

Questions

Q1.

The curve C has parametric equations

$$x = 3t - 4, y = 5 - \frac{6}{t}, \quad t > 0$$

(a) Find $\frac{dy}{dx}$ in terms of t

(2)

The point P lies on C where $t = \frac{1}{2}$

(b) Find the equation of the tangent to C at the point P . Give your answer in the form $y = px + q$, where p and q are integers to be determined.

(3)

(c) Show that the cartesian equation for C can be written in the form

$$y = \frac{ax + b}{x + 4}, \quad x > -4$$

where a and b are integers to be determined.

(3)

(Total for question = 8 marks)

Q2.

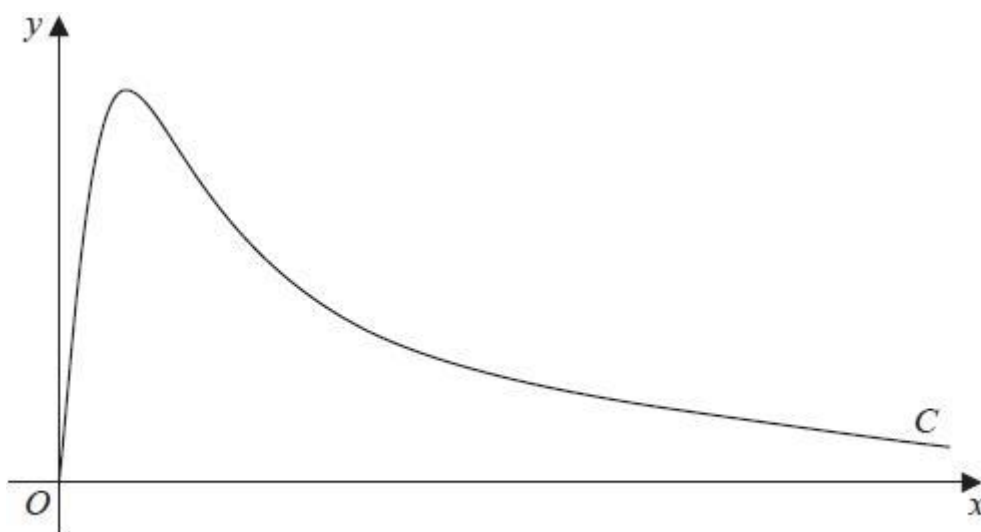


Figure 2

Figure 2 shows a sketch of the curve C with parametric equations

$$x = 4 \tan t, \quad y = 5\sqrt{3} \sin 2t, \quad 0 \leq t < \frac{\pi}{2}$$

The point P lies on C and has coordinates $\left(4\sqrt{3}, \frac{15}{2}\right)$.

- (a) Find the exact value of $\frac{dy}{dx}$ at the point P .
Give your answer as a simplified surd.

(4)

- The point Q lies on the curve C , where $\frac{dy}{dx} = 0$
(b) Find the exact coordinates of the point Q .

(2)

(Total for question = 6 marks)

Q3.

The curve C has parametric equations

$$x = 2 \cos t, \quad y = \sqrt{3} \cos 2t, \quad 0 \leq t \leq \pi$$

- (a) Find an expression for $\frac{dy}{dx}$ in terms of t .

(2)

The point P lies on C where $t = \frac{2\pi}{3}$

The line l is the normal to C at P .

- (b) Show that an equation for l is

$$2x - 2\sqrt{3}y - 1 = 0$$

(5)

The line l intersects the curve C again at the point Q .

- (c) Find the exact coordinates of Q .

You must show clearly how you obtained your answers.

(6)

(Total for question = 13 marks)

Q4.

A curve C has parametric equations

$$x = 2 \sin t, \quad y = 1 - \cos 2t, \quad -\frac{\pi}{2} \leq t \leq \frac{\pi}{2}$$

(a) Find $\frac{dy}{dx}$ at the point where $t = \frac{\pi}{6}$

(4)

(b) Find a cartesian equation for C in the form

$$y = f(x), \quad -k \leq x \leq k,$$

stating the value of the constant k .

(3)

(c) Write down the range of $f(x)$.

(2)

(Total 9 marks)

Q5.

A curve has parametric equations

$$x = \tan^2 t, \quad y = \sin t, \quad 0 < t < \frac{\pi}{2}.$$

(a) Find an expression for $\frac{dy}{dx}$ in terms of t . You need not simplify your answer.

(3)

(b) Find an equation of the tangent to the curve at the point where $t = \frac{\pi}{4}$.

Give your answer in the form $y = ax + b$, where a and b are constants to be determined.

(5)

(c) Find a cartesian equation of the curve in the form $y^2 = f(x)$.

(4)

(Total 12 marks)

Q6.

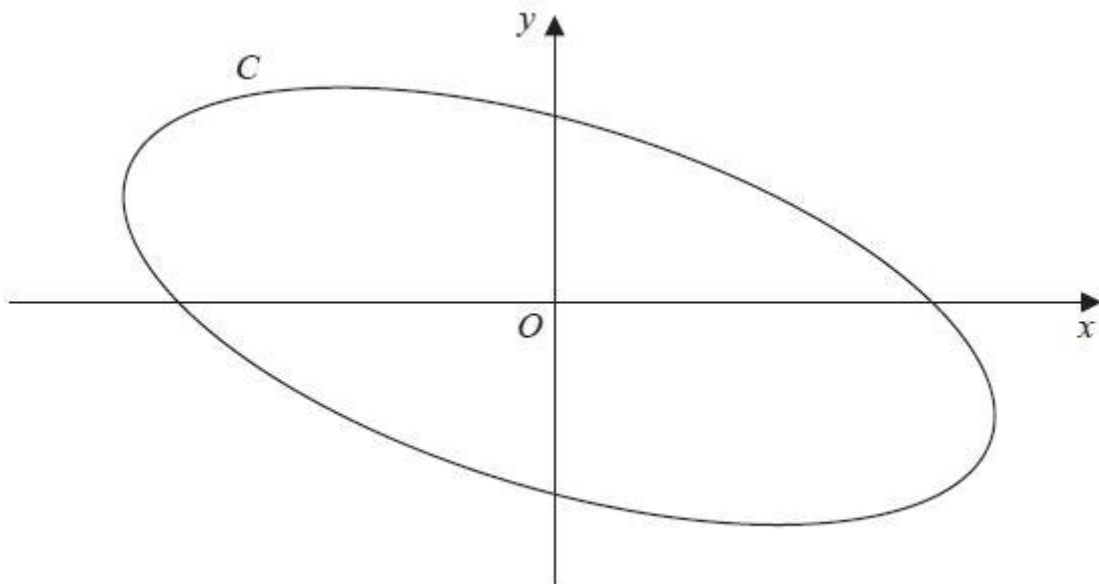


Figure 3

Figure 3 shows a sketch of the curve C with parametric equations

$$x = 4\cos\left(t + \frac{\pi}{6}\right), \quad y = 2\sin t, \quad 0 \leq t < 2\pi$$

(a) Show that

$$x + y = \sqrt{3} \cos t$$

(3)

(b) Show that a cartesian equation of C is

$$(x + y)^2 + ay^2 = b$$

where a and b are integers to be determined.

(2)

(Total 5 marks)

Q7.

A curve C has parametric equations

$$x = \frac{t^2 + 5}{t^2 + 1}, \quad y = \frac{4t}{t^2 + 1}, \quad t \in \mathbb{R}$$

Show that all points on C satisfy

$$(x - 3)^2 + y^2 = 4$$

(Total for question = 3 marks)

Q8.

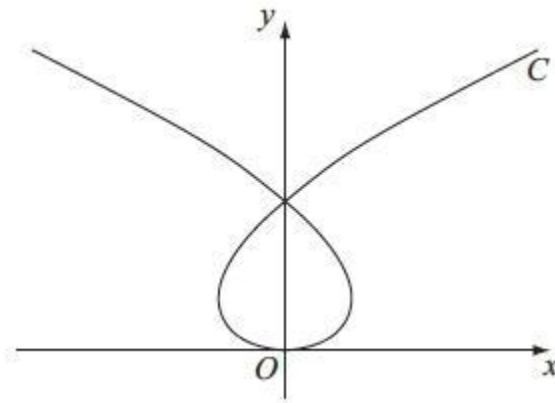


Figure 3

The curve C shown in Figure 3 has parametric equations

$$x = t^3 - 8t, \quad y = t^2$$

where t is a parameter. Given that the point A has parameter $t = -1$,

(a) find the coordinates of A .

(1)

The line l is the tangent to C at A .

(b) Show that an equation for l is $2x - 5y - 9 = 0$.

(5)

The line l also intersects the curve at the point B .

(c) Find the coordinates of B .

(6)

(Total 12 marks)

Q9.

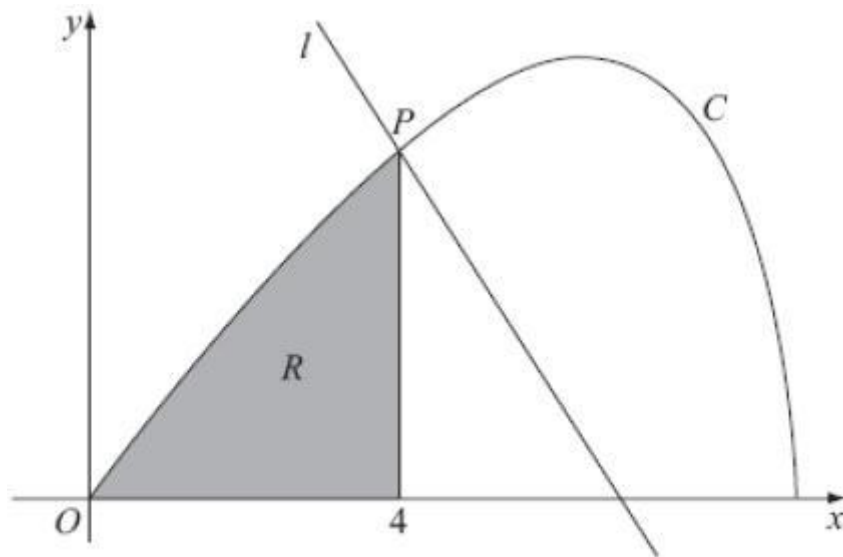


Figure 3

Figure 3 shows the curve C with parametric equations

$$x = 8 \cos t, \quad y = 4 \sin 2t, \quad 0 \leq t \leq \frac{\pi}{2}.$$

The point P lies on C and has coordinates $(4, 2\sqrt{3})$.

(a) Find the value of t at the point P .

(2)

The line l is a normal to C at P .

(b) Show that an equation for l is $y = -x\sqrt{3} + 6\sqrt{3}$.

(6)

The finite region R is enclosed by the curve C , the x -axis and the line $x = 4$, as shown shaded in Figure 3.

$$\int_{\frac{\pi}{3}}^{\frac{\pi}{2}} 64 \sin^2 t \cos t \, dt.$$

(c) Show that the area of R is given by the integral

(4)

(d) Use this integral to find the area of R , giving your answer in the form $a + b\sqrt{3}$, where a and b are constants to be determined.

(4)

(Total 16 marks)

Q10.

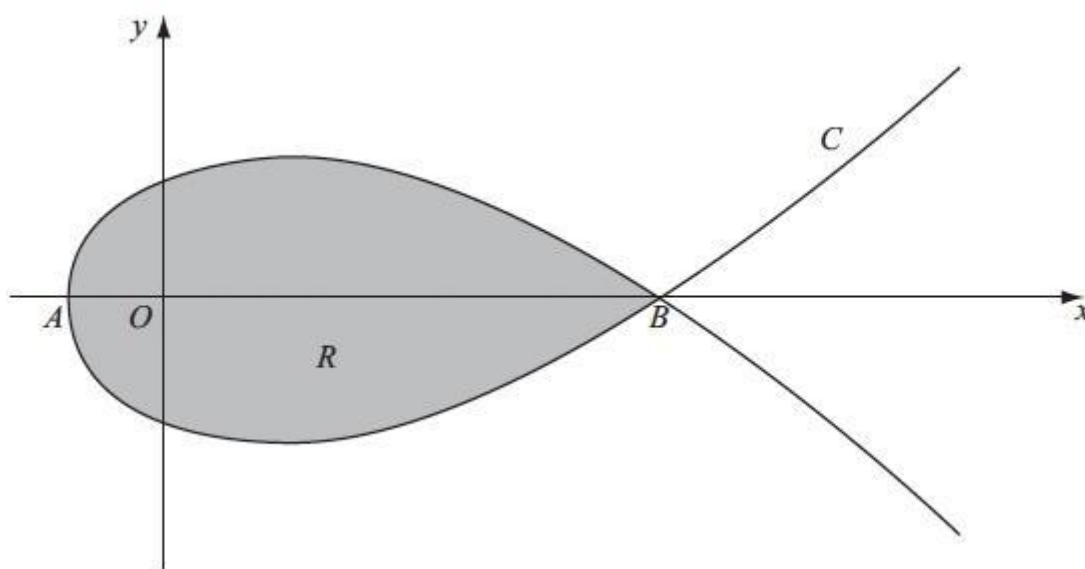


Figure 2

Figure 2 shows a sketch of the curve C with parametric equations

$$x = 5t^2 - 4, \quad y = t(9 - t^2)$$

The curve C cuts the x -axis at the points A and B .

(a) Find the x -coordinate at the point A and the x -coordinate at the point B .

(3)

The region R , as shown shaded in Figure 2, is enclosed by the loop of the curve.

(b) Use integration to find the area of R .

(6)

(Total 9 marks)

Q11.

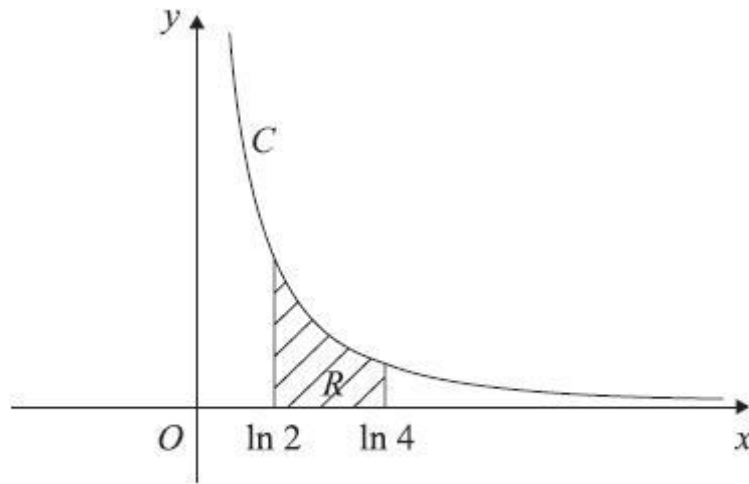


Figure 3

The curve C has parametric equations

$$x = \ln(t+2), \quad y = \frac{1}{(t+1)}, \quad t > -1.$$

The finite region R between the curve C and the x -axis, bounded by the lines with equations $x = \ln 2$ and $x = \ln 4$, is shown shaded in Figure 3.

(a) Show that the area of R is given by the integral

$$\int_0^2 \frac{1}{(t+1)(t+2)} dt.$$

(4)

(b) Hence find an exact value for this area.

(6)

(c) Find a cartesian equation of the curve C , in the form $y = f(x)$.

