) \ \Rightarrow\ 4 = t+2 \ \Rightarrow t = 2$	changes limits $x \to t$ so that $\ln 2 \to 0$ and $\ln 4 \to 2$	B1
	Hence, $\text{Area}(R) = \int_0^2 \frac{1}{(t+1)(t+2)}\,dt$		
			[4]
(b)	$\left(\frac{1}{(t+1)(t+2)}\right) = \frac{A}{(t+1)} + \frac{B}{(t+2)}$	$\frac{A}{(t+1)} + \frac{B}{(t+2)}$ with A and B found	M1
	$1 = A(t+2) + B(t+1)$		
	Let $t = -1$, $1 = A(1) \ \Rightarrow\ \underline{A = 1}$ Let $t = -2$, $1 = B(-1) \ \Rightarrow\ \underline{B = -1}$	Finds both A and B correctly. Can be implied. (See note below)	A1
	$\int_0^2 \frac{1}{(t+1)(t+2)}\,dt = \int_0^2 \frac{1}{(t+1)} - \frac{1}{(t+2)}\,dt$		
	$= \left[\ln(t+1) - \ln(t+2)\right]_0^2$	Either $\pm a\ln(t+1)$ or $\pm b\ln(t+2)$	dM1
		Both ln terms correctly ft.	A1 $\sqrt{}$
	$= (\ln 3 - \ln 4) - (\ln 1 - \ln 2)$	Substitutes *both* limits of 2 and 0 and subtracts the correct way round.	ddM1
	$= \ln 3 - \ln 4 + \ln 2 = \ln 3 - \ln 2 = \ln\left(\frac{3}{2}\right)$	$\underline{\ln 3 - \ln 4 + \ln 2}$ or $\underline{\ln\left(\frac{3}{4}\right) - \ln\left(\frac{1}{2}\right)}$ or $\underline{\ln 3 - \ln 2}$ or $\underline{\ln\left(\frac{3}{2}\right)}$ (must deal with ln 1)	A1 aef isw
			[6]

Takes out brackets.

Writing down $\frac{1}{(t+1)(t+2)} = \frac{1}{(t+1)} + \frac{1}{(t+2)}$ means first M1A0 in (b).

Writing down $\frac{1}{(t+1)(t+2)} = \frac{1}{(t+1)} - \frac{1}{(t+2)}$ means first M1A1 in (b).

Question Number	Scheme		Marks
	$x = \ln(t+2), \quad y = \frac{1}{t+1}$		
(c)	$e^x = t+2 \Rightarrow t = e^x - 2$	Attempt to make $t = \ldots$ the subject giving $t = e^x - 2$	M1 A1
	$y = \frac{1}{e^x - 2 + 1} \Rightarrow y = \frac{1}{e^x - 1}$	Eliminates t by substituting in y giving $y = \frac{1}{e^x - 1}$	dM1 A1
			[4]
Aliter (c) **Way 2**	$t + 1 = \frac{1}{y} \Rightarrow t = \frac{1}{y} - 1$ or $t = \frac{1-y}{y}$ $y(t+1) = 1 \Rightarrow yt + y = 1 \Rightarrow yt = 1 - y \Rightarrow t = \frac{1-y}{y}$	Attempt to make $t = \ldots$ the subject Giving either $t = \frac{1}{y} - 1$ or $t = \frac{1-y}{y}$	M1 A1
	$x = \ln\left(\frac{1}{y} - 1 + 2\right)$ or $x = \ln\left(\frac{1-y}{y} + 2\right)$	Eliminates t by substituting in x	dM1
	$x = \ln\left(\frac{1}{y} + 1\right)$		
	$e^x = \frac{1}{y} + 1$		
	$e^x - 1 = \frac{1}{y}$		
	$y = \frac{1}{e^x - 1}$	giving $y = \frac{1}{e^x - 1}$	A1
			[4]
(d)	Domain : $\underline{x > 0}$	$\underline{x > 0}$ or just > 0	B1
			[1]
			15 marks

Question Number	Scheme		Marks
Aliter (c) **Way 3**	$e^x = t+2 \;\Rightarrow\; t+1 = e^x - 1$	Attempt to make $t+1 = \ldots$ the subject giving $t+1 = e^x - 1$	M1 A1
	$y = \dfrac{1}{t+1} \;\Rightarrow\; y = \dfrac{1}{e^x - 1}$	Eliminates t by substituting in y giving $y = \dfrac{1}{e^x - 1}$	dM1 A1
			[4]
Aliter (c) **Way 4**	$t+1 = \dfrac{1}{y} \;\Rightarrow\; t+2 = \dfrac{1}{y}+1$ or $t+2 = \dfrac{1+y}{y}$	Attempt to make $t+2 = \ldots$ the subject Either $t+2 = \dfrac{1}{y}+1$ or $t+2 = \dfrac{1+y}{y}$	M1 A1
	$x = \ln\left(\dfrac{1}{y}+1\right)$ or $x = \ln\left(\dfrac{1+y}{y}\right)$	Eliminates t by substituting in x	dM1
	$x = \ln\left(\dfrac{1}{y}+1\right)$		
	$e^x = \dfrac{1}{y}+1 \;\Rightarrow\; e^x - 1 = \dfrac{1}{y}$		
	$y = \dfrac{1}{e^x - 1}$	giving $y = \dfrac{1}{e^x - 1}$	A1
			[4]

Q12.