(4)

(d) State the domain of values for x for this curve.

(1)

(Total 15 marks)

Q12.

A curve C has parametric equations

$$x = 4t + 3, \quad y = 4t + 8 + \frac{5}{2t}, \quad t \neq 0$$

$\frac{dy}{dx}$

(a) Find the value of $\frac{dy}{dx}$ at the point on C where $t = 2$, giving your answer as a fraction in its simplest form.

(3)

(b) Show that the cartesian equation of the curve C can be written in the form

$$y = \frac{x^2 + ax + b}{x - 3}, \quad x \neq 3$$

where a and b are integers to be determined.

(3)

(Total for question = 6 marks)

Q13.

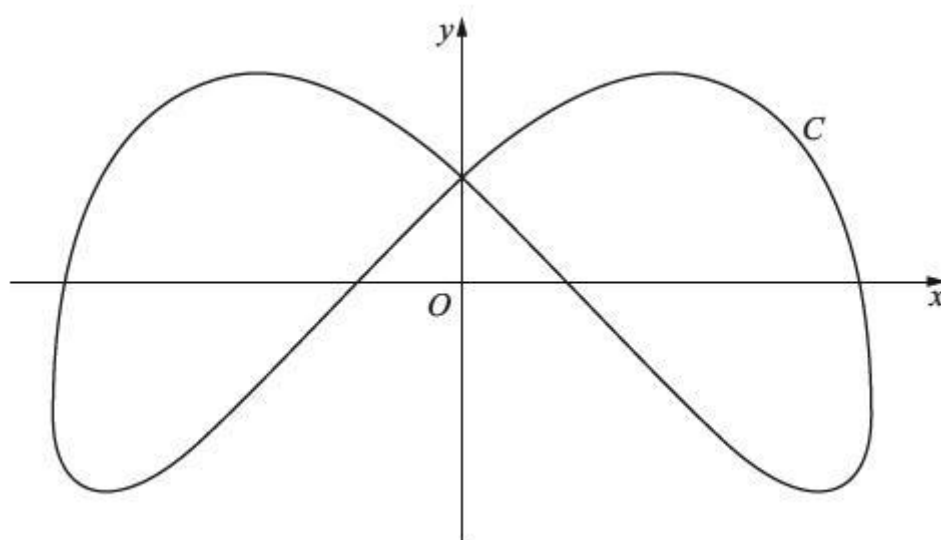


Figure 2

Figure 2 shows a sketch of the curve C with parametric equations

$$x = 4 \sin\left(t + \frac{\pi}{6}\right), \quad y = 3 \cos 2t, \quad 0 \leq t < 2\pi$$

(a) Find an expression for $\frac{dy}{dx}$ in terms of t .

(3)

Find the coordinates of all the points on C where $\frac{dy}{dx} = 0$

(5)

(Total 8 marks)

Q14.

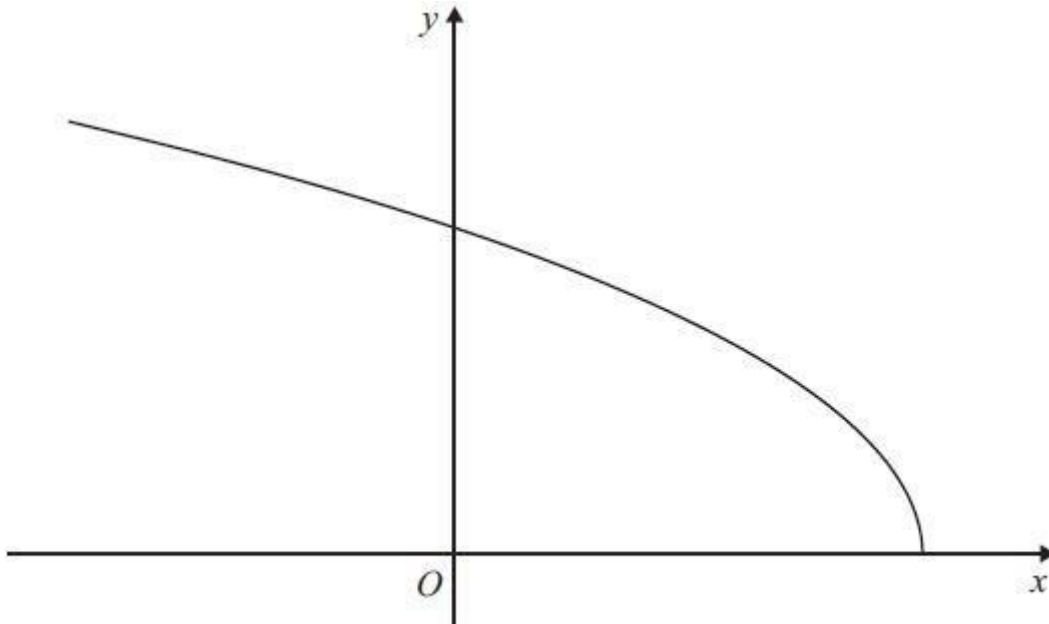


Figure 2

Figure 2 shows a sketch of the curve with parametric equations

$$x = 2 \cos 2t, \quad y = 6 \sin t, \quad 0 \leq t \leq \frac{\pi}{2}$$

- (a) Find the gradient of the curve at the point where $t = \frac{\pi}{3}$.

(4)

- (b) Find a cartesian equation of the curve in the form

$$y = f(x), \quad -k \leq x \leq k,$$

stating the value of the constant k .

(4)

- (c) Write down the range of $f(x)$.

(2)

(Total 10 marks)

Q15.

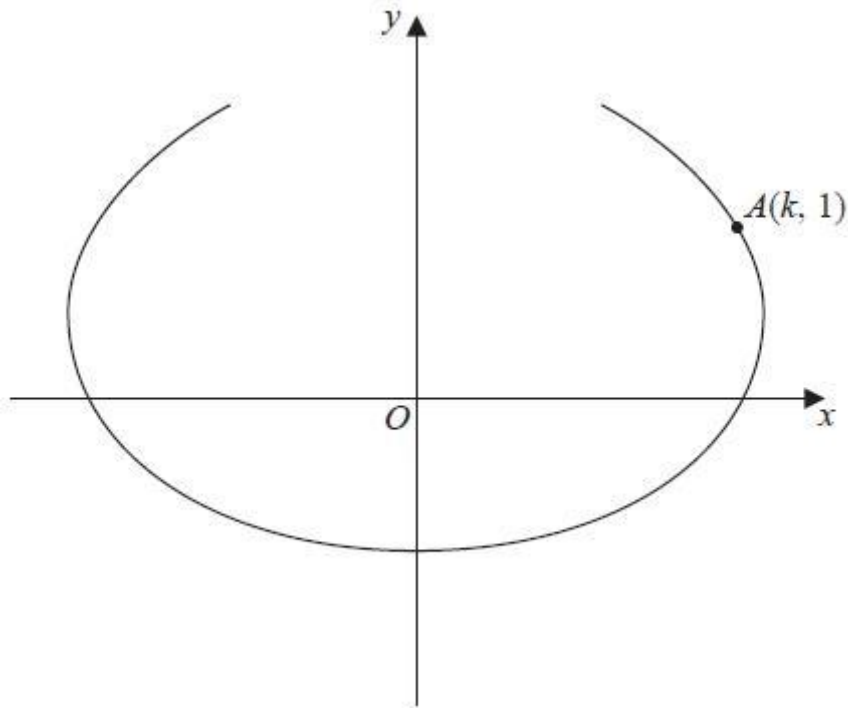


Figure 3

The curve shown in Figure 3 has parametric equations

$$x = t - 4 \sin t, \quad y = 1 - 2 \cos t, \quad -\frac{2\pi}{3} \leq t \leq \frac{2\pi}{3}$$

The point A, with coordinates $(k, 1)$, lies on the curve.

Given that $k > 0$

(a) find the exact value of k ,

(2)

(b) find the gradient of the curve at the point A.

(4)

There is one point on the curve where the gradient is equal to $-\frac{1}{2}$

(c) Find the value of t at this point, showing each step in your working and giving your answer to 4 decimal places.

[Solutions based entirely on graphical or numerical methods are not acceptable.]

(6)

(Total 12 marks)

Q16.

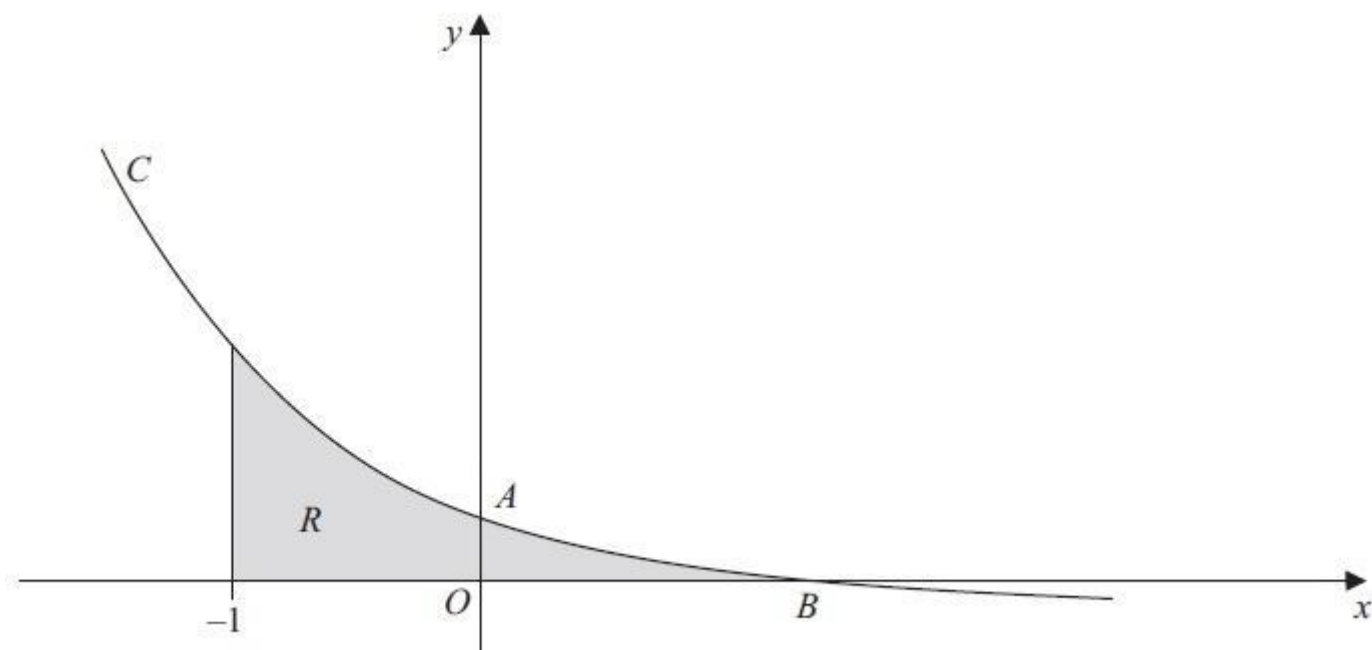


Figure 2

Figure 2 shows a sketch of part of the curve C with parametric equations

$$x = 1 - \frac{1}{2}t, \quad y = 2^t - 1$$

The curve crosses the y -axis at the point A and crosses the x -axis at the point B .

(a) Show that A has coordinates $(0, 3)$.

(2)

(b) Find the x coordinate of the point B .

(2)

(c) Find an equation of the normal to C at the point A .

(5)

The region R , as shown shaded in Figure 2, is bounded by the curve C , the line $x = -1$ and the x -axis.

(d) Use integration to find the exact area of R .

(6)

(Total 15 marks)

Mark Scheme

Q1.

Question Number	Scheme	Notes	Marks
	$x = 3t - 4, y = 5 - \frac{6}{t}, t > 0$		
(a)	$\frac{dx}{dt} = 3, \frac{dy}{dt} = 6t^{-2}$		
	$\frac{dy}{dx} = \frac{6t^{-2}}{3} \left\{ = \frac{6}{3t^2} = 2t^{-2} = \frac{2}{t^2} \right\}$	their $\frac{dy}{dt}$ divided by their $\frac{dx}{dt}$ to give $\frac{dy}{dx}$ in terms of t or their $\frac{dy}{dt}$ multiplied by their $\frac{dt}{dx}$ to give $\frac{dy}{dx}$ in terms of t	M1
		$\frac{6t^{-2}}{3}$, simplified or un-simplified, in terms of t . See note.	A1 isw
	Award Special Case 1 st M1 if both $\frac{dx}{dt}$ and $\frac{dy}{dt}$ are stated correctly and explicitly.		[2]
Note: You can recover the work for part (a) in part (b).			
(a) Way 2	$y = 5 - \frac{18}{x+4} \Rightarrow \frac{dy}{dx} = \frac{18}{(x+4)^2} = \frac{18}{(3t)^2}$	Writes $\frac{dy}{dx}$ in the form $\frac{\pm \lambda}{(x+4)^2}$, and writes $\frac{dy}{dx}$ as a function of t .	M1
		Correct un-simplified or simplified answer, in terms of t . See note.	A1 isw
			[2]
(b)	$\left\{ t = \frac{1}{2} \Rightarrow \right\} P\left(-\frac{5}{2}, -7\right)$	$x = -\frac{5}{2}, y = -7$ or $P\left(-\frac{5}{2}, -7\right)$ seen or implied.	B1
	$\frac{dy}{dx} = \frac{2}{\left(\frac{1}{2}\right)^2}$ and either • $y - "-7" = "8"(x - "-\frac{5}{2}")$ • $"-7" = ("8")("-\frac{5}{2"}) + c$ So, $y = (\text{their } m_t)x + "c"$	Some attempt to substitute $t = 0.5$ into their $\frac{dy}{dx}$ which contains t in order to find m_t and either applies $y - (\text{their } y_p) = (\text{their } m_t)(x - \text{their } x_p)$ or finds c from $(\text{their } y_p) = (\text{their } m_t)(\text{their } x_p) + c$ and uses their numerical c in $y = (\text{their } m_t)x + c$	M1
	T: $y = 8x + 13$	$y = 8x + 13$ or $y = 13 + 8x$	A1 cso
	Note: their x_p , their y_p and their m_t must be numerical values in order to award M1		[3]
(c) Way 1	$\left\{ t = \frac{x+4}{3} \Rightarrow \right\} y = 5 - \frac{6}{\left(\frac{x+4}{3}\right)}$	An attempt to eliminate t . See notes.	M1
		Achieves a correct equation in x and y only	A1 o.e.
	$\Rightarrow y = 5 - \frac{18}{x+4} \Rightarrow y = \frac{2(x+4) - 18}{x+4}$		
	So, $y = \frac{5x+2}{x+4}, \{x > -4\}$	$y = \frac{5x+2}{x+4}$ (or implied equation)	A1 cso
		[3]	
(c) Way 2	$\left\{ t = \frac{6}{5-y} \Rightarrow \right\} x = \frac{18}{5-y} - 4$	An attempt to eliminate t . See notes.	M1
		Achieves a correct equation in x and y only	A1 o.e.
	$\Rightarrow (x+4)(5-y) = 18 \Rightarrow 5x - xy + 20 - 4y = 18$		
	$\left\{ \Rightarrow 5x + 2 = y(x+4) \right\}$ So, $y = \frac{5x+2}{x+4}, \{x > -4\}$	$y = \frac{5x+2}{x+4}$ (or implied equation)	A1 cso
			[3]
Note: Some or all of the work for part (c) can be recovered in part (a) or part (b)			
8			

Question Number	Scheme		Notes	Marks
(c) Way 3	$y = \frac{3at - 4a + b}{3t - 4 + 4} = \frac{3at}{3t} - \frac{4a - b}{3t} = a - \frac{4a - b}{3t} \Rightarrow a = 5$		A full method leading to the value of a being found	M1
			$y = a - \frac{4a - b}{3t}$ and $a = 5$	A1
	$\frac{4a - b}{3} = 6 \Rightarrow b = 4(5) - 6(3) = 2$		Both $a = 5$ and $b = 2$	A1
				[3]
Question Notes				
(a)	Note	Condone $\frac{dy}{dx} = \frac{\left(\frac{6}{t^2}\right)}{3}$ for A1		
	Note	You can ignore subsequent working following on from a correct expression for $\frac{dy}{dx}$ in terms of t .		
(b)	Note	Using a changed gradient (i.e. applying $\frac{-1}{\text{their } \frac{dy}{dx}}$ or $\frac{1}{\text{their } \frac{dy}{dx}}$ or $-\left(\text{their } \frac{dy}{dx}\right)$) is M0.		
	Note	Final A1: A correct solution is required from a correct $\frac{dy}{dx}$.		
	Note	Final A1: You can ignore subsequent working following on from a correct solution.		
(c)	Note	1st M1: A full attempt to eliminate t is defined as either - rearranging one of the parametric equations to make t the subject and substituting for t in the other parametric equation (only the RHS of the equation required for M mark) - rearranging both parametric equations to make t the subject and putting the results equal to each other.		
	Note	Award M1A1 for $\frac{6}{5 - y} = \frac{x + 4}{3}$ or equivalent.		