Question Number	Scheme		Marks
	Note: You can mark parts (a) and (b) together.		
(a)	$x = 4t + 3,\ \ y = 4t + 8 + \dfrac{5}{2t}$		
	$\dfrac{dx}{dt} = 4,\ \ \dfrac{dy}{dt} = 4 - \dfrac{5}{2}t^{-2}$	Both $\dfrac{dx}{dt} = 4$ or $\dfrac{dt}{dx} = \dfrac{1}{4}$ and $\dfrac{dy}{dt} = 4 - \dfrac{5}{2}t^{-2}$	B1
	So, $\dfrac{dy}{dx} = \dfrac{4 - \frac{5}{2}t^{-2}}{4}\ \left\{= 1 - \dfrac{5}{8}t^{-2} = 1 - \dfrac{5}{8t^2}\right\}$	Candidate's $\dfrac{dy}{dt}$ divided by a candidate's $\dfrac{dx}{dt}$	M1 o.e.
	$\{\text{When } t = 2,\}\ \dfrac{dy}{dx} = \dfrac{27}{32}$	$\dfrac{27}{32}$ or 0.84375 cao	A1
			[3]
	Way 2: Cartesian Method		
	$\dfrac{dy}{dx} = 1 - \dfrac{10}{(x-3)^2}$	$\dfrac{dy}{dx} = 1 - \dfrac{10}{(x-3)^2}$, simplified or un-simplifed.	B1
		$\dfrac{dy}{dx} = \pm\lambda \pm \dfrac{\mu}{(x-3)^2},\ \ \lambda \neq 0, \mu \neq 0$	M1
	$\{\text{When } t = 2, x = 11\}\ \dfrac{dy}{dx} = \dfrac{27}{32}$	$\dfrac{27}{32}$ or 0.84375 cao	A1
			[3]
	Way 3: Cartesian Method		
	$\dfrac{dy}{dx} = \dfrac{(2x+2)(x-3) - (x^2+2x-5)}{(x-3)^2}$	Correct expression for $\dfrac{dy}{dx}$, simplified or un-simplified.	B1
	$\left\{= \dfrac{x^2 - 6x - 1}{(x-3)^2}\right\}$	$\dfrac{dy}{dx} = \dfrac{f'(x)(x-3) - 1f(x)}{(x-3)^2}$, where $f(x) =$ their "$x^2 + ax + b$", $g(x) = x - 3$	M1
	$\{\text{When } t = 2, x = 11\}\ \dfrac{dy}{dx} = \dfrac{27}{32}$	$\dfrac{27}{32}$ or 0.84375 cao	A1
			[3]
(b)	$\left\{t = \dfrac{x-3}{4} \Rightarrow\right\}\ y = 4\left(\dfrac{x-3}{4}\right) + 8 + \dfrac{5}{2\left(\dfrac{x-3}{4}\right)}$	Eliminates t to achieve an equation in only x and y	M1
	$y = x - 3 + 8 + \dfrac{10}{x-3}$		
	$y = \dfrac{(x-3)(x-3) + 8(x-3) + 10}{x-3}$ *or* $y(x-3) = (x-3)(x-3) + 8(x-3) + 10$ *or* $y = \dfrac{(x+5)(x-3) + 10}{x-3}$ *or* $y = \dfrac{(x+5)(x-3)}{x-3} + \dfrac{10}{x-3}$	See notes	dM1
	$\Rightarrow y = \dfrac{x^2 + 2x - 5}{x-3},\ \ \{a = 2 \text{ and } b = -5\}$	Correct algebra leading to $y = \dfrac{x^2 + 2x - 5}{x-3}$ or $a = 2$ and $b = -5$	A1 cso
			[3] 6

Question Number	Scheme		Marks
(b)	**Alternative Method 1 of Equating Coefficients**		
	$y = \dfrac{x^2 + ax + b}{x - 3} \Rightarrow y(x-3) = x^2 + ax + b$ $y(x-3) = (4t+3)^2 + 2(4t+3) - 5 = 16t^2 + 32t + 10$ $x^2 + ax + b = (4t+3)^2 + a(4t+3) + b$		
	$(4t+3)^2 + a(4t+3) + b = 16t^2 + 32t + 10$	Correct method of obtaining an equation in only t, a and b	M1
	t: $\quad 24 + 4a = 32 \quad \Rightarrow a = 2$ constant: $9 + 3a + b = 10 \quad \Rightarrow b = -5$	Equates their coefficients in t and finds both $a = ...$ and $b = ...$	dM1
		$a = 2$ and $b = -5$	A1
			[3]
(b)	**Alternative Method 2 of Equating Coefficients**		
	$\left\{t = \dfrac{x-3}{4} \Rightarrow\right\} y = 4\left(\dfrac{x-3}{4}\right) + 8 + \dfrac{5}{2\left(\dfrac{x-3}{4}\right)}$	Eliminates t to achieve an equation in only x and y	M1
	$y = x - 3 + 8 + \dfrac{10}{x-3} \Rightarrow y = x + 5 + \dfrac{10}{(x-3)}$ $\underline{\underline{y(x-3)}} = (x+5)(x-3) + 10 \Rightarrow x^2 + ax + b = \underline{\underline{(x+5)(x-3) + 10}}$		dM1
	$\Rightarrow y = \dfrac{x^2 + 2x - 5}{x - 3}$ or equating coefficients to give $a = 2$ and $b = -5$	Correct algebra leading to $y = \dfrac{x^2 + 2x - 5}{x - 3}$ or $a = 2$ and $b = -5$	A1 cso
			[3]

		Question Notes
(a)	B1	$\frac{dx}{dt} = 4$ and $\frac{dy}{dt} = 4 - \frac{5}{2}t^{-2}$ or $\frac{dy}{dt} = \frac{8t^2 - 5}{2t^2}$ or $\frac{dy}{dt} = 4 - 5(2t)^{-2}(2)$, etc.
	Note	$\frac{dy}{dt}$ can be simplified or un-simplified.
	Note	**You can imply the B1 mark by later working.**
	M1	Candidate's $\frac{dy}{dt}$ divided by a candidate's $\frac{dx}{dt}$ **or** $\frac{dy}{dt}$ multiplied by a candidate's $\frac{dt}{dx}$
	Note	M1 can be also be obtained by substituting $t = 2$ into both their $\frac{dy}{dt}$ and their $\frac{dx}{dt}$ and then dividing their values the correct way round.
	A1	$\frac{27}{32}$ or 0.84375 **cao**
(b)	M1	Eliminates t to achieve an equation in only x and y.
	dM1	**dependent on the first method mark being awarded.** **Either: (ignoring sign slips or constant slips, noting that k can be 1)** • Combining all three parts of their $\underline{x-3} + \underline{\underline{8}} + \underline{\underline{\left(\frac{10}{x-3}\right)}}$ to form a single fraction with a common denominator of $\pm k(x-3)$. Accept three separate fractions with the same denominator. • Combining both parts of their $\underline{x+5} + \underline{\underline{\left(\frac{10}{x-3}\right)}}$, (where $\underline{x+5}$ is their $4\left(\frac{x-3}{4}\right) + 8$), to form a single fraction with a common denominator of $\pm k(x-3)$. Accept two separate fractions with the same denominator. • Multiplies both sides of their $y = \underline{x-3} + \underline{\underline{8}} + \underline{\underline{\left(\frac{10}{x-3}\right)}}$ or their $y = \underline{x+5} + \underline{\underline{\left(\frac{10}{x-3}\right)}}$ by $\pm k(x-3)$. Note that all terms in their equation must be multiplied by $\pm k(x-3)$.
	Note	Condone "invisible" brackets for dM1.
	A1	Correct algebra with no incorrect working leading to $y = \frac{x^2 + 2x - 5}{x - 3}$ or $a = 2$ and $b = -5$
	Note	**Some examples for the award of dM1 in (b):** **dM0** for $y = x - 3 + 8 + \frac{10}{x-3} \rightarrow y = \frac{(x-3)(x-3) + 8 + 10}{x-3}$. Should be $\ldots + 8(x-3) + \ldots$ **dM0** for $y = x - 3 + \frac{10}{x-3} \rightarrow y = \frac{(x-3)(x-3) + 10}{x-3}$. The "8" part has been omitted. **dM0** for $y = x + 5 + \frac{10}{x-3} \rightarrow y = \frac{x(x-3) + 5 + 10}{x-3}$. Should be $\ldots + 5(x-3) + \ldots$ **dM0** for $y = x + 5 + \frac{10}{x-3} \rightarrow y(x-3) = x(x-3) + 5(x-3) + 10(x-3)$. Should be just 10.
	Note	$y = x + 5 + \frac{10}{x-3} \rightarrow y = \frac{x^2 + 2x - 5}{x-3}$ with no intermediate working is dM1A1.

Q13.

Question Number	Scheme	Marks
	$x = 4\sin\left(t+\frac{\pi}{6}\right),\quad y = 3\cos 2t,\quad 0 \leqslant t < 2\pi$	
(a)	$\frac{dx}{dt} = 4\cos\left(t+\frac{\pi}{6}\right),\ \frac{dy}{dt} = -6\sin 2t$	B1 B1
	So, $\frac{dy}{dx} = \frac{-6\sin 2t}{4\cos\left(t+\frac{\pi}{6}\right)}$	B1$\sqrt{\ }$ oe
		[3]
(b)	$\left\{\frac{dy}{dx} = 0 \Rightarrow\right\} -6\sin 2t = 0$	M1 oe
	@ $t=0,\ x = 4\sin\left(\frac{\pi}{6}\right) = 2,\quad y = 3\cos 0 = 3 \quad \to (2, 3)$	M1
	@ $t=\frac{\pi}{2},\ x = 4\sin\left(\frac{2\pi}{3}\right) = \frac{4\sqrt{3}}{2},\ y = 3\cos\pi = -3 \to (2\sqrt{3}, -3)$	
	@ $t=\pi,\ x = 4\sin\left(\frac{7\pi}{6}\right) = -2,\ y = 3\cos 2\pi = 3 \to (-2, 3)$	
	@ $t=\frac{3\pi}{2}, x = 4\sin\left(\frac{5\pi}{3}\right) = \frac{4(-\sqrt{3})}{2},\ y = 3\cos 3\pi = -3 \to (-2\sqrt{3}, -3)$	A1A1A1
		[5] 8

(a)

B1: Either one of $\frac{dx}{dt} = 4\cos\left(t+\frac{\pi}{6}\right)$ or $\frac{dy}{dt} = -6\sin 2t$. They do not have to be simplified.

B1: Both $\frac{dx}{dt}$ and $\frac{dy}{dt}$ correct. They do not have to be simplified.

Any or both of the first two marks can be implied.
Don't worry too much about their notation for the first two B1 marks.

B1: Their $\frac{dy}{dt}$ divided by their $\frac{dx}{dt}$ or their $\frac{dy}{dt}\times\frac{1}{\text{their}\left(\frac{dx}{dt}\right)}$. **Note:** This is a follow through mark.