Q2.

Question Number	Scheme	Notes	Marks
	$x = 4 \tan t, \quad y = 5\sqrt{3} \sin 2t, \quad 0 \leq t < \frac{\pi}{2}$		
(a) Way 1	$\frac{dx}{dt} = 4 \sec^2 t, \quad \frac{dy}{dt} = 10\sqrt{3} \cos 2t$ $\Rightarrow \frac{dy}{dx} = \frac{10\sqrt{3} \cos 2t}{4 \sec^2 t} \left\{ = \frac{5}{2} \sqrt{3} \cos 2t \cos^2 t \right\}$	Either both x and y are differentiated correctly with respect to t or their $\frac{dy}{dt}$ divided by their $\frac{dx}{dt}$ or applies $\frac{dy}{dt}$ multiplied by their $\frac{dt}{dx}$	M1
		Correct $\frac{dy}{dx}$ (Can be implied)	A1 oe
	$\left\{ \text{At } P \left(4\sqrt{3}, \frac{15}{2} \right), t = \frac{\pi}{3} \right\}$		
	$\frac{dy}{dx} = \frac{10\sqrt{3} \cos \left(\frac{2\pi}{3} \right)}{4 \sec^2 \left(\frac{\pi}{3} \right)}$	dependent on the previous M mark Some evidence of substituting $t = \frac{\pi}{3}$ or $t = 60^\circ$ into their $\frac{dy}{dx}$	dM1
	$\frac{dy}{dx} = -\frac{5}{16} \sqrt{3} \text{ or } -\frac{15}{16\sqrt{3}}$	$-\frac{5}{16} \sqrt{3} \text{ or } -\frac{15}{16\sqrt{3}}$ from a correct solution only	A1 cso
[4]			
(b)	$\left\{ 10\sqrt{3} \cos 2t = 0 \Rightarrow t = \frac{\pi}{4} \right\}$		
	So $x = 4 \tan \left(\frac{\pi}{4} \right), y = 5\sqrt{3} \sin \left(2 \left(\frac{\pi}{4} \right) \right)$	At least one of either $x = 4 \tan \left(\frac{\pi}{4} \right)$ or $y = 5\sqrt{3} \sin \left(2 \left(\frac{\pi}{4} \right) \right)$ or $x = 4$ or $y = 5\sqrt{3}$ or $y = \text{awrt } 8.7$	M1
	Coordinates are $(4, 5\sqrt{3})$	$(4, 5\sqrt{3})$ or $x = 4, y = 5\sqrt{3}$	A1
			[2]
6			

Question Notes		
(a)	1 st A1	Correct $\frac{dy}{dx}$. E.g. $\frac{10\sqrt{3} \cos 2t}{4 \sec^2 t}$ or $\frac{5}{2} \sqrt{3} \cos 2t \cos^2 t$ or $\frac{5}{2} \sqrt{3} \cos^2 t (\cos^2 t - \sin^2 t)$ or any equivalent form.
	Note	Give the final A0 for a final answer of $-\frac{10}{32} \sqrt{3}$ without reference to $-\frac{5}{16} \sqrt{3}$ or $-\frac{15}{16\sqrt{3}}$
	Note	Give the final A0 for more than one value stated for $\frac{dy}{dx}$
(b)	Note	Also allow M1 for either $x = 4 \tan(45)$ or $y = 5\sqrt{3} \sin(2(45))$
	Note	M1 can be gained by ignoring previous working in part (a) and/or part (b)
	Note	Give A0 for stating more than one set of coordinates for Q.
	Note	Writing $x = 4, y = 5\sqrt{3}$ followed by $(5\sqrt{3}, 4)$ is A0.

Question Number	Scheme	Notes	Marks
	$x = 4 \tan t, \quad y = 5\sqrt{3} \sin 2t, \quad 0 \leq t < \frac{\pi}{2}$		
(a) Way 2	$\tan t = \frac{x}{4} \Rightarrow \sin t = \frac{x}{\sqrt{x^2+16}}, \cos t = \frac{4}{\sqrt{x^2+16}} \Rightarrow y = \frac{40\sqrt{3}x}{x^2+16}$		
	$\begin{cases} u = 40\sqrt{3}x & v = x^2 + 16 \\ \frac{du}{dx} = 40\sqrt{3} & \frac{dv}{dx} = 2x \end{cases}$		
	$\frac{dy}{dx} = \frac{40\sqrt{3}(x^2+16) - 2x(40\sqrt{3}x)}{(x^2+16)^2} \left\{ = \frac{40\sqrt{3}(16-x^2)}{(x^2+16)^2} \right\}$	$\frac{\pm A(x^2+16) \pm Bx^2}{(x^2+16)^2}$	M1
		Correct $\frac{dy}{dx}$; simplified or un-simplified	A1
	$\frac{dy}{dx} = \frac{40\sqrt{3}(48+16) - 80\sqrt{3}(48)}{(48+16)^2}$	dependent on the previous M mark Some evidence of substituting $x = 4\sqrt{3}$ into their $\frac{dy}{dx}$	dM1
	$\frac{dy}{dx} = -\frac{5}{16}\sqrt{3} \text{ or } -\frac{15}{16\sqrt{3}}$	$-\frac{5}{16}\sqrt{3} \text{ or } -\frac{15}{16\sqrt{3}}$ from a correct solution only	A1 cso
			[4]
(a) Way 3	$y = 5\sqrt{3} \sin\left(2 \tan^{-1}\left(\frac{x}{4}\right)\right)$		
	$\frac{dy}{dx} = 5\sqrt{3} \cos\left(2 \tan^{-1}\left(\frac{x}{4}\right)\right) \left(\frac{2}{1+\left(\frac{x}{4}\right)^2}\right) \left(\frac{1}{4}\right)$	$\frac{dy}{dx} = \pm A \cos\left(2 \tan^{-1}\left(\frac{x}{4}\right)\right) \left(\frac{1}{1+x^2}\right)$	M1
		Correct $\frac{dy}{dx}$; simplified or un-simplified.	A1
	$\frac{dy}{dx} = 5\sqrt{3} \cos\left(2 \tan^{-1}(\sqrt{3})\right) \left(\frac{2}{1+3}\right) \left(\frac{1}{4}\right) \left\{ = 5\sqrt{3} \left(-\frac{1}{2}\right) \left(\frac{1}{2}\right) \left(\frac{1}{4}\right) \right\}$	dependent on the previous M mark Some evidence of substituting $x = 4\sqrt{3}$ into their $\frac{dy}{dx}$	dM1
	$\frac{dy}{dx} = -\frac{5}{16}\sqrt{3} \text{ or } -\frac{15}{16\sqrt{3}}$	$-\frac{5}{16}\sqrt{3} \text{ or } -\frac{15}{16\sqrt{3}}$ from a correct solution only	A1 cso
			[4]

Q3.

Question	Scheme	Marks	AOs
(a)	Attempts $\frac{dy}{dx} = \frac{dy/dt}{dx/dt}$	M1	1.1b
	$\frac{dy}{dx} = \frac{\sqrt{3} \sin 2t}{\sin t} \quad (= 2\sqrt{3} \cos t)$	A1	1.1b
		(2)	
(b)	Substitutes $t = \frac{2\pi}{3}$ in $\frac{dy}{dx} = \frac{\sqrt{3} \sin 2t}{\sin t} = (-\sqrt{3})$	M1	2.1
	Uses gradient of normal $= -\frac{1}{dy/dx} = \left(\frac{1}{\sqrt{3}}\right)$	M1	2.1
	Coordinates of $P = \left(-1, -\frac{\sqrt{3}}{2}\right)$	B1	1.1b
	Correct form of normal $y + \frac{\sqrt{3}}{2} = \frac{1}{\sqrt{3}}(x+1)$	M1	2.1
	Completes proof $\Rightarrow 2x - 2\sqrt{3}y - 1 = 0$ *	A1*	1.1b
		(5)	
(c)	Substitutes $x = 2\cos t$ and $y = \sqrt{3} \cos 2t$ into $2x - 2\sqrt{3}y - 1 = 0$	M1	3.1a
	Uses the identity $\cos 2t = 2\cos^2 t - 1$ to produce a quadratic in $\cos t$	M1	3.1a
	$\Rightarrow 12\cos^2 t - 4\cos t - 5 = 0$	A1	1.1b
	Finds $\cos t = \frac{5}{6}, \cancel{\frac{1}{2}}$	M1	2.4
	Substitutes their $\cos t = \frac{5}{6}$ into $x = 2\cos t, y = \sqrt{3} \cos 2t,$	M1	1.1b
	$Q = \left(\frac{5}{3}, \frac{7}{18}\sqrt{3}\right)$	A1	1.1b
		(6)	
(13 marks)			

Notes:**(a)**

M1: Attempts $\frac{dy}{dx} = \frac{dy/dt}{dx/dt}$ and achieves a form $k \frac{\sin 2t}{\sin t}$ Alternatively candidates may apply the

double angle identity for $\cos 2t$ and achieve a form $k \frac{\sin t \cos t}{\sin t}$

A1: Scored for a correct answer, either $\frac{\sqrt{3} \sin 2t}{\sin t}$ or $2\sqrt{3} \cos t$

(b)

M1: For substituting $t = \frac{2\pi}{3}$ in their $\frac{dy}{dx}$ which must be in terms of t

M1: Uses the gradient of the normal is the negative reciprocal of the value of $\frac{dy}{dx}$. This may be seen in the equation of l .

B1: States or uses (in their tangent or normal) that $P = \left(-1, -\frac{\sqrt{3}}{2}\right)$

M1: Uses their numerical value of $-1/\frac{dy}{dx}$ with their $\left(-1, -\frac{\sqrt{3}}{2}\right)$ to form an equation of the normal at P

A1*: This is a proof and all aspects need to be correct. Correct answer only $2x - 2\sqrt{3}y - 1 = 0$

(c)

M1: For substituting $x = 2\cos t$ and $y = \sqrt{3}\cos 2t$ into $2x - 2\sqrt{3}y - 1 = 0$ to produce an equation in t . Alternatively candidates could use $\cos 2t = 2\cos^2 t - 1$ to set up an equation of the form $y = Ax^2 + B$.

M1: Uses the identity $\cos 2t = 2\cos^2 t - 1$ to produce a quadratic equation in $\cos t$
In the alternative method it is for combining their $y = Ax^2 + B$ with $2x - 2\sqrt{3}y - 1 = 0$ to get an equation in just one variable

A1: For the correct quadratic equation $12\cos^2 t - 4\cos t - 5 = 0$

Alternatively the equations in x and y are $3x^2 - 2x - 5 = 0$ $12\sqrt{3}y^2 + 4y - 7\sqrt{3} = 0$

M1: Solves the quadratic equation in $\cos t$ (or x or y) and rejects the value corresponding to P .

M1: Substitutes their $\cos t = \frac{5}{6}$ or their $t = \arccos\left(\frac{5}{6}\right)$ in $x = 2\cos t$ and $y = \sqrt{3}\cos 2t$

If a value of x or y has been found it is for finding the other coordinate.

A1: $Q = \left(\frac{5}{3}, \frac{7}{18}\sqrt{3}\right)$. Allow $x = \frac{5}{3}, y = \frac{7}{18}\sqrt{3}$ but do not allow decimal equivalents.

Q4.