Alternative differentiation in part (a)

$$x = 2\sqrt{3}\sin t + 2\cos t \Rightarrow \frac{dx}{dt} = 2\sqrt{3}\cos t - 2\sin t$$

$$y = 3(2\cos^2 t - 1) \Rightarrow \frac{dy}{dt} = 3(-4\cos t\sin t)$$

$$\text{or } y = 3\cos^2 t - 3\sin^2 t \Rightarrow \frac{dy}{dt} = -6\cos t\sin t - 6\sin t\cos t$$

$$\text{or } y = 3(1-2\sin^2 t) \Rightarrow \frac{dy}{dt} = 3(-4\cos t\sin t)$$

(b)

M1: Candidate sets their numerator from part (a) or their $\frac{dy}{dt}$ equal to 0.

Note that their numerator must be a trig function. Ignore $\frac{dx}{dt}$ equal to 0 at this stage.

M1: Candidate substitutes a found value of t, to attempt to find either one of x or y.
The first two method marks can be implied by ONE correct set of coordinates for (x, y) or (y, x) interchanged.
A correct point coming from NO WORKING can be awarded M1M1.
A1: At least TWO sets of coordinates.
A1: At least THREE sets of coordinates.
A1: ONLY FOUR correct sets of coordinates. If there are more than 4 sets of coordinates then award A0.
Note: Candidate can use the diagram's symmetry to write down some of their coordinates.

Note: When $x = 4\sin\left(\frac{\pi}{6}\right) = 2$, $y = 3\cos 0 = 3$ is acceptable for a pair of coordinates.

Also it is fine for candidates to display their coordinates on a table of values.
Note: The coordinates must be exact for the accuracy marks. Ie (3.46..., -3) or (-3.46..., -3) is A0.

Note: $\frac{dy}{dx} = 0 \Rightarrow \sin t = 0$ ONLY is fine for the first M1, and potentially the following M1A1A0A0.

Note: $\frac{dy}{dx} = 0 \Rightarrow \cos t = 0$ ONLY is fine for the first M1 and potentially the following M1A1A0A0.

Note: $\frac{dy}{dx} = 0 \Rightarrow \sin t = 0\ \&\ \cos t = 0$ has the potential to achieve all five marks.

Note: It is possible for a candidate to gain full marks in part (b) if they make sign errors in part (a).

(b) An alternative method for finding the coordinates of the two maximum points.
Some candidates may use $y = 3\cos 2t$ to write down that the y-coordinate of a maximum point is 3.
They will then deduce that $t = 0$ or π and proceed to find the x-coordinate of their maximum point. These candidates will receive no credit until they attempt to find one of the x-coordinates for the maximum point.

M1M1: Candidate states $y = 3$ and attempts to substitute $t = 0$ or π into $x = 4\sin\left(t+\frac{\pi}{6}\right)$.

M1M1 can be implied by candidate stating either $(2, 3)$ or $(2, -3)$.
Note: these marks can only be awarded together for a candidate using this method.

A1: For both $(2, 3)$ or $(-2, 3)$.

A0A0: Candidate cannot achieve the final two marks by using this method. They can, however, achieve these marks by subsequently solving their numerator equal to 0.

Q14.

Question Number	Scheme	Marks
Q (a)	$\frac{dx}{dt} = -4\sin 2t$, $\frac{dy}{dt} = 6\cos t$	B1, B1
	$\frac{dy}{dx} = -\frac{6\cos t}{4\sin 2t}$ $\left(= -\frac{3}{4\sin t}\right)$	M1
	At $t = \frac{\pi}{3}$, $m = -\frac{3}{4\times\frac{\sqrt{3}}{2}} = -\frac{\sqrt{3}}{2}$ accept equivalents, awrt -0.87	A1 (4)
(b)	Use of $\cos 2t = 1 - 2\sin^2 t$	M1
	$\cos 2t = \frac{x}{2}$, $\sin t = \frac{y}{6}$	
	$\frac{x}{2} = 1 - 2\left(\frac{y}{6}\right)^2$	M1
	Leading to $y = \sqrt{(18-9x)}$ $\left(= 3\sqrt{(2-x)}\right)$ cao	A1
	$-2 \le x \le 2$ $k = 2$	B1 (4)
(c)	$0 \le f(x) \le 6$ either $0 \le f(x)$ or $f(x) \le 6$	B1
	Fully correct. Accept $0 \le y \le 6$, $[0, 6]$	B1 (2)
		[10]
	Alternatives to (a) *where the parameter is eliminated*	
	① $y = (18-9x)^{\frac{1}{2}}$	
	$\frac{dy}{dx} = \frac{1}{2}(18-9x)^{-\frac{1}{2}} \times (-9)$	B1
	At $t = \frac{\pi}{3}$, $x = \cos\frac{2\pi}{3} = -1$	B1
	$\frac{dy}{dx} = \frac{1}{2} \times \frac{1}{\sqrt{(27)}} \times -9 = -\frac{\sqrt{3}}{2}$	M1 A1 (4)
	② $y^2 = 18 - 9x$	
	$2y\frac{dy}{dx} = -9$	B1
	At $t = \frac{\pi}{3}$, $y = 6\sin\frac{\pi}{3} = 3\sqrt{3}$	B1
	$\frac{dy}{dx} = -\frac{9}{2\times 3\sqrt{3}} = -\frac{\sqrt{3}}{2}$	M1 A1 (4)

Q15.

Question Number	Scheme		Marks
	$x = t - 4\sin t, \quad y = 1 - 2\cos t, \quad -\frac{2\pi}{3} \leqslant t \leqslant \frac{2\pi}{3} \quad A(k, 1)$ lies on the curve, $k > 0$		
(a)	$\{\text{When } y = 1,\}\ 1 = 1 - 2\cos t \Rightarrow t = -\frac{\pi}{2}, \frac{\pi}{2}$ $k \text{ (or } x) = \frac{\pi}{2} - 4\sin\left(\frac{\pi}{2}\right)$ **or** $x = -\frac{\pi}{2} - 4\sin\left(-\frac{\pi}{2}\right)$	Sets $y = 1$ to find t and uses their t to find x.	M1
	$\left\{\text{When } t = -\frac{\pi}{2}, k > 0,\right\}$ so $k = 4 - \frac{\pi}{2}$ or $\frac{8-\pi}{2}$	x or $k = 4 - \frac{\pi}{2}$	A1
			[2]
(b)	$\frac{dx}{dt} = 1 - 4\cos t, \quad \frac{dy}{dt} = 2\sin t$	At least one of $\frac{dx}{dt}$ or $\frac{dy}{dt}$ correct.	B1
		Both $\frac{dx}{dt}$ and $\frac{dy}{dt}$ are correct.	B1
	So, $\frac{dy}{dx} = \frac{2\sin t}{1 - 4\cos t}$	Applies their $\frac{dy}{dt}$ divided by their $\frac{dx}{dt}$ and substitutes their t into their $\frac{dy}{dx}$.	M1;
	At $t = -\frac{\pi}{2}$, $\frac{dy}{dx} = \frac{2\sin\left(-\frac{\pi}{2}\right)}{1 - 4\cos\left(-\frac{\pi}{2}\right)}$; $= -2$	Correct value for $\frac{dy}{dx}$ of -2	A1 cao cso
			[4]
(c)	$\frac{2\sin t}{1 - 4\cos t} = -\frac{1}{2}$	Sets their $\frac{dy}{dx} = -\frac{1}{2}$	M1
	gives $4\sin t - 4\cos t = -1$	**See notes**	A1
	So $4\sqrt{2}\sin\left(t - \frac{\pi}{4}\right); = -1$ or $-4\sqrt{2}\cos\left(t + \frac{\pi}{4}\right); = -1$	**See notes**	M1; A1
	$t = \sin^{-1}\left(\frac{-1}{4\sqrt{2}}\right) + \frac{\pi}{4}$ or $t = \cos^{-1}\left(\frac{1}{4\sqrt{2}}\right) - \frac{\pi}{4}$	**See notes**	dM1
	$t = 0.6076875626... = 0.6077$ (4 dp)	anything that rounds to 0.6077	A1
			[6] **12**

	Question Notes	
(c)	**NOTE**	**<u>VERY IMPORTANT NOTE FOR PART (c)</u>** Candidates who state $t = 0.6077$ with no intermediate working from $4\sin t - 4\cos t = -1$ will get 2nd M0, 2nd A0, 3rd M0, 3rd A0. They will not express $4\sin t - 4\cos t$ as either $4\sqrt{2}\sin\left(t - \frac{\pi}{4}\right)$ or $-4\sqrt{2}\cos\left(t + \frac{\pi}{4}\right)$. OR use any acceptable alternative method to achieve $t = 0.6077$
	NOTE	Alternative methods for part (c) are given on the next page.

	Question Alternative Methods for Part (c)		
(c)	**Alternative Method 1:**		
	$$\frac{2\sin t}{1-4\cos t} = -\frac{1}{2}$$	Sets their $\frac{\mathrm{d}y}{\mathrm{d}x} = -\frac{1}{2}$	M1
	eg. $\left(\frac{2\sin t}{1-4\cos t}\right)^2 = \frac{1}{4}$ or $(4\sin t)^2 = (4\cos t - 1)^2$ or $(4\sin t + 1)^2 = (4\cos t)^2$ etc.	Squaring to give a correct equation. This mark can be implied by a "squared" correct equation.	A1
		Note: You can also give 1st A1 in this method for $4\sin t - 4\cos t = -1$ as in the main scheme.	
		Squares their equation, applies $\sin^2 t + \cos^2 t = 1$ and achieves a **three term quadratic equation** of the form $\pm a\cos^2 t \pm b\cos t \pm c = 0$ or $\pm a\sin^2 t \pm b\sin t \pm c = 0$ or eg. $\pm a\cos^2 t \pm b\cos t = \pm c$ where $a \neq 0, b \neq 0$ and $c \neq 0$.	M1
	• **Either** $32\cos^2 t - 8\cos t - 15 = 0$ • **or** $32\sin^2 t + 8\sin t - 15 = 0$	For a correct **three term quadratic equation.**	A1
	• **Either** $\cos t = \frac{8 \pm \sqrt{1984}}{64} = \frac{1+\sqrt{31}}{8} \Rightarrow t = \cos^{-1}(...)$ • **or** $\sin t = \frac{-8 \pm \sqrt{1984}}{64} = \frac{-1 \pm \sqrt{31}}{8} \Rightarrow t = \sin^{-1}(...)$	Uses correct algebraic processes to give $t = ...$ **which is dependent on the 2nd M1 mark.**	dM1
	$t = 0.6076875626... = 0.6077$ (4 dp)	anything that rounds to 0.6077	A1
			[6]
(c)	**Alternative Method 2:**		
	$$\frac{2\sin t}{1-4\cos t} = -\frac{1}{2}$$	Sets their $\frac{\mathrm{d}y}{\mathrm{d}x} = -\frac{1}{2}$	M1
	eg. $(4\sin t - 4\cos t)^2 = (-1)^2$	Squaring to give a correct equation. This mark can be implied by a correct equation. **Note:** You can also give 1st A1 in this method for $4\sin t - 4\cos t = -1$ as in the main scheme.	A1
	So $16\sin^2 t - 32\sin t\cos t + 16\cos^2 t = 1$		
	leading to $16 - 16\sin 2t = 1$	Squares their equation, applies both $\sin^2 t + \cos^2 t = 1$ and $\sin 2t = 2\sin t\cos t$ and then achieves an equation of the form $\pm a \pm b\sin 2t = \pm c$	M1
		$16 - 16\sin 2t = 1$ or equivalent.	A1
	$\left\{\sin 2t = \frac{15}{16} \Rightarrow\right\} t = \frac{\sin^{-1}(....)}{2}$	Uses correct algebraic processes to give $t = ...$ **which is dependent on the 2nd M1 mark.**	dM1
	$t = 0.6076875626... = 0.6077$ (4 dp)	anything that rounds to 0.6077	A1
			[6]