Question Number	Scheme	Marks
(a)	$x = 2 \sin t, \quad y = 1 - \cos 2t \quad \{ = 2 \sin^2 t \}, \quad -\frac{\pi}{2} \leq t \leq \frac{\pi}{2}$ $\frac{dx}{dt} = 2 \cos t, \quad \frac{dy}{dt} = 2 \sin 2t \quad \text{or} \quad \frac{dy}{dt} = 4 \sin t \cos t$ So, $\frac{dy}{dx} = \frac{2 \sin 2t}{2 \cos t} \left\{ = \frac{4 \cos t \sin t}{2 \cos t} = 2 \sin t \right\}$ At $t = \frac{\pi}{6}, \quad \frac{dy}{dx} = \frac{2 \sin\left(\frac{2\pi}{6}\right)}{2 \cos\left(\frac{\pi}{6}\right)} = 1$	B1 B1 M1; A1 cao cso [4]
(b)	$y = 1 - \cos 2t = 1 - (1 - 2 \sin^2 t)$ $= 2 \sin^2 t$ So, $y = 2 \left(\frac{x}{2}\right)^2 \quad \text{or} \quad y = \frac{x^2}{2} \quad \text{or} \quad y = 2 - 2 \left(1 - \left(\frac{x}{2}\right)^2\right)$ Either $k = 2$ or $-2 \leq x \leq 2$	M1 A1 cso isw B1 [3]
(c)	Range: $0 \leq f(x) \leq 2$ or $0 \leq y \leq 2$ or $0 \leq f \leq 2$	See notes B1 B1 [2]

Notes for Question

(a)	B1: At least one of $\frac{dx}{dt}$ or $\frac{dy}{dt}$ correct. Note: that this mark can be implied from their working. B1: Both $\frac{dx}{dt}$ and $\frac{dy}{dt}$ are correct. Note: that this mark can be implied from their working. M1: Applies their $\frac{dy}{dx}$ divided by their $\frac{dx}{dt}$ and attempts to substitute $t = \frac{\pi}{6}$ into their expression for $\frac{dy}{dx}$. This mark may be implied by their final answer. I.e. $\frac{dy}{dx} = \frac{\sin 2t}{2 \cos t}$ followed by an answer of $\frac{1}{2}$ would be M1 (implied). A1: For an answer of 1 by correct solution only. Note: Don't just look at the answer! A number of candidates are finding $\frac{dy}{dx} = 1$ from incorrect methods. Note: Applying $\frac{dx}{dt}$ divided by their $\frac{dy}{dt}$ is M0, even if they state $\frac{dy}{dx} = \frac{dy}{dt} + \frac{dx}{dt}$. Special Case: Award SC: B0B0M1A1 for $\frac{dx}{dt} = -2 \cos t, \quad \frac{dy}{dt} = -2 \sin 2t$ leading to $\frac{dy}{dx} = \frac{-2 \sin 2t}{-2 \cos t}$ which after substitution of $t = \frac{\pi}{6}$, yields $\frac{dy}{dx} = 1$ Note: It is possible for you to mark part(a), part (b) and part (c) together. Ignore labelling!
-----	--

Notes for Question Continued																																
(b)	M1: Uses the correct double angle formula $\cos 2t = 1 - 2\sin^2 t$ or $\cos 2t = 2\cos^2 t - 1$ or $\cos 2t = \cos^2 t - \sin^2 t$ in an attempt to get y in terms of $\sin^2 t$ or get y in terms of $\cos^2 t$ or get y in terms of $\sin^2 t$ and $\cos^2 t$. Writing down $y = 2\sin^2 t$ is fine for M1. A1: Achieves $y = \frac{x^2}{2}$ or un-simplified equivalents in the form $y = f(x)$. For example: $y = \frac{2x^2}{4} \quad \text{or} \quad y = 2\left(\frac{x}{2}\right)^2 \quad \text{or} \quad y = 2 - 2\left(1 - \left(\frac{x}{2}\right)^2\right) \quad \text{or} \quad y = 1 - \frac{4 - x^2}{4} + \frac{x^2}{4}$ and you can ignore subsequent working if a candidate states a correct version of the Cartesian equation. IMPORTANT: Please check working as this result can be fluked from an incorrect method. Award A0 if there is a $+c$ added to their answer. B1: Either $k = 2$ or a candidate writes down $-2 \leq x \leq 2$. Note: $-2 \leq k \leq 2$ unless k stated as 2 is B0.																															
(c)	Note: The values of 0 and/or 2 need to be evaluated in this part B1: Achieves an inclusive upper or lower limit, using acceptable notation. Eg: $f(x) \geq 0$ or $f(x) \leq 2$ B1: $0 \leq f(x) \leq 2$ or $0 \leq y \leq 2$ or $0 \leq f \leq 2$ Special Case: SC: B1B0 for either $0 < f(x) < 2$ or $0 < f < 2$ or $0 < y < 2$ or $(0, 2)$ Special Case: SC: B1B0 for $0 \leq x \leq 2$. IMPORTANT: Note that: Therefore candidates can use either y or f in place of $f(x)$ Examples: <table border="0"> <tr> <td>$0 \leq x \leq 2$ is SC: B1B0</td> <td>$0 < x < 2$ is B0B0</td> </tr> <tr> <td>$x \geq 0$ is B0B0</td> <td>$x \leq 2$ is B0B0</td> </tr> <tr> <td>$f(x) > 0$ is B0B0</td> <td>$f(x) < 2$ is B0B0</td> </tr> <tr> <td>$x > 0$ is B0B0</td> <td>$x < 2$ is B0B0</td> </tr> <tr> <td>$0 \geq f(x) \geq 2$ is B0B0</td> <td>$0 < f(x) \leq 2$ is B1B0</td> </tr> <tr> <td>$0 \leq f(x) < 2$ is B1B0.</td> <td>$f(x) \geq 0$ is B1B0</td> </tr> <tr> <td>$f(x) \leq 2$ is B1B0</td> <td>$f(x) \geq 0$ and $f(x) \leq 2$ is B1B1. Must state AND {or} $\cap$</td> </tr> <tr> <td>$2 \leq f(x) \leq 2$ is B0B0</td> <td>$f(x) \geq 0$ or $f(x) \leq 2$ is B1B0.</td> </tr> <tr> <td>$f(x) \leq 2$ is B1B0</td> <td>$f(x) \geq 2$ is B0B0</td> </tr> <tr> <td>$1 \leq f(x) \leq 2$ is B1B0</td> <td>$1 < f(x) < 2$ is B0B0</td> </tr> <tr> <td>$0 \leq f(x) \leq 4$ is B1B0</td> <td>$0 < f(x) < 4$ is B0B0</td> </tr> <tr> <td>$0 \leq \text{Range} \leq 2$ is B1B0</td> <td>Range is in between 0 and 2 is B1B0</td> </tr> <tr> <td>$0 < \text{Range} < 2$ is B0B0.</td> <td>Range ≥ 0 is B1B0</td> </tr> <tr> <td>Range ≤ 2 is B1B0</td> <td>Range ≥ 0 and Range ≤ 2 is B1B0.</td> </tr> <tr> <td>$[0, 2]$ is B1B1</td> <td>$(0, 2)$ is SC B1B0</td> </tr> </table>	$0 \leq x \leq 2$ is SC: B1B0	$0 < x < 2$ is B0B0	$x \geq 0$ is B0B0	$x \leq 2$ is B0B0	$f(x) > 0$ is B0B0	$f(x) < 2$ is B0B0	$x > 0$ is B0B0	$x < 2$ is B0B0	$0 \geq f(x) \geq 2$ is B0B0	$0 < f(x) \leq 2$ is B1B0	$0 \leq f(x) < 2$ is B1B0.	$f(x) \geq 0$ is B1B0	$f(x) \leq 2$ is B1B0	$f(x) \geq 0$ and $f(x) \leq 2$ is B1B1. Must state AND {or} $\cap$	$2 \leq f(x) \leq 2$ is B0B0	$f(x) \geq 0$ or $f(x) \leq 2$ is B1B0.	$ f(x) \leq 2$ is B1B0	$ f(x) \geq 2$ is B0B0	$1 \leq f(x) \leq 2$ is B1B0	$1 < f(x) < 2$ is B0B0	$0 \leq f(x) \leq 4$ is B1B0	$0 < f(x) < 4$ is B0B0	$0 \leq \text{Range} \leq 2$ is B1B0	Range is in between 0 and 2 is B1B0	$0 < \text{Range} < 2$ is B0B0.	Range ≥ 0 is B1B0	Range ≤ 2 is B1B0	Range ≥ 0 and Range ≤ 2 is B1B0.	$[0, 2]$ is B1B1	$(0, 2)$ is SC B1B0	
$0 \leq x \leq 2$ is SC: B1B0	$0 < x < 2$ is B0B0																															
$x \geq 0$ is B0B0	$x \leq 2$ is B0B0																															
$f(x) > 0$ is B0B0	$f(x) < 2$ is B0B0																															
$x > 0$ is B0B0	$x < 2$ is B0B0																															
$0 \geq f(x) \geq 2$ is B0B0	$0 < f(x) \leq 2$ is B1B0																															
$0 \leq f(x) < 2$ is B1B0.	$f(x) \geq 0$ is B1B0																															
$f(x) \leq 2$ is B1B0	$f(x) \geq 0$ and $f(x) \leq 2$ is B1B1. Must state AND {or} $\cap$																															
$2 \leq f(x) \leq 2$ is B0B0	$f(x) \geq 0$ or $f(x) \leq 2$ is B1B0.																															
$ f(x) \leq 2$ is B1B0	$ f(x) \geq 2$ is B0B0																															
$1 \leq f(x) \leq 2$ is B1B0	$1 < f(x) < 2$ is B0B0																															
$0 \leq f(x) \leq 4$ is B1B0	$0 < f(x) < 4$ is B0B0																															
$0 \leq \text{Range} \leq 2$ is B1B0	Range is in between 0 and 2 is B1B0																															
$0 < \text{Range} < 2$ is B0B0.	Range ≥ 0 is B1B0																															
Range ≤ 2 is B1B0	Range ≥ 0 and Range ≤ 2 is B1B0.																															
$[0, 2]$ is B1B1	$(0, 2)$ is SC B1B0																															
Aliter (a) Way 2	$\frac{dx}{dt} = 2\cos t, \quad \frac{dy}{dt} = 2\sin 2t,$ $\text{At } t = \frac{\pi}{6}, \quad \frac{dx}{dt} = 2\cos\left(\frac{\pi}{6}\right) = \sqrt{3}, \quad \frac{dy}{dt} = 2\sin\left(\frac{2\pi}{6}\right) = \sqrt{3}$ Hence $\frac{dy}{dx} = 1$	So B1, B1. So implied M1, A1.																														

Notes for Question Continued			
Aliter (a) Way 3	$y = \frac{1}{2}x^2 \Rightarrow \frac{dy}{dx} = x$		B1ft
	Finds $\frac{dy}{dx} = x$, using the correct Cartesian equation only.		B1
	At $t = \frac{\pi}{6}$, $\frac{dy}{dx} = 2 \sin\left(\frac{\pi}{6}\right)$		M1
	$= 1$		A1
Aliter (b) Way 2	$y = 1 - \cos 2t = 1 - (2\cos^2 t - 1)$		M1
	$y = 2 - 2\cos^2 t \Rightarrow \cos^2 t = \frac{2-y}{2} \Rightarrow 1 - \sin^2 t = \frac{2-y}{2}$		
	$1 - \left(\frac{x}{2}\right)^2 = \frac{2-y}{2}$		(Must be in the form $y = f(x)$).
	$y = 2 - 2\left(1 - \left(\frac{x}{2}\right)^2\right)$		A1
Aliter (b) Way 3	$x = 2 \sin t \Rightarrow t = \sin^{-1}\left(\frac{x}{2}\right)$		
	So, $y = 1 - \cos\left(2 \sin^{-1}\left(\frac{x}{2}\right)\right)$		M1
	Rearranges to make t the subject and substitutes the result into y .		
	$y = 1 - \cos\left(2 \sin^{-1}\left(\frac{x}{2}\right)\right)$		A1 oe
Aliter (b) Way 4	$y = 1 - \cos 2t \Rightarrow \cos 2t = 1 - y \Rightarrow t = \frac{1}{2} \cos^{-1}(1 - y)$		
	So, $x = \pm 2 \sin\left(\frac{1}{2} \cos^{-1}(1 - y)\right)$		M1
	Rearranges to make t the subject and substitutes the result into y .		
	So, $y = 1 - \cos\left(2 \sin^{-1}\left(\frac{x}{2}\right)\right)$		A1 oe
Aliter (b) Way 5	$\frac{dy}{dx} = 2 \sin t = x \Rightarrow y = \frac{1}{2}x^2 + c$		M1
	Eg: when eg: $t = 0$ (nb: $-\frac{\pi}{2} \leq t \leq \frac{\pi}{2}$),		
	$x = 0, y = 1 - 1 = 0 \Rightarrow c = 0 \Rightarrow y = \frac{1}{2}x^2$		A1
	Note: $\frac{dy}{dx} = 2 \sin t = x \Rightarrow y = \frac{1}{2}x^2$, with no attempt to find c is M1A0.		

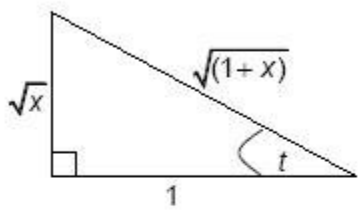
Q5.

Question Number	Scheme	Marks
(a)	$x = \tan^2 t, \quad y = \sin t$ $\frac{dx}{dt} = 2(\tan t)\sec^2 t, \quad \frac{dy}{dt} = \cos t$ $\therefore \frac{dy}{dx} = \frac{\cos t}{2\tan t \sec^2 t} \quad \left(= \frac{\cos^4 t}{2\sin t} \right)$	B1 M1 A1 $\sqrt{\quad}$ [3]
(b)	When $t = \frac{\pi}{4}, \quad x = 1, \quad y = \frac{1}{\sqrt{2}}$ (need values) When $t = \frac{\pi}{4}, \quad m(T) = \frac{dy}{dx} = \frac{\cos \frac{\pi}{4}}{2\tan \frac{\pi}{4} \sec^2 \frac{\pi}{4}}$	B1, B1 [5]
	$= \frac{\frac{1}{\sqrt{2}}}{2.(1)\left(\frac{1}{\sqrt{2}}\right)^2} = \frac{\frac{1}{\sqrt{2}}}{2.(1)\left(\frac{1}{2}\right)} = \frac{\frac{1}{\sqrt{2}}}{2.(1)(2)} = \frac{1}{4\sqrt{2}} = \frac{\sqrt{2}}{8}$ T: $y - \frac{1}{\sqrt{2}} = \frac{1}{4\sqrt{2}}(x - 1)$ T: $y = \frac{1}{4\sqrt{2}}x + \frac{3}{4\sqrt{2}}$ or $y = \frac{\sqrt{2}}{8}x + \frac{3\sqrt{2}}{8}$ or $\frac{1}{\sqrt{2}} = \frac{1}{4\sqrt{2}}(1) + c \Rightarrow c = \frac{1}{\sqrt{2}} - \frac{1}{4\sqrt{2}} = \frac{3}{4\sqrt{2}}$ Hence T: $y = \frac{1}{4\sqrt{2}}x + \frac{3}{4\sqrt{2}}$ or $y = \frac{\sqrt{2}}{8}x + \frac{3\sqrt{2}}{8}$	B1 aef M1 $\sqrt{\quad}$ aef A1 aef cso [5]
Note: The x and y coordinates must be the right way round. A candidate who incorrectly differentiates $\tan^2 t$ to give $\frac{dx}{dt} = 2\sec^2 t$ or $\frac{dx}{dt} = \sec^4 t$ is then able to fluke the correct answer in part (b). Such candidates can potentially get: (a) B0M1A1 $\sqrt{\quad}$ (b) B1B1B1M1A0 cso. Note: cso means "correct solution only". Note: part (a) not fully correct implies candidate can achieve a maximum of 4 out of 5 marks in part (b).		

(c)	$x = \tan^2 t = \frac{\sin^2 t}{\cos^2 t}$	$y = \sin t$		
Way 1	$x = \frac{\sin^2 t}{1 - \sin^2 t}$		Uses $\cos^2 t = 1 - \sin^2 t$	M1
	$x = \frac{y^2}{1 - y^2}$		Eliminates 't' to write an equation involving x and y.	M1
	$x(1 - y^2) = y^2 \Rightarrow x - xy^2 = y^2$			
	$x = y^2 + xy^2 \Rightarrow x = y^2(1 + x)$		Rearranging and factorising with an attempt to make y^2 the subject.	ddM1
	$y^2 = \frac{x}{1+x}$		$\frac{x}{1+x}$	A1
				[4]
Aliter				
(c)	$1 + \cot^2 t = \operatorname{cosec}^2 t$		Uses $1 + \cot^2 t = \operatorname{cosec}^2 t$	M1
Way 2	$= \frac{1}{\sin^2 t}$		Uses $\operatorname{cosec}^2 t = \frac{1}{\sin^2 t}$	M1 implied
	Hence, $1 + \frac{1}{x} = \frac{1}{y^2}$		Eliminates 't' to write an equation involving x and y.	ddM1
	Hence, $y^2 = 1 - \frac{1}{(1+x)}$ or $\frac{x}{1+x}$		$1 - \frac{1}{(1+x)}$ or $\frac{x}{1+x}$	A1
				[4]

$\frac{1}{1+\frac{1}{x}}$ is an acceptable response for the final accuracy A1 mark.

Aliter (c) Way 3	$x = \tan^2 t \quad y = \sin t$ $1 + \tan^2 t = \sec^2 t$ $= \frac{1}{\cos^2 t}$ $= \frac{1}{1 - \sin^2 t}$ Hence, $1 + x = \frac{1}{1 - y^2}$ Hence, $y^2 = 1 - \frac{1}{(1+x)}$ or $\frac{x}{1+x}$	Uses $1 + \tan^2 t = \sec^2 t$ M1 Uses $\sec^2 t = \frac{1}{\cos^2 t}$ M1 Eliminates 't' to write an equation involving x and y. ddM1 $1 - \frac{1}{(1+x)}$ or $\frac{x}{1+x}$ A1 [4]
Aliter (c) Way 4	$y^2 = \sin^2 t = 1 - \cos^2 t$ $= 1 - \frac{1}{\sec^2 t}$ $= 1 - \frac{1}{(1 + \tan^2 t)}$ Hence, $y^2 = 1 - \frac{1}{(1+x)}$ or $\frac{x}{1+x}$	Uses $\sin^2 t = 1 - \cos^2 t$ M1 Uses $\cos^2 t = \frac{1}{\sec^2 t}$ M1 then uses $\sec^2 t = 1 + \tan^2 t$ ddM1 $1 - \frac{1}{(1+x)}$ or $\frac{x}{1+x}$ A1 [4]
$\frac{1}{1 + \frac{1}{x}}$ is an acceptable response for the final accuracy A1 mark.		

Aliter (c) Way 5	$x = \tan^2 t \quad y = \sin t$ $x = \tan^2 t \Rightarrow \tan t = \sqrt{x}$  Hence, $y = \sin t = \frac{\sqrt{x}}{\sqrt{1+x}}$ Hence, $y^2 = \frac{x}{1+x}$ Draws a right-angled triangle and places both $\sqrt{x}$ and 1 on the triangle Uses Pythagoras to deduce the hypotenuse Eliminates 't' to write an equation involving x and y.	M1 M1 ddM1 $\frac{x}{1+x}$ A1 [4] 12 marks
$\frac{1}{1+\frac{1}{x}}$ is an acceptable response for the final accuracy A1 mark. There are so many ways that a candidate can proceed with part (c). If a candidate produces a correct solution then please award all four marks. If they use a method commensurate with the five ways as detailed on the mark scheme then award the marks appropriately. If you are unsure of how to apply the scheme please escalate your response up to your team leader.		

Q6.

Question Number	Scheme	Marks
	$x = 4\cos\left(t + \frac{\pi}{6}\right), \quad y = 2\sin t$	
(a)	Main Scheme $x = 4\left(\cos t \cos\left(\frac{\pi}{6}\right) - \sin t \sin\left(\frac{\pi}{6}\right)\right) \quad \cos\left(t + \frac{\pi}{6}\right) \rightarrow \cos t \cos\left(\frac{\pi}{6}\right) \pm \sin t \sin\left(\frac{\pi}{6}\right)$ So, $\{x + y\} = 4\left(\cos t \cos\left(\frac{\pi}{6}\right) - \sin t \sin\left(\frac{\pi}{6}\right)\right) + 2\sin t$ Adds their expanded x (which is in terms of t) to $2\sin t$ $= 4\left(\left(\frac{\sqrt{3}}{2}\right)\cos t - \left(\frac{1}{2}\right)\sin t\right) + 2\sin t$ $= 2\sqrt{3}\cos t \quad *$ Correct proof	M1 oe dM1 A1 * [3]
(a)	Alternative Method 1 $x = 4\left(\cos t \cos\left(\frac{\pi}{6}\right) - \sin t \sin\left(\frac{\pi}{6}\right)\right) \quad \cos\left(t + \frac{\pi}{6}\right) \rightarrow \cos t \cos\left(\frac{\pi}{6}\right) \pm \sin t \sin\left(\frac{\pi}{6}\right)$ $= 4\left(\left(\frac{\sqrt{3}}{2}\right)\cos t - \left(\frac{1}{2}\right)\sin t\right) = 2\sqrt{3}\cos t - 2\sin t$ So, $x = 2\sqrt{3}\cos t - y$ Forms an equation in x, y and t. $x + y = 2\sqrt{3}\cos t \quad *$ Correct proof	M1 oe dM1 A1 * [3]
(b)	Main Scheme $\left(\frac{x+y}{2\sqrt{3}}\right)^2 + \left(\frac{y}{2}\right)^2 = 1$ $\Rightarrow \frac{(x+y)^2}{12} + \frac{y^2}{4} = 1$ $\Rightarrow (x+y)^2 + 3y^2 = 12$ $(x+y)^2 + 3y^2 = 12$ $\{a = 3, b = 12\}$	M1 A1 [2]
(b)	Alternative Method 1 $(x+y)^2 = 12\cos^2 t = 12(1 - \sin^2 t) = 12 - 12\sin^2 t$ So, $(x+y)^2 = 12 - 3y^2$ Applies $\cos^2 t + \sin^2 t = 1$ to achieve an equation containing only x's and y's. $\Rightarrow (x+y)^2 + 3y^2 = 12$ $(x+y)^2 + 3y^2 = 12$ A1	M1 A1 [2]
(b)	Alternative Method 2 $(x+y)^2 = 12\cos^2 t$ As $12\cos^2 t + 12\sin^2 t = 12$ then $(x+y)^2 + 3y^2 = 12$	M1, A1 [2]
		5

Question Notes		
(a)	M1	$\cos\left(t + \frac{\pi}{6}\right) \rightarrow \cos t \cos\left(\frac{\pi}{6}\right) \pm \sin t \sin\left(\frac{\pi}{6}\right)$ or $\cos\left(t + \frac{\pi}{6}\right) \rightarrow \left(\frac{\sqrt{3}}{2}\right)\cos t \pm \left(\frac{1}{2}\right)\sin t$
	Note	If a candidate states $\cos(A + B) = \cos A \cos B \pm \sin A \sin B$, but there is an error in its application then give M1. <u>Awarding the dM1 mark which is dependent on the first method mark</u>
Main	dM1	Adds their expanded x (which is in terms of t) to $2 \sin t$
	Note	Writing $x + y = \dots$ is not needed in the Main Scheme method.
Alt 1	dM1	Forms an equation in x , y and t .
	A1*	Evidence of $\cos\left(\frac{\pi}{6}\right)$ and $\sin\left(\frac{\pi}{6}\right)$ evaluated and the proof is correct with no errors.
	Note	$\{x + y\} = 4 \cos\left(t + \frac{\pi}{6}\right) + 2 \sin t$, by itself is M0M0A0.
(b)	M1	Applies $\cos^2 t + \sin^2 t = 1$ to achieve an equation containing only x 's and y 's.
	A1	leading $(x + y)^2 + 3y^2 = 12$
	SC	Award Special Case B1B0 for a candidate who writes down either $(x + y)^2 + 3y^2 = 12$ from no working $a = 3, b = 12$, but <u>does not provide a correct proof</u>.
	Note	Alternative method 2 is fine for M1 A1
	Note	Writing $(x + y)^2 = 12 \cos^2 t$ followed by $12 \cos^2 t + a(4 \sin^2 t) = b \Rightarrow a = 3, b = 12$ is SC: B1B0
	Note	Writing $(x + y)^2 = 12 \cos^2 t$ followed by $12 \cos^2 t + a(4 \sin^2 t) = b$ - states $a = 3, b = 12$ - and refers to either $\cos^2 t + \sin^2 t = 1$ or $12 \cos^2 t + 12 \sin^2 t = 12$ - and there is no incorrect working would get M1A1

Q7.

Question	Scheme	Marks	AOs
	$(x-3)^2 + y^2 = \left(\frac{t^2+5}{t^2+1} - 3\right)^2 + \left(\frac{4t}{t^2+1}\right)^2$	M1	3.1a
	$= \frac{(2-2t^2)^2 + 16t^2}{(t^2+1)^2} = \frac{4+8t^2+4t^4}{(t^2+1)^2}$	dM1	1.1b
	$\frac{4(t^4+2t^2+1)}{(t^2+1)^2} = \frac{4(t^2+1)^2}{(t^2+1)^2} = 4^*$	A1*	2.1
		(3)	

M1: Attempts to substitute the given parametric forms into the Cartesian equation or the lhs of the Cartesian equation. There may have been an (incorrect) attempt to multiply out the $(x-3)^2$ term.
 dM1: Attempts to combine (at least the lhs) using correct processing into a single fraction, multiplies out and collects terms on the numerator.
 A1*: Fully correct proof showing all key steps

Question	Scheme	Marks	AOs
Alt	$x = \frac{t^2 + 5}{t^2 + 1} \Rightarrow xt^2 + x = t^2 + 5 \Rightarrow t^2 = \frac{5-x}{x-1}$ $y = \frac{4t}{t^2 + 1} \Rightarrow y^2 = \frac{16t^2}{(t^2 + 1)^2} = \frac{16\left(\frac{5-x}{x-1}\right)}{\left(\frac{5-x}{x-1} + 1\right)^2}$	M1	3.1a
	$y^2 = \frac{16\left(\frac{5-x}{x-1}\right)}{\left(\frac{5-x}{x-1} + 1\right)^2} = 16\left(\frac{5-x}{x-1}\right) \times \left(\frac{(x-1)}{5-x+x-1}\right)^2 \Rightarrow y^2 = (5-x)(x-1)$	dM1	1.1b
	$y^2 = (5-x)(x-1) \Rightarrow y^2 = 6x - x^2 - 5$ $\Rightarrow y^2 = 4 - (x-3)^2 \text{ or other intermediate step}$ $\Rightarrow (x-3)^2 + y^2 = 4^*$	A1*	2.1
		(3)	
(3 marks)			
Notes			

M1: Adopts a correct strategy for eliminating t to obtain an equation in terms of x and y only. See scheme.
 Other methods exist which also lead to an appropriate equation. E.g using $t = \frac{y}{x-1}$
 dM1: Uses correct processing to eliminate the fractions and start to simplify
 A1*: Fully correct proof showing all key steps

Q8.

Question Number	Scheme	Marks
(a)	At A, $x = -1 + 8 = 7$ & $y = (-1)^2 = 1 \Rightarrow A(7,1)$	B1
(b)	$x = t^3 - 8t, \quad y = t^2,$ $\frac{dx}{dt} = 3t^2 - 8, \quad \frac{dy}{dt} = 2t$ $\therefore \frac{dy}{dx} = \frac{2t}{3t^2 - 8}$ At A, $m(T) = \frac{2(-1)}{3(-1)^2 - 8} = \frac{-2}{3-8} = \frac{-2}{-5} = \frac{2}{5}$ T: $y - (\text{their } 1) = m_T(x - (\text{their } 7))$ or $1 = \frac{2}{5}(7) + c \Rightarrow c = 1 - \frac{14}{5} = -\frac{9}{5}$ Hence T: $y = \frac{2}{5}x - \frac{9}{5}$ gives T: $2x - 5y - 9 = 0$ AG	(1) M1 A1 Substitutes for t to give any of the four underlined oe: Finding an equation of a tangent with their point and their tangent gradient or finds c and uses $y = (\text{their gradient})x + "c"$. dM1 A1 cso (5)
(c)	$2(t^3 - 8t) - 5t^2 - 9 = 0$ $2t^3 - 5t^2 - 16t - 9 = 0$ $(t+1)\{(2t^2 - 7t - 9) = 0\}$ $(t+1)\{(t+1)(2t-9) = 0\}$ $\{t = -1 \text{ (at A)}\} \quad t = \frac{9}{2} \text{ at B}$ $x = \left(\frac{9}{2}\right)^2 - 8\left(\frac{9}{2}\right) = \frac{729}{4} - 36 = \frac{441}{4} = 55.125 \text{ or awrt } 55.1$ $y = \left(\frac{9}{2}\right)^2 = \frac{81}{4} = 20.25 \text{ or awrt } 20.3$ Hence B $\left(\frac{441}{4}, \frac{81}{4}\right)$	Substitution of both $x = t^3 - 8t$ and $y = t^2$ into T M1 A realisation that $(t+1)$ is a factor. dM1 A1 Candidate uses their value of t to find either the x or y coordinate One of either x or y correct. Both x and y correct. awrt A1 A1 (6)
		[12]

Q9.

Question Number	Scheme	Marks
(a)	At $P(4, 2\sqrt{3})$ either $4 = 8\cos t$ or $2\sqrt{3} = 4\sin 2t$ $\Rightarrow$ only solution is $t = \frac{\pi}{3}$ where $0 \leq t \leq \frac{\pi}{2}$	M1 A1
(b)	$x = 8\cos t, \quad y = 4\sin 2t$ $\frac{dx}{dt} = -8\sin t, \quad \frac{dy}{dt} = 8\cos 2t$ At $P, \quad \frac{dy}{dx} = \frac{8\cos(\frac{2\pi}{3})}{-8\sin(\frac{\pi}{3})}$ $\left\{ = \frac{8(-\frac{1}{2})}{(-8)(\frac{\sqrt{3}}{2})} = \frac{1}{\sqrt{3}} = \text{awrt } 0.58 \right\}$ Hence $m(N) = -\sqrt{3}$ or $-\frac{1}{\sqrt{3}}$ N: $y - 2\sqrt{3} = -\sqrt{3}(x - 4)$ N: $y = -\sqrt{3}x + 6\sqrt{3}$ (*)	M1 A1 M1 M1 A1 cso (6)
(c)	$A = \int_0^4 y \, dx = \int_{\frac{\pi}{2}}^{\frac{\pi}{4}} 4\sin 2t \cdot (-8\sin t) \, dt$ $A = \int_{\frac{\pi}{2}}^{\frac{\pi}{4}} -32 \sin 2t \cdot \sin t \, dt = \int_{\frac{\pi}{2}}^{\frac{\pi}{4}} -32(2\sin t \cos t) \cdot \sin t \, dt$ $A = \int_{\frac{\pi}{2}}^{\frac{\pi}{4}} -64 \sin^2 t \cos t \, dt$ $A = \int_{\frac{\pi}{2}}^{\frac{\pi}{4}} 64 \sin^2 t \cos t \, dt$ (*)	M1 A1 M1 A1 (4)
(d)	$A = 64 \left[\frac{\sin^3 t}{3} \right]_{\frac{\pi}{2}}^{\frac{\pi}{4}}$ or $A = 64 \left[\frac{u^3}{3} \right]_{\frac{\pi}{2}}^{\frac{\pi}{4}}$ $A = 64 \left[\frac{1}{3} - \left(\frac{1}{3} \cdot \frac{\sqrt{3}}{2} \cdot \frac{\sqrt{3}}{2} \cdot \frac{\sqrt{3}}{2} \right) \right]$ $A = 64 \left(\frac{1}{3} - \frac{1}{8}\sqrt{3} \right) = \frac{64}{3} - 8\sqrt{3}$	M1 A1 M1 A1 (4)
		(16 marks)

Question Number	Scheme	Marks
	(a) $y = 0 \Rightarrow t(9 - t^2) = t(3 - t)(3 + t) = 0$ $t = 0, 3, -3$ Any one correct value B1 At $t = 0$, $x = 5(0)^2 - 4 = -4$ Method for finding one value of x M1 At $t = 3$, $x = 5(3)^2 - 4 = 41$ (At $t = -3$, $x = 5(-3)^2 - 4 = 41$) At A, $x = -4$; at B, $x = 41$ Both A1 (3) (b) $\frac{dx}{dt} = 10t$ Seen or implied B1 $\int y \, dx = \int y \frac{dx}{dt} \, dt = \int t(9 - t^2)10t \, dt$ M1 A1 $= \int (90t^2 - 10t^4) \, dt$ $= \frac{90t^3}{3} - \frac{10t^5}{5} (+C) \quad (= 30t^3 - 2t^5 (+C))$ A1 $\left[\frac{90t^3}{3} - \frac{10t^5}{5} \right]_0^3 = 30 \times 3^3 - 2 \times 3^5 \quad (= 324)$ M1 $A = 2 \int y \, dx = 648 \quad (\text{units}^2)$ A1 (6) [9]	

Q11.