	Question Notes	
(a)	M1	Sets $y=1$ to find t and uses their t to find x.
	Note	M1 can be implied by either x or $k=4-\frac{\pi}{2}$ or 2.429... or $\frac{\pi}{2}-4$ or -2.429...
	A1	x or $k=4-\frac{\pi}{2}$ or $\frac{8-\pi}{2}$
	Note	A decimal answer of 2.429... (without a correct **exact** answer) is A0.
	Note	Allow A1 for a candidate using $t=\frac{\pi}{2}$ to find $x=\frac{\pi}{2}-4$ **and** then stating that k must be $4-\frac{\pi}{2}$ o.e.
(b)	B1	At least one of $\frac{\mathrm{d}x}{\mathrm{d}t}$ or $\frac{\mathrm{d}y}{\mathrm{d}t}$ correct. **Note:** that this mark can be implied from their working.
	B1	Both $\frac{\mathrm{d}x}{\mathrm{d}t}$ and $\frac{\mathrm{d}y}{\mathrm{d}t}$ are correct. **Note:** that this mark can be implied from their working.
	M1	Applies their $\frac{\mathrm{d}y}{\mathrm{d}t}$ divided by their $\frac{\mathrm{d}x}{\mathrm{d}t}$ **and** attempts to substitute their t into their expression for $\frac{\mathrm{d}y}{\mathrm{d}x}$.
	Note	This mark may be implied by their final answer. i.e. $\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{2\sin t}{1-4\cos t}$ followed by an answer of -2 (from $t=-\frac{\pi}{2}$) or 2 (from $t=\frac{\pi}{2}$)
	Note	**Applying** $\frac{\mathrm{d}x}{\mathrm{d}t}$ divided by their $\frac{\mathrm{d}y}{\mathrm{d}t}$ is M0, even if they state $\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{\mathrm{d}y}{\mathrm{d}t}\div\frac{\mathrm{d}x}{\mathrm{d}t}$.
	A1	Using $t=-\frac{\pi}{2}$ $\left(\text{and not } t=\frac{3\pi}{2}\right)$ to find a correct $\frac{\mathrm{d}y}{\mathrm{d}x}$ of -2 *by correct solution only.*
(c)		
	NOTE	If a candidate uses an incorrect $\frac{\mathrm{d}y}{\mathrm{d}x}$ expression in part (c) then the accuracy marks are not obtainable.
	1st M1	Sets their $\frac{\mathrm{d}y}{\mathrm{d}x}=-\frac{1}{2}$
	1st A1	Rearranges to give the correct equation with $\sin t$ and $\cos t$ **on the same side.** eg. $4\sin t-4\cos t=-1$ or $4\cos t-4\sin t=1$ or $\sin t-\cos t=-\frac{1}{4}$ or $\cos t-\sin t=\frac{1}{4}$ or $4\sin t-4\cos t+1=0$ or $4\cos t-4\sin t-1=0$ or $\sin t-\cos t+\frac{1}{4}=0$ etc. are fine for A1.
	2nd M1	Rewrites $\pm\lambda\sin t\pm\mu\cos t$ in the form of either $R\cos(t\pm\alpha)$ or $R\sin(t\pm\alpha)$ where $R\neq 1$ or 0 and $\alpha\neq 0$
	2nd A1	Correct equation. Eg. $4\sqrt{2}\sin\left(t-\frac{\pi}{4}\right)=-1$ or $-4\sqrt{2}\cos\left(t+\frac{\pi}{4}\right)=-1$ or $\sqrt{2}\sin\left(t-\frac{\pi}{4}\right)=-\frac{1}{4}$ or $\sqrt{2}\cos\left(t+\frac{\pi}{4}\right)=\frac{1}{4}$, etc.
	Note	Unless recovered, give A0 for $4\sqrt{2}\sin(t-45^\circ)=-1$ or $-4\sqrt{2}\cos(t+45^\circ)=-1$, etc.
	3rd M1	**which is dependent on the 2nd M1 mark.** Uses correct algebraic processes to give $t=\ldots$
	4th A1	anything that rounds to 0.6077
	Note	**Do not give** the final A1 mark in (c) if there any extra solutions given in the range $-\frac{2\pi}{3}\leqslant t\leqslant\frac{2\pi}{3}$.
	Note	You can give the final A1 mark in (c) if extra solutions are given outside of $-\frac{2\pi}{3}\leqslant t\leqslant\frac{2\pi}{3}$.

Q16.

Question Number	Scheme		Marks
	Working parametrically: $x = 1 - \frac{1}{2}t,\quad y = 2^t - 1$ or $y = e^{t\ln 2} - 1$		
(a)	$\{x = 0 \Rightarrow\}\ 0 = 1 - \frac{1}{2}t \Rightarrow t = 2$	Applies $x = 0$ to obtain a value for t.	M1
	When $t = 2$, $y = 2^2 - 1 = 3$	Correct value for y.	A1
			[2]
(b)	$\{y = 0 \Rightarrow\}\ 0 = 2^t - 1 \Rightarrow t = 0$	Applies $y = 0$ to obtain a value for t. (Must be seen in part (b)).	M1
	When $t = 0$, $x = 1 - \frac{1}{2}(0) = 1$	$x = 1$	A1
			[2]
(c)	$\frac{dx}{dt} = -\frac{1}{2}$ and either $\frac{dy}{dt} = 2^t\ln 2$ or $\frac{dy}{dt} = e^{t\ln 2}\ln 2$		B1
	$\frac{dy}{dx} = \frac{2^t\ln 2}{-\frac{1}{2}}$	Attempts their $\frac{dy}{dt}$ divided by their $\frac{dx}{dt}$	M1
	At A, $t = $ "2", so $m(\mathrm{T}) = -8\ln 2 \Rightarrow m(\mathrm{N}) = \frac{1}{8\ln 2}$	Applies $t = $ "2" and $m(\mathrm{N}) = \frac{-1}{m(\mathrm{T})}$	M1
	$y - 3 = \frac{1}{8\ln 2}(x - 0)$ or $y = 3 + \frac{1}{8\ln 2}x$ or equivalent.	See notes.	M1 A1 oe cso
			[5]
(d)	$\text{Area}(R) = \int (2^t - 1)\left(-\frac{1}{2}\right)dt$	Complete substitution for both y and dx	M1
	$x = -1 \to t = 4$ and $x = 1 \to t = 0$		B1
	$= \left(-\frac{1}{2}\right)\left(\frac{2^t}{\ln 2} - t\right)$	Either $2^t \to \frac{2^t}{\ln 2}$ or $(2^t - 1) \to \frac{(2^t)}{\pm\alpha(\ln 2)} - t$ or $(2^t - 1) \to \pm\alpha(\ln 2)(2^t) - t$	M1*
		$(2^t - 1) \to \frac{2^t}{\ln 2} - t$	A1
	$\left\{-\frac{1}{2}\left[\frac{2^t}{\ln 2} - t\right]_4^0\right\} = -\frac{1}{2}\left(\left(\frac{1}{\ln 2}\right) - \left(\frac{16}{\ln 2} - 4\right)\right)$	**Depends on the previous method mark.** Substitutes their changed limits in t and subtracts either way round.	dM1*
	$= \frac{15}{2\ln 2} - 2$	$\frac{15}{2\ln 2} - 2$ or equivalent	A1
			[6] 15
(a)	**M1:** Applies $x = 0$ and obtains a value of t. **A1:** For $y = 2^2 - 1 = 3$ or $y = 4 - 1 = 3$. ***Alternative Solution 1:*** **M1:** For substituting $t = 2$ into either x or y. **A1:** $x = 1 - \frac{1}{2}(2) = 0$ and $y = 2^2 - 1 = 3$. ***Alternative Solution 2:*** **M1:** Applies $y = 3$ and obtains a value of t. **A1:** For $x = 1 - \frac{1}{2}(2) = 0$ or $x = 1 - 1 = 0$. ***Alternative Solution 3:*** **M1:** Applies $y = 3$ or $x = 0$ and obtains a value of t. **A1:** Shows that $t = 2$ for both $y = 3$ and $x = 0$.		
(b)	**M1:** Applies $y = 0$ and obtains a value of t. Working must be seen in part (b). **A1:** For finding $x = 1$. Note: Award M1A1 for $x = 1$.		
(c)	**B1:** Both $\frac{dx}{dt}$ and $\frac{dy}{dt}$ correct. This mark can be implied by later working. **M1:** Their $\frac{dy}{dt}$ divided by their $\frac{dx}{dt}$ or their $\frac{dy}{dt} \times \frac{1}{\text{their}\left(\frac{dx}{dt}\right)}$. Note: their $\frac{dy}{dx}$ must be a function of t. **M1:** Uses their value of t found in part (a) and applies $m(\mathrm{N}) = \frac{-1}{m(\mathrm{T})}$. **M1:** $y - 3 = (\text{their normal gradient})x$ or $y = (\text{their normal gradient})x + 3$ or equivalent. **A1:** $y - 3 = \frac{1}{8\ln 2}(x - 0)$ or $y = 3 + \frac{1}{8\ln 2}x$ or $y - 3 = \frac{1}{\ln 256}(x - 0)$ or $(8\ln 2)y - 24\ln 2 = x$ or $\frac{y - 3}{(x - 0)} = \frac{1}{8\ln 2}$. You can apply cso here. Working in decimals is ok for the three method marks. B1, A1 require exact values.		
(d)	**M1:** Complete substitution for both y and dx. So candidate should write down $\int (2^t - 1)\left(\text{their }\frac{dx}{dt}\right)$. **B1:** Changes limits from $x \to t$: $x = -1 \to t = 4$ and $x = 1 \to t = 0$. Note $t = 4$ and $t = 0$ seen is B1. **M1*:** Integrates 2^t correctly to give $\frac{2^t}{\ln 2}$ … or integrates $(2^t - 1)$ to give either $\frac{(2^t)}{\pm\alpha(\ln 2)} - t$ or $\pm\alpha(\ln 2)(2^t) - t$. **A1:** Correct integration of $(2^t - 1)$ with respect to t to give $\frac{2^t}{\ln 2} - t$. **dM1*:** **Depends upon the previous method mark.** Substitutes their limits in t and subtracts either way round. **A1:** Exact answer of $\frac{15}{2\ln 2} - 2$ or $\frac{15}{\ln 4} - 2$ or $\frac{15 - 4\ln 2}{2\ln 2}$ or $\frac{7.5}{\ln 2} - 2$ or $\frac{15}{2}\log_2 e - 2$ or equivalent.		