Question Number	Scheme	Marks
(a)	$\left[x = \ln(t+2), y = \frac{1}{t+1} \right], \Rightarrow \frac{dx}{dt} = \frac{1}{t+2}$ $\text{Area}(R) = \int_{\ln 2}^{\ln 4} \frac{1}{t+1} dx = \int_0^2 \left(\frac{1}{t+1} \right) \left(\frac{1}{t+2} \right) dt$ Changing limits, when: $x = \ln 2 \Rightarrow \ln 2 = \ln(t+2) \Rightarrow 2 = t+2 \Rightarrow t = 0$ $x = \ln 4 \Rightarrow \ln 4 = \ln(t+2) \Rightarrow 4 = t+2 \Rightarrow t = 2$ Hence, $\text{Area}(R) = \int_0^2 \frac{1}{(t+1)(t+2)} dt$	Must state $\frac{dx}{dt} = \frac{1}{t+2}$ B1 $\text{Area} = \int \frac{1}{t+1} dx$. M1; Ignore limits. $\int \left(\frac{1}{t+1} \right) \times \left(\frac{1}{t+2} \right) dt$. Ignore limits. A1 AG changes limits $x \rightarrow t$ so that $\ln 2 \rightarrow 0$ and $\ln 4 \rightarrow 2$ B1
(b)	$\left(\frac{1}{(t+1)(t+2)} \right) = \frac{A}{(t+1)} + \frac{B}{(t+2)}$ $1 = A(t+2) + B(t+1)$ Let $t = -1, 1 = A(1) \Rightarrow \underline{A = 1}$ Let $t = -2, 1 = B(-1) \Rightarrow \underline{B = -1}$ $\int_0^2 \frac{1}{(t+1)(t+2)} dt = \int_0^2 \frac{1}{(t+1)} - \frac{1}{(t+2)} dt$ $= [\ln(t+1) - \ln(t+2)]_0^2$ $= (\ln 3 - \ln 4) - (\ln 1 - \ln 2)$ $= \ln 3 - \ln 4 + \ln 2 = \ln 3 - \ln 2 = \ln\left(\frac{3}{2}\right)$	$\frac{A}{(t+1)} + \frac{B}{(t+2)}$ with A and B found M1 Finds both A and B correctly. Can be implied. (See note below) A1 Either $\pm a \ln(t+1)$ or $\pm b \ln(t+2)$ dM1 Both $\ln$ terms correctly ft. A1 ✓ Substitutes <i>both</i> limits of 2 and 0 and subtracts the correct way round. ddM1 $\frac{\ln 3 - \ln 4 + \ln 2}{\text{or } \ln\left(\frac{3}{4}\right) - \ln\left(\frac{1}{2}\right)}$ $\text{or } \underline{\ln 3 - \ln 2} \text{ or } \underline{\ln\left(\frac{3}{2}\right)}$ (must deal with $\ln 1$) A1 aef isw

[4]

[6]

Takes out brackets.

Writing down $\frac{1}{(t+1)(t+2)} = \frac{1}{(t+1)} + \frac{1}{(t+2)}$ means first M1A0 in (b).Writing down $\frac{1}{(t+1)(t+2)} = \frac{1}{(t+1)} - \frac{1}{(t+2)}$ means first M1A1 in (b).

Question Number	Scheme	Marks
(c)	$x = \ln(t+2), \quad y = \frac{1}{t+1}$ $e^x = t+2 \Rightarrow t = e^x - 2$ $y = \frac{1}{e^x - 2 + 1} \Rightarrow y = \frac{1}{e^x - 1}$	Attempt to make $t = \dots$ the subject giving $t = e^x - 2$ M1 Eliminates t by substituting in y giving $y = \frac{1}{e^x - 1}$ A1 dM1 [4]
<i>Aliter</i> (c) Way 2	$t+1 = \frac{1}{y} \Rightarrow t = \frac{1}{y} - 1 \text{ or } t = \frac{1-y}{y}$ $y(t+1) = 1 \Rightarrow yt + y = 1 \Rightarrow yt = 1 - y \Rightarrow t = \frac{1-y}{y}$ $x = \ln\left(\frac{1}{y} - 1 + 2\right) \text{ or } x = \ln\left(\frac{1-y}{y} + 2\right)$ $x = \ln\left(\frac{1}{y} + 1\right)$ $e^x = \frac{1}{y} + 1$ $e^x - 1 = \frac{1}{y}$ $y = \frac{1}{e^x - 1}$	Attempt to make $t = \dots$ the subject M1 Giving either $t = \frac{1}{y} - 1$ or $t = \frac{1-y}{y}$ A1 Eliminates t by substituting in x dM1 giving $y = \frac{1}{e^x - 1}$ A1 [4]
(d)	Domain : $x > 0$	$x > 0$ or just > 0 B1 [1]
		15 marks

Question Number	Scheme	Marks
<i>Aliter</i> (c) Way 3	$e^x = t + 2 \Rightarrow t + 1 = e^x - 1$	Attempt to make $t + 1 = \dots$ the subject giving $t + 1 = e^x - 1$ M1 A1
	$y = \frac{1}{t+1} \Rightarrow y = \frac{1}{e^x - 1}$	Eliminates t by substituting in y giving $y = \frac{1}{e^x - 1}$ dM1 A1 [4]
<i>Aliter</i> (c) Way 4	$t + 1 = \frac{1}{y} \Rightarrow t + 2 = \frac{1}{y} + 1 \text{ or } t + 2 = \frac{1+y}{y}$	Attempt to make $t + 2 = \dots$ the subject Either $t + 2 = \frac{1}{y} + 1$ or $t + 2 = \frac{1+y}{y}$ M1 A1
	$x = \ln\left(\frac{1}{y} + 1\right) \quad \text{or} \quad x = \ln\left(\frac{1+y}{y}\right)$ $x = \ln\left(\frac{1}{y} + 1\right)$ $e^x = \frac{1}{y} + 1 \Rightarrow e^x - 1 = \frac{1}{y}$ $y = \frac{1}{e^x - 1}$	Eliminates t by substituting in x giving $y = \frac{1}{e^x - 1}$ dM1 A1 [4]

Q12.

Question Number	Scheme	Marks
(a)	Note: You can mark parts (a) and (b) together.	
	$x = 4t + 3, y = 4t + 8 + \frac{5}{2t}$	
	$\frac{dx}{dt} = 4, \frac{dy}{dt} = 4 - \frac{5}{2}t^{-2}$ Both $\frac{dx}{dt} = 4$ or $\frac{dt}{dx} = \frac{1}{4}$ and $\frac{dy}{dt} = 4 - \frac{5}{2}t^{-2}$	B1
	So, $\frac{dy}{dx} = \frac{4 - \frac{5}{2}t^{-2}}{4} \left\{ = 1 - \frac{5}{8}t^{-2} = 1 - \frac{5}{8t^2} \right\}$ Candidate's $\frac{dy}{dt}$ divided by a candidate's $\frac{dx}{dt}$	M1 o.e.
	$\left\{ \text{When } t = 2, \right\} \frac{dy}{dx} = \frac{27}{32}$ $\frac{27}{32}$ or 0.84375 cao	A1
		[3]
	Way 2: Cartesian Method	
	$\frac{dy}{dx} = 1 - \frac{10}{(x-3)^2}$ $\frac{dy}{dx} = 1 - \frac{10}{(x-3)^2}$, simplified or un-simplified.	B1
	$\frac{dy}{dx} = \pm \lambda \pm \frac{\mu}{(x-3)^2}, \lambda \neq 0, \mu \neq 0$	M1
	$\left\{ \text{When } t = 2, x = 11 \right\} \frac{dy}{dx} = \frac{27}{32}$ $\frac{27}{32}$ or 0.84375 cao	A1
		[3]
(b)	Way 3: Cartesian Method	
	$\frac{dy}{dx} = \frac{(2x+2)(x-3) - (x^2 + 2x - 5)}{(x-3)^2}$ Correct expression for $\frac{dy}{dx}$, simplified or un-simplified.	B1
	$\left\{ = \frac{x^2 - 6x - 1}{(x-3)^2} \right\}$ $\frac{dy}{dx} = \frac{f'(x)(x-3) - 1f(x)}{(x-3)^2}$, where $f(x) = \text{their } "x^2 + ax + b", g(x) = x - 3$	M1
	$\left\{ \text{When } t = 2, x = 11 \right\} \frac{dy}{dx} = \frac{27}{32}$ $\frac{27}{32}$ or 0.84375 cao	A1
		[3]
	$\left\{ t = \frac{x-3}{4} \Rightarrow \right\} y = 4 \left(\frac{x-3}{4} \right) + 8 + \frac{5}{2 \left(\frac{x-3}{4} \right)}$ Eliminates t to achieve an equation in only x and y	M1
	$y = x - 3 + 8 + \frac{10}{x-3}$	
	$y = \frac{(x-3)(x-3) + 8(x-3) + 10}{x-3}$ or $y(x-3) = (x-3)(x-3) + 8(x-3) + 10$ See notes	dM1
	or $y = \frac{(x+5)(x-3) + 10}{x-3}$ or $y = \frac{(x+5)(x-3)}{x-3} + \frac{10}{x-3}$	
	$\Rightarrow y = \frac{x^2 + 2x - 5}{x-3}, \{a=2 \text{ and } b=-5\}$ Correct algebra leading to $y = \frac{x^2 + 2x - 5}{x-3}$ or $a=2$ and $b=-5$	A1 cso
		[3] 6

Question Number	Scheme	Marks
(b)	Alternative Method 1 of Equating Coefficients $y = \frac{x^2 + ax + b}{x - 3} \Rightarrow y(x - 3) = x^2 + ax + b$ $y(x - 3) = (4t + 3)^2 + 2(4t + 3) - 5 = 16t^2 + 32t + 10$ $x^2 + ax + b = (4t + 3)^2 + a(4t + 3) + b$	
	$(4t + 3)^2 + a(4t + 3) + b = 16t^2 + 32t + 10$	Correct method of obtaining an equation in only t , a and b M1
	$t: 24 + 4a = 32 \Rightarrow a = 2$ constant: $9 + 3a + b = 10 \Rightarrow b = -5$	Equates their coefficients in t and finds both $a = \dots$ and $b = \dots$ dM1
		$a = 2$ and $b = -5$ A1
		[3]
(b)	Alternative Method 2 of Equating Coefficients $\left\{ t = \frac{x - 3}{4} \Rightarrow \right\} y = 4\left(\frac{x - 3}{4}\right) + 8 + \frac{5}{2\left(\frac{x - 3}{4}\right)}$	Eliminates t to achieve an equation in only x and y M1
	$y = x - 3 + 8 + \frac{10}{x - 3} \Rightarrow y = x + 5 + \frac{10}{(x - 3)}$ $y(x - 3) = (x + 5)(x - 3) + 10 \Rightarrow x^2 + ax + b = (x + 5)(x - 3) + 10$	dM1
	$\Rightarrow y = \frac{x^2 + 2x - 5}{x - 3}$ or equating coefficients to give $a = 2$ and $b = -5$	Correct algebra leading to A1 cso
		[3]

Question Notes		
(a)	B1	$\frac{dx}{dt} = 4$ and $\frac{dy}{dt} = 4 - \frac{5}{2}t^{-2}$ or $\frac{dy}{dt} = \frac{8t^2 - 5}{2t^2}$ or $\frac{dy}{dt} = 4 - 5(2t)^{-2}(2)$, etc.
	Note	$\frac{dy}{dt}$ can be simplified or un-simplified.
	Note	You can imply the B1 mark by later working.
	M1	Candidate's $\frac{dy}{dt}$ divided by a candidate's $\frac{dx}{dt}$ or $\frac{dy}{dt}$ multiplied by a candidate's $\frac{dt}{dx}$
(b)	Note	M1 can be also be obtained by substituting $t = 2$ into both their $\frac{dy}{dt}$ and their $\frac{dx}{dt}$ and then dividing their values the correct way round.
	A1	$\frac{27}{32}$ or 0.84375 cao
	M1	Eliminates t to achieve an equation in only x and y .
	dM1	dependent on the first method mark being awarded. Either: (ignoring sign slips or constant slips, noting that k can be 1) - Combining all three parts of their $\underline{x-3} + \bar{8} + \left(\frac{10}{\underline{x-3}}\right)$ to form a single fraction with a common denominator of $\pm k(x-3)$. Accept three separate fractions with the same denominator. - Combining both parts of their $\underline{x+5} + \left(\frac{10}{\underline{x-3}}\right)$, (where $\underline{x+5}$ is their $4\left(\frac{x-3}{4}\right) + 8$), to form a single fraction with a common denominator of $\pm k(x-3)$. Accept two separate fractions with the same denominator. - Multiplies both sides of their $y = \underline{x-3} + \bar{8} + \left(\frac{10}{\underline{x-3}}\right)$ or their $y = \underline{x+5} + \left(\frac{10}{\underline{x-3}}\right)$ by $\pm k(x-3)$. Note that all terms in their equation must be multiplied by $\pm k(x-3)$.
(c)	Note	Condone "invisible" brackets for dM1.
	A1	Correct algebra with no incorrect working leading to $y = \frac{x^2 + 2x - 5}{x - 3}$ or $a = 2$ and $b = -5$
	Note	Some examples for the award of dM1 in (b): dM0 for $y = x - 3 + 8 + \frac{10}{x-3} \rightarrow y = \frac{(x-3)(x-3) + 8 + 10}{x-3}$. Should be $\dots + 8(x-3) + \dots$ dM0 for $y = x - 3 + \frac{10}{x-3} \rightarrow y = \frac{(x-3)(x-3) + 10}{x-3}$. The "8" part has been omitted. dM0 for $y = x + 5 + \frac{10}{x-3} \rightarrow y = \frac{x(x-3) + 5 + 10}{x-3}$. Should be $\dots + 5(x-3) + \dots$ dM0 for $y = x + 5 + \frac{10}{x-3} \rightarrow y(x-3) = x(x-3) + 5(x-3) + 10(x-3)$. Should be just 10.
	Note	$y = x + 5 + \frac{10}{x-3} \rightarrow y = \frac{x^2 + 2x - 5}{x - 3}$ with no intermediate working is dM1A1.