	Alternative: Converting to a Cartesian equation: $t = 2 - 2x \Rightarrow y = 2^{2-2x} - 1$		
(a)	$\{x = 0 \Rightarrow\}\ y = 2^2 - 1$	Applies $x = 0$ in their Cartesian equation…	M1
	$y = 3$	…to arrive at a correct answer of 3.	A1
			[2]
(b)	$\{y = 0 \Rightarrow\}\ 0 = 2^{2-2x} - 1 \Rightarrow 0 = 2 - 2x \Rightarrow x = \ldots$	Applies $y = 0$ to obtain a value for x. (Must be seen in part (b)).	M1
	$x = 1$	$x = 1$	A1
			[2]
(c)	$\frac{dy}{dx} = -2\left(2^{2-2x}\right)\ln 2$	$\pm k\,2^{2-2x}$, $k \neq 1$	M1
		$-2\left(2^{2-2x}\right)\ln 2$ or equivalent	A1
	At A, $x = 0$, so $m(\mathrm{T}) = -8\ln 2 \Rightarrow m(\mathrm{N}) = \frac{1}{8\ln 2}$	Applies $x = 0$ and $m(\mathrm{N}) = \frac{-1}{m(\mathrm{T})}$	M1
	$y - 3 = \frac{1}{8\ln 2}(x - 0)$ or $y = 3 + \frac{1}{8\ln 2}x$ or equivalent.	***As in the original scheme.***	M1 A1 oe
			[5]
(d)	$\text{Area}(R) = \int (2^{2-2x} - 1)\,dx$	Forms the integral of their Cartesian equation of C.	M1
	$= \int_{-1}^{1} (2^{2-2x} - 1)\,dx$	For $2^{2-2x} - 1$ with limits of $x = -1$ and $x = 1$. ie. $\int_{-1}^{1} (2^{2-2x} - 1)$	B1
	$= \left(\frac{2^{2-2x}}{-2\ln 2} - x\right)$	Either $2^{2-2x} \to \frac{2^{2-2x}}{-2\ln 2}$ or $(2^{2-2x} - 1) \to \frac{2^{2-2x}}{\pm\alpha(\ln 2)} - x$ or $(2^{2-2x} - 1) \to \pm\alpha(\ln 2)(2^{2-2x}) - x$	M1*
		$(2^{2-2x} - 1) \to \frac{2^{2-2x}}{-2\ln 2} - x$	A1
	$\left[\frac{2^{2-2x}}{-2\ln 2} - x\right]_{-1}^{1} = \left(\left(\frac{1}{-2\ln 2} - 1\right) - \left(\frac{16}{-2\ln 2} + 1\right)\right)$	**Depends on the previous method mark.** Substitutes limits of -1 and their x_B and subtracts either way round.	dM1*
	$= \frac{15}{2\ln 2} - 2$	$\frac{15}{2\ln 2} - 2$ or equivalent.	A1
			[6] 15
(d)	**Alternative methods: In Cartesian and applying $u = 2 - 2x$** $\text{Area}(R) = \int \{(2^u - 1)\}\{dx\}$, where $u = 2 - 2x$ $= \int_{4}^{0} (2^u - 1)(-\frac{1}{2})\{du\}$	**M0**: Unless a candidate writes $\int \{(2^{2-2x} - 1)\}\{dx\}$. Then apply the "working parametrically" mark scheme.	
(d)	***Alternative method: For substitution $u = 2^t$*** $\text{Area}(R) = \int (2^t - 1)\left(-\frac{1}{2}\right)dt$	Complete substitution for both y and dx	M1
	where $u = 2^t \Rightarrow \frac{du}{dt} = 2^t\ln 2 \Rightarrow \frac{du}{dt} = u\ln 2$		
	$x = -1 \to t = 4 \to u = 16$ and $x = 1 \to t = 0 \to u = 1$	Both correct limits in t or both correct limits in u.	B1
	So $\text{area}(R) = -\frac{1}{2}\int \frac{u - 1}{u\ln 2}du$ $= -\frac{1}{2}\int \frac{1}{\ln 2} - \frac{1}{u\ln 2}du$	If not awarded above, you can award M1 for this integral	
	$= \left(-\frac{1}{2}\right)\left(\frac{u}{\ln 2} - \frac{\ln u}{\ln 2}\right)$	Either $2^t \to \frac{u}{\ln 2}$ or $(2^t - 1) \to \frac{u}{\pm\alpha(\ln 2)} - \frac{\ln u}{\ln 2}$ or $(2^t - 1) \to \pm\alpha(\ln 2)(u) - \frac{\ln u}{\ln 2}$	M1*
		$(2^t - 1) \to \frac{u}{\ln 2} - \frac{\ln u}{\ln 2}$	A1
	$\left\{-\frac{1}{2}\left[\frac{u}{\ln 2} - \frac{\ln u}{\ln 2}\right]_{16}^{1}\right\} = -\frac{1}{2}\left(\left(\frac{1}{\ln 2}\right) - \left(\frac{16}{\ln 2} - \frac{\ln 16}{\ln 2}\right)\right)$	**Depends on the previous method mark.** Substitutes their changed limits in u and subtracts either way round.	dM1*
	$= \frac{15}{2\ln 2} - \frac{\ln 16}{2\ln 2}$ or $\frac{15}{2\ln 2} - 2$	$\frac{15}{2\ln 2} - \frac{\ln 16}{2\ln 2}$ or $\frac{15}{2\ln 2} - 2$ or equivalent.	A1
			[6]