Question Number	Scheme	Marks
(a)	$x = 4 \sin\left(t + \frac{\pi}{6}\right), \quad y = 3 \cos 2t, \quad 0 \leq t < 2\pi$ $\frac{dx}{dt} = 4 \cos\left(t + \frac{\pi}{6}\right), \quad \frac{dy}{dt} = -6 \sin 2t$ So, $\frac{dy}{dx} = \frac{-6 \sin 2t}{4 \cos\left(t + \frac{\pi}{6}\right)}$	B1 B1 B1 $\sqrt{\quad}$ oe [3]
(b)	$\left\{ \frac{dy}{dx} = 0 \Rightarrow \right\} -6 \sin 2t = 0$ @ $t = 0, \quad x = 4 \sin\left(\frac{\pi}{6}\right) = 2, \quad y = 3 \cos 0 = 3 \rightarrow (2, 3)$ @ $t = \frac{\pi}{2}, \quad x = 4 \sin\left(\frac{2\pi}{3}\right) = \frac{4\sqrt{3}}{2}, \quad y = 3 \cos \pi = -3 \rightarrow (2\sqrt{3}, -3)$ @ $t = \pi, \quad x = 4 \sin\left(\frac{7\pi}{6}\right) = -2, \quad y = 3 \cos 2\pi = 3 \rightarrow (-2, 3)$ @ $t = \frac{3\pi}{2}, \quad x = 4 \sin\left(\frac{5\pi}{3}\right) = \frac{4(-\sqrt{3})}{2}, \quad y = 3 \cos 3\pi = -3 \rightarrow (-2\sqrt{3}, -3)$	M1 oe M1 A1A1A1 [5] 8
(a)	B1: Either one of $\frac{dx}{dt} = 4 \cos\left(t + \frac{\pi}{6}\right)$ or $\frac{dy}{dt} = -6 \sin 2t$. They do not have to be simplified. B1: Both $\frac{dx}{dt}$ and $\frac{dy}{dt}$ correct. They do not have to be simplified. Any or both of the first two marks can be implied. Don't worry too much about their notation for the first two B1 marks. B1: Their $\frac{dy}{dt}$ divided by their $\frac{dx}{dt}$ or their $\frac{dy}{dt} \times \frac{1}{\text{their}\left(\frac{dx}{dt}\right)}$. Note: This is a follow through mark. <u>Alternative differentiation in part (a)</u> $x = 2\sqrt{3} \sin t + 2 \cos t \Rightarrow \frac{dx}{dt} = 2\sqrt{3} \cos t - 2 \sin t$ $y = 3(2 \cos^2 t - 1) \Rightarrow \frac{dy}{dt} = 3(-4 \cos t \sin t)$ or $y = 3 \cos^2 t - 3 \sin^2 t \Rightarrow \frac{dy}{dt} = -6 \cos t \sin t - 6 \sin t \cos t$ or $y = 3(1 - 2 \sin^2 t) \Rightarrow \frac{dy}{dt} = 3(-4 \cos t \sin t)$	
(b)	M1: Candidate sets their numerator from part (a) or their $\frac{dy}{dt}$ equal to 0. Note that their numerator must be a trig function. Ignore $\frac{dx}{dt}$ equal to 0 at this stage. M1: Candidate substitutes a found value of t, to attempt to find either one of x or y. The first two method marks can be implied by ONE correct set of coordinates for (x, y) or (y, x) interchanged. A correct point coming from NO WORKING can be awarded M1M1. A1: At least TWO sets of coordinates. A1: At least THREE sets of coordinates. A1: ONLY FOUR correct sets of coordinates. If there are more than 4 sets of coordinates then award A0. Note: Candidate can use the diagram's symmetry to write down some of their coordinates. Note: When $x = 4 \sin\left(\frac{\pi}{6}\right) = 2, \quad y = 3 \cos 0 = 3$ is acceptable for a pair of coordinates. Also it is fine for candidates to display their coordinates on a table of values. Note: The coordinates must be exact for the accuracy marks. I.e. (3.46..., -3) or (-3.46..., -3) is A0. Note: $\frac{dy}{dx} = 0 \Rightarrow \sin t = 0$ ONLY is fine for the first M1, and potentially the following M1A1A0A0. Note: $\frac{dy}{dx} = 0 \Rightarrow \cos t = 0$ ONLY is fine for the first M1 and potentially the following M1A1A0A0. Note: $\frac{dy}{dx} = 0 \Rightarrow \sin t = 0 \& \cos t = 0$ has the potential to achieve all five marks. Note: It is possible for a candidate to gain full marks in part (b) if they make sign errors in part (a). <u>(b) An alternative method for finding the coordinates of the two maximum points.</u> Some candidates may use $y = 3 \cos 2t$ to write down that the y-coordinate of a maximum point is 3. They will then deduce that $t = 0$ or π and proceed to find the x-coordinate of their maximum point. These candidates will receive no credit until they attempt to find one of the x-coordinates for the maximum point. M1M1: Candidate states $y = 3$ and attempts to substitute $t = 0$ or π into $x = 4 \sin\left(t + \frac{\pi}{6}\right)$. M1M1 can be implied by candidate stating either (2, 3) or (2, -3). Note: these marks can only be awarded together for a candidate using this method. A1: For both (2, 3) or (-2, 3). A0A0: Candidate cannot achieve the final two marks by using this method. They can, however, achieve these marks by subsequently solving their numerator equal to 0.	

Q14.

Question Number	Scheme	Marks
Q (a)	$\frac{dx}{dt} = -4 \sin 2t, \quad \frac{dy}{dt} = 6 \cos t$ $\frac{dy}{dx} = -\frac{6 \cos t}{4 \sin 2t} \left(= -\frac{3}{4 \sin t} \right)$ At $t = \frac{\pi}{3}$, $m = -\frac{3}{4 \times \frac{\sqrt{3}}{2}} = -\frac{\sqrt{3}}{2}$ accept equivalents, awrt -0.87	B1, B1 M1 A1 (4)
(b)	Use of $\cos 2t = 1 - 2 \sin^2 t$ $\cos 2t = \frac{x}{2}, \quad \sin t = \frac{y}{6}$ $\frac{x}{2} = 1 - 2 \left(\frac{y}{6} \right)^2$ Leading to $y = \sqrt{(18 - 9x)} \quad (= 3\sqrt{(2 - x)})$ cao $-2 \leq x \leq 2 \quad k = 2$	M1 M1 A1 B1 (4)
(c)	$0 \leq f(x) \leq 6 \quad \text{either } 0 \leq f(x) \text{ or } f(x) \leq 6$ Fully correct. Accept $0 \leq y \leq 6, [0, 6]$	B1 B1 (2)
[10]		
<i>Alternatives to (a) where the parameter is eliminated</i>		
①	$y = (18 - 9x)^{\frac{1}{2}}$ $\frac{dy}{dx} = \frac{1}{2}(18 - 9x)^{-\frac{1}{2}} \times (-9)$ At $t = \frac{\pi}{3}, x = \cos \frac{2\pi}{3} = -1$ $\frac{dy}{dx} = \frac{1}{2} \times \frac{1}{\sqrt{(27)}} \times -9 = -\frac{\sqrt{3}}{2}$	B1 B1 M1 A1 (4)
②	$y^2 = 18 - 9x$ $2y \frac{dy}{dx} = -9$ At $t = \frac{\pi}{3}, y = 6 \sin \frac{\pi}{3} = 3\sqrt{3}$ $\frac{dy}{dx} = -\frac{9}{2 \times 3\sqrt{3}} = -\frac{\sqrt{3}}{2}$	B1 B1 M1 A1 (4)

Q15.

Question Number	Scheme		Marks
(a)	$x = t - 4 \sin t, \quad y = 1 - 2 \cos t, \quad -\frac{2\pi}{3} \leq t \leq \frac{2\pi}{3} \quad A(k, 1) \text{ lies on the curve, } k > 0$ $\{ \text{When } y = 1, \} 1 = 1 - 2 \cos t \Rightarrow t = -\frac{\pi}{2}, \frac{\pi}{2}$ $k \text{ (or } x) = \frac{\pi}{2} - 4 \sin\left(\frac{\pi}{2}\right) \text{ or } x = -\frac{\pi}{2} - 4 \sin\left(-\frac{\pi}{2}\right)$ $\left\{ \text{When } t = -\frac{\pi}{2}, k > 0, \right\} \text{ so } k = 4 - \frac{\pi}{2} \text{ or } \frac{8 - \pi}{2}$	Sets $y = 1$ to find t and uses their t to find x. $x \text{ or } k = 4 - \frac{\pi}{2}$	M1 A1 [2]
(b)	$\frac{dx}{dt} = 1 - 4 \cos t, \quad \frac{dy}{dt} = 2 \sin t$	At least one of $\frac{dx}{dt}$ or $\frac{dy}{dt}$ correct.	B1
		Both $\frac{dx}{dt}$ and $\frac{dy}{dt}$ are correct.	B1
(c)	So, $\frac{dy}{dx} = \frac{2 \sin t}{1 - 4 \cos t}$ At $t = -\frac{\pi}{2}, \frac{dy}{dx} = \frac{2 \sin\left(-\frac{\pi}{2}\right)}{1 - 4 \cos\left(-\frac{\pi}{2}\right)}; = -2$	Applies their $\frac{dy}{dt}$ divided by their $\frac{dx}{dt}$ and substitutes their t into their $\frac{dy}{dx}$. Correct value for $\frac{dy}{dx}$ of -2	M1; A1 cao cso [4]
	$\frac{2 \sin t}{1 - 4 \cos t} = -\frac{1}{2}$ gives $4 \sin t - 4 \cos t = -1$ So $4\sqrt{2} \sin\left(t - \frac{\pi}{4}\right); = -1 \text{ or } -4\sqrt{2} \cos\left(t + \frac{\pi}{4}\right); = -1$ $t = \sin^{-1}\left(\frac{-1}{4\sqrt{2}}\right) + \frac{\pi}{4} \text{ or } t = \cos^{-1}\left(\frac{1}{4\sqrt{2}}\right) - \frac{\pi}{4}$ $t = 0.6076875626... = 0.6077 \text{ (4 dp)}$	Sets their $\frac{dy}{dx} = -\frac{1}{2}$ See notes See notes See notes anything that rounds to 0.6077	M1 A1 M1; A1 dM1 A1
	Question Notes VERY IMPORTANT NOTE FOR PART (c) Candidates who state $t = 0.6077$ with no intermediate working from $4 \sin t - 4 \cos t = -1$ will get 2nd M0, 2nd A0, 3rd M0, 3rd A0. They will not express $4 \sin t - 4 \cos t$ as either $4\sqrt{2} \sin\left(t - \frac{\pi}{4}\right)$ or $-4\sqrt{2} \cos\left(t + \frac{\pi}{4}\right)$. OR use any acceptable alternative method to achieve $t = 0.6077$ NOTE Alternative methods for part (c) are given on the next page.		

	Question	Alternative Methods for Part (c)	
(c)	Alternative Method 1:		
	$\frac{2 \sin t}{1 - 4 \cos t} = -\frac{1}{2}$	Sets their $\frac{dy}{dx} = -\frac{1}{2}$	M1
	eg. $\left(\frac{2 \sin t}{1 - 4 \cos t}\right)^2 = \frac{1}{4}$ or $(4 \sin t)^2 = (4 \cos t - 1)^2$ or $(4 \sin t + 1)^2 = (4 \cos t)^2$ etc.	Squaring to give a correct equation. This mark can be implied by a "squared" correct equation.	A1
		Note: You can also give 1 st A1 in this method for $4 \sin t - 4 \cos t = -1$ as in the main scheme.	
	Squares their equation, applies $\sin^2 t + \cos^2 t = 1$ and achieves a three term quadratic equation of the form $\pm a \cos^2 t \pm b \cos t \pm c = 0$ or $\pm a \sin^2 t \pm b \sin t \pm c = 0$ or eg. $\pm a \cos^2 t \pm b \cos t = \pm c$ where $a \neq 0, b \neq 0$ and $c \neq 0$.		M1
	• Either $32 \cos^2 t - 8 \cos t - 15 = 0$ • or $32 \sin^2 t + 8 \sin t - 15 = 0$	For a correct three term quadratic equation.	A1
	• Either $\cos t = \frac{8 \pm \sqrt{1984}}{64} = \frac{1 \pm \sqrt{31}}{8} \Rightarrow t = \cos^{-1}(\dots)$	which is dependent on the 2 nd M1 mark. Uses correct algebraic processes to give $t = \dots$	dM1
	• or $\sin t = \frac{-8 \pm \sqrt{1984}}{64} = \frac{-1 \pm \sqrt{31}}{8} \Rightarrow t = \sin^{-1}(\dots)$ $t = 0.6076875626\dots = 0.6077$ (4 dp)	anything that rounds to 0.6077	A1
			[6]
(c)	Alternative Method 2:		
	$\frac{2 \sin t}{1 - 4 \cos t} = -\frac{1}{2}$	Sets their $\frac{dy}{dx} = -\frac{1}{2}$	M1
	eg. $(4 \sin t - 4 \cos t)^2 = (-1)^2$	Squaring to give a correct equation. This mark can be implied by a correct equation. Note: You can also give 1 st A1 in this method for $4 \sin t - 4 \cos t = -1$ as in the main scheme.	A1
	So $16 \sin^2 t - 32 \sin t \cos t + 16 \cos^2 t = 1$		
	leading to $16 - 16 \sin 2t = 1$	Squares their equation, applies both $\sin^2 t + \cos^2 t = 1$ and $\sin 2t = 2 \sin t \cos t$ and then achieves an equation of the form $\pm a \pm b \sin 2t = \pm c$	M1
		$16 - 16 \sin 2t = 1$ or equivalent.	A1
	$\left\{ \sin 2t = \frac{15}{16} \Rightarrow \right\} t = \frac{\sin^{-1}(\dots)}{2}$	which is dependent on the 2 nd M1 mark. Uses correct algebraic processes to give $t = \dots$	dM1
	$t = 0.6076875626\dots = 0.6077$ (4 dp)	anything that rounds to 0.6077	A1
			[6]

		Question	Notes
(a)	M1	Sets $y=1$ to find t and uses their t to find x .	
	Note	M1 can be implied by either x or $k = 4 - \frac{\pi}{2}$ or 2.429... or $\frac{\pi}{2} - 4$ or $-2.429...$	
	A1	x or $k = 4 - \frac{\pi}{2}$ or $\frac{8 - \pi}{2}$	
	Note	A decimal answer of 2.429... (without a correct exact answer) is A0.	
	Note	Allow A1 for a candidate using $t = \frac{\pi}{2}$ to find $x = \frac{\pi}{2} - 4$ and then stating that k must be $4 - \frac{\pi}{2}$ o.e.	
(b)	B1	At least one of $\frac{dx}{dt}$ or $\frac{dy}{dt}$ correct. Note: that this mark can be implied from their working.	
	B1	Both $\frac{dx}{dt}$ and $\frac{dy}{dt}$ are correct. Note: that this mark can be implied from their working.	
	M1	Applies their $\frac{dy}{dt}$ divided by their $\frac{dx}{dt}$ and attempts to substitute their t into their expression for $\frac{dy}{dx}$.	
	Note	This mark may be implied by their final answer. i.e. $\frac{dy}{dx} = \frac{2 \sin t}{1 - 4 \cos t}$ followed by an answer of -2 (from $t = -\frac{\pi}{2}$) or 2 (from $t = \frac{\pi}{2}$)	
	Note	Applying $\frac{dx}{dt}$ divided by their $\frac{dy}{dt}$ is M0, even if they state $\frac{dy}{dx} = \frac{dy}{dt} + \frac{dx}{dt}$.	
	A1	Using $t = -\frac{\pi}{2}$ (and not $t = \frac{3\pi}{2}$) to find a correct $\frac{dy}{dx}$ of -2 by correct solution only.	
(c)	NOTE	If a candidate uses an incorrect $\frac{dy}{dx}$ expression in part (c) then the accuracy marks are not obtainable.	
	1 st M1	Sets their $\frac{dy}{dx} = -\frac{1}{2}$	
	1 st A1	Rearranges to give the correct equation with $\sin t$ and $\cos t$ on the same side. eg. $4 \sin t - 4 \cos t = -1$ or $4 \cos t - 4 \sin t = 1$ or $\sin t - \cos t = -\frac{1}{4}$ or $\cos t - \sin t = \frac{1}{4}$ or $4 \sin t - 4 \cos t + 1 = 0$ or $4 \cos t - 4 \sin t - 1 = 0$ or $\sin t - \cos t + \frac{1}{4} = 0$ etc. are fine for A1.	
	2 nd M1	Rewrites $\pm \lambda \sin t \pm \mu \cos t$ in the form of either $R \cos(t \pm \alpha)$ or $R \sin(t \pm \alpha)$ where $R \neq 1$ or 0 and $\alpha \neq 0$	
	2 nd A1	Correct equation. Eg. $4\sqrt{2} \sin\left(t - \frac{\pi}{4}\right) = -1$ or $-4\sqrt{2} \cos\left(t + \frac{\pi}{4}\right) = -1$ or $\sqrt{2} \sin\left(t - \frac{\pi}{4}\right) = -\frac{1}{4}$ or $\sqrt{2} \cos\left(t + \frac{\pi}{4}\right) = \frac{1}{4}$, etc.	
	Note	Unless recovered, give A0 for $4\sqrt{2} \sin(t - 45^\circ) = -1$ or $-4\sqrt{2} \cos(t + 45^\circ) = -1$, etc.	
	3 rd M1	which is dependent on the 2 nd M1 mark. Uses correct algebraic processes to give $t = \dots$	
	4 th A1	anything that rounds to 0.6077	
	Note	Do not give the final A1 mark in (c) if there any extra solutions given in the range $-\frac{2\pi}{3} \leq t \leq \frac{2\pi}{3}$.	
	Note	You can give the final A1 mark in (c) if extra solutions are given outside of $-\frac{2\pi}{3} \leq t \leq \frac{2\pi}{3}$.	

Question Number	Solution	Marks	
	Working parametrically: $t = 1 - \frac{1}{x}$, $y = 2^x - 1$ or $y = e^{2^x - 1}$		
06	$\{y = 0 \Rightarrow 0 = 1 - \frac{1}{x} \Rightarrow x = 1$ When $x = 2$, $y = 2^2 - 1 = 3$	Applies $x = 0$ to obtain a value for y Correct value for y	M1 A1
08	$\{y = 0 \Rightarrow 0 = 2^x - 1 \Rightarrow x = 0$ When $x = 0$, $x = 1 - \frac{1}{2(0) + 1}$	Applies $y = 0$ to obtain a value for x (Must be seen in part (b)) $x = 1$	M1 A1
09	$\frac{dy}{dx} = \frac{1}{x-1}$ and either $\frac{dy}{dx} = 2 \ln 2$ or $\frac{dy}{dx} = e^{2^x - 1} \ln 2$	Although their $\frac{dy}{dx}$ divided by their $\frac{dx}{dt}$	M1 M1
10	At $x = 1 = 2^0$, so $w(T) = -\ln 1 = w(N) = \frac{1}{\ln 2}$ $y = 3 = \frac{1}{\ln 2} - (1 - 0)$ or $y = 3 = \frac{1}{\ln 2} - 1$ or equivalent	Applies $x = 2^0$ and $w(N) = \frac{1}{\ln 2}$ See notes	M1 M1 A1 or A1 or A1
16	Area(R) = $\int_{-1}^2 \left(2^x - 1 \right) - \left(-\frac{1}{2} \right) dx$ $x = -1 \Rightarrow x = 4$ and $x = 1 \Rightarrow x = 4$	Complete calculation for both y and x	M1 M1
	$= \left(\frac{1}{2} \right) \left(\frac{2^x}{\ln 2} - x \right)$	Either $x = \frac{2^x}{\ln 2}$ or $(x - 1) = \frac{(2^x)}{2 \ln 2} - 1$ or $(x - 1) = 2 \ln 2(2^x - 1)$	M1*
	$\left(-\frac{1}{2} \ln 2, -\frac{1}{2} \right) = \left(-\frac{1}{2} \left(\frac{1}{\ln 2} \right), -\frac{1}{2} \left(\ln 2 - 4 \right) \right)$	Depends on the previous method mark. Substitutes their changed limits in x and subtracts either way round	EM1*
	$= \frac{15}{2 \ln 2} - 2$	$\frac{15}{2 \ln 2} - 2$ or equivalent	A1
19			
01	M1: Applies $x = 0$ and obtains a value of f A1: For $y = 2^x - 1 = 0$ or $y = 4 - 1 = 3$ Alternative Solution 1 M1: For substituting $x = 2$ into either x or y A1: $x = 1 - \frac{1}{2(2)} = 0$ or $y = 2^2 - 1 = 3$ Alternative Solution 2 M1: Applies $y = 1$ and obtains a value of f A1: For $x = 1 - \frac{1}{2(2)} = 0$ or $x = 1 - 1 = 0$ Alternative Solution 3 M1: Applies $y = 3$ or $x = 0$ and obtains a value of f A1: Shows that $x = 2$ for both $y = 3$ and $x = 0$		
03	M1: Applies $y = 0$ and obtains a value of f . Working must be seen in part (b) A1: For finding $x = 1$ Note: Award M1A1 for correct.		
05	B1: Both $\frac{dy}{dx}$ and $\frac{dx}{dt}$ correct. This mark can be applied by late marking M1: Then $\frac{dy}{dx}$ divided by their $\frac{dx}{dt}$ or their $\frac{dy}{dx} \times \left(\frac{1}{\text{their } \frac{dx}{dt}} \right)$ Note: their $\frac{dy}{dx}$ must be a function of t		
	M1: Uses their value of f found in part (a) and applies $w(N) = \frac{-1}{\ln 2}$ M1: $y = 3 = \text{(their normal gradient)} \times w = y = \text{(their normal gradient)} \times x = 3$ or equivalent A1: $y = 3 = \frac{1}{\ln 2} - (x - 0)$ or $y = 3 = \frac{1}{\ln 2} - x$ or $y = 3 = \frac{1}{\ln 2} - (x - 0)$ or $(\ln 2)(y - 3) = \ln 2(2^x - 1)$ or $y = 3 = \frac{1}{\ln 2} - x$ You can apply late here		
06	Working in decimals is ok for these method marks. B1: All square root values M1: Complete calculation for both y and x . The candidate should write down $\int (x - 1) \left(\text{their } \frac{dy}{dx} \right)$ B1: Changes limits from $x = 1$, $x = -1 \rightarrow x = 4$ and $x = 1 \rightarrow x = 6$. State $x = 4$ and $x = 6$ seen in B1 M1*: Integrates $\int$ correctly to give $\frac{2^x}{\ln 2}$ or integrates $\int (x - 1)$ to give either $\frac{(x^2)}{2 \ln 2} - x$ or $-2 \ln 2(2^x) - x$		
	A1: Correct integration of $\int (x - 1)$ with respect to x to give $\frac{x^2}{2 \ln 2} - x$ EM1*: Depends upon the previous method mark. Substitutes their limits in x and subtracts either way round		
	A1: Exact answer of $\frac{15}{2 \ln 2} - 2$ or $\frac{15}{2 \ln 2} - 2 = \frac{15 - 2 \ln 2}{2 \ln 2}$ or $\frac{7.5}{\ln 2} - 2$ or $\frac{15}{2} \log_e x = 2$ or equivalent		
07	Alternative: Constructing a Cartesian equation: $\{x = 0 \Rightarrow y = 2^{2^x - 1}$ $y = 3$	Applies $x = 0$ on their Cartesian equation to arrive at a correct answer of 5	M1 A1
08	$\{y = 0 \Rightarrow 0 = 2^{2^x - 1} - 1 \Rightarrow 0 = 2^x - 2 \Rightarrow x = 1$ $x = 1$	Applies $y = 0$ to obtain a value for x (Must be seen in part (b)) $x = 1$	M1 A1
09	$\frac{dy}{dx} = -2(2^{2^x - 1}) \ln 2$ $-2(2^{2^x - 1}) \ln 2$ or equivalent	$\pm 2^{2^x - 1} \ln 2$ $-2(2^{2^x - 1}) \ln 2$ or equivalent	M1 A1
	At $x = 1 = 0$, so $w(T) = -\ln 2 = w(N) = \frac{1}{\ln 2}$ $y = 5 = \frac{1}{\ln 2} - (x - 0)$ or $y = 5 = \frac{1}{\ln 2} - x$	Applies $x = 0$ and $w(N) = \frac{-1}{\ln 2}$ As in the original scheme	M1 A1 or A1
16	Area(R) = $\int_{-1}^2 \left(2^{2^x - 1} \right) dx$ $= \int_{-1}^2 \left(2^{2^x - 1} - 1 \right) dx$ $= \left(\frac{2^{2^x - 1}}{-2 \ln 2} - x \right)$ $= \left(\frac{2^{2^x - 1}}{-2 \ln 2} - 1 \right) = \left(\frac{1}{-2 \ln 2} - 1 \right) = \left(\frac{16}{-2 \ln 2} - 1 \right)$ $= \frac{15}{2 \ln 2} - 2$	For the integral of their Cartesian equation of C For $2^{2^x - 1}$ with limits of $x = 1$ and $x = 1$ or $x = 1$ or $\int_{-1}^2 \left(2^{2^x - 1} \right) dx$ Either $2^{2^x - 1} = \frac{2^{2^x}}{-2 \ln 2}$ or $(2^{2^x - 1}) = \frac{2^{2^x}}{2 \ln 2} - x$ or $(2^{2^x - 1}) = 2 \ln 2(2^{2^x - 1}) - x$ $(2^{2^x - 1}) = \frac{2^{2^x}}{-2 \ln 2} - x$ Depends on the previous method mark. Substitutes limits of x and finds y and subtracts either way round	M1 M1*
	$\left(\frac{2^{2^x - 1}}{-2 \ln 2}, -1 \right) = \left(\frac{1}{-2 \ln 2}, -1 \right) = \left(\frac{16}{-2 \ln 2}, -1 \right)$	Depends on the previous method mark. Substitutes their changed limits in x and subtracts either way round	EM1*
	$= \frac{15}{2 \ln 2} - 2$	$\frac{15}{2 \ln 2} - 2$ or equivalent	A1
19			
01	Alternative method: In C function and applying $w = \frac{1}{\ln 2}$ Area(C) = $\int_{-1}^2 \left(2^x - 1 \right) dx$, where $x = 2 - 2x$ M1: Writes a candidate notes $\int (2^{2^x - 1}) dx$ Then apply the "working parametrically" work scheme		
03	Alternative method: For substitution $u = 2^x$ Area(R) = $\int_{-1}^2 \left(2^x - 1 \right) - \left(-\frac{1}{2} \right) dx$ Complete calculation for both y and x where $u = 2^x = \frac{dx}{du} = 2^x \ln 2 \Rightarrow \frac{dx}{du} = \frac{1}{u \ln 2}$ $x = -1 \Rightarrow x = 4$ and $x = 1 \Rightarrow x = 6$ So area(R) = $\frac{1}{2} \int_{-1}^2 \left(\frac{u}{\ln 2} - 1 \right) du$ $= \frac{1}{2} \left(\frac{u^2}{2 \ln 2} - u \right)$ $= \left(\frac{1}{2} \right) \left(\frac{u}{\ln 2} - \ln u \right)$ $= \left(\frac{1}{2} \right) \left(\frac{u}{\ln 2} - \ln u \right)$ $= \left(\frac{1}{2} \right) \left(\frac{u}{\ln 2} - \ln u \right)$ $= \left(\frac{1}{2} \right) \left(\frac{u}{\ln 2} - \ln u \right)$ $= \left(\frac{1}{2} \right) \left(\frac{u}{\ln 2} - \ln u \right)$ $= \left(\frac{1}{2} \right) \left(\frac{u}{\ln 2} - \ln u \right)$ $= \left(\frac{1}{2} \right) \left(\frac{u}{\ln 2} - \ln u \right)$ $= \left(\frac{1}{2} \right) \left(\frac{u}{\ln 2} - \ln u \right)$ $= \left(\frac{1}{2} \right) \left(\frac{u}{\ln 2} - \ln u \right)$ $= \left(\frac{1}{2} \right) \left(\frac{u}{\ln 2} - \ln u \right)$ $= \left(\frac{1}{2} \right) \left(\frac{u}{\ln 2} - \ln u \right)$ $= \left(\frac{1}{2} \right) \left(\frac{u}{\ln 2} - \ln u \right)$ $= \left(\frac{1}{2} \right) \left(\frac{u}{\ln 2} - \ln u \right)$ $= \left(\frac{1}{2} \right) \left(\frac{u}{\ln 2} - \ln u \right)$ $= \left(\frac{1}{2} \right) \left(\frac{u}{\ln 2} - \ln u \right)$ $= \left(\frac{1}{2} \right) \left(\frac{u}{\ln 2} - \ln u \right)$ $= \left(\frac{1}{2} \right) \left(\frac{u}{\ln 2} - \ln u \right)$ $= \left(\frac{1}{2} \right) \left(\frac{u}{\ln 2} - \ln u \right)$ $= \left(\frac{1}{2} \right) \left(\frac{u}{\ln 2} - \ln u \right)$ $= \left(\frac{1}{2} \right) \left(\frac{u}{\ln 2} - \ln u \right)$ $= \left(\frac{1}{2} \right) \left(\frac{u}{\ln 2} - \ln u \right)$ $= \left(\frac{1}{2} \right) \left(\frac{u}{\ln 2} - \ln u \right)$ $= \left(\frac{1}{2} \right) \left(\frac{u}{\ln 2} - \ln u \right)$ $= \left(\frac{1}{2} \right) \left(\frac{u}{\ln 2} - \ln u \right)$ $= \left(\frac{1}{2} \right) \left(\frac{u}{\ln 2} - \ln u \right)$ $= \left(\frac{1}{2} \right) \left(\frac{u}{\ln 2} - \ln u \right)$ $= \left(\frac{1}{2} \right) \left(\frac{u}{\ln 2} - \ln u \right)$ $= \left(\frac{1}{2} \right) \left(\frac{u}{\ln 2} - \ln u \right)$ $= \left(\frac{1}{2} \right) \left(\frac{u}{\ln 2} - \ln u \right)$ $= \left(\frac{1}{2} \right) \left(\frac{u}{\ln 2} - \ln u \right)$ $= \left(\frac{1}{2} \right) \left(\frac{u}{\ln 2} - \ln u \right)$ $= \left(\frac{1}{2} \right) \left(\frac{u}{\ln 2} - \ln u \right)$ $= \left(\frac{1}{2} \right) \left(\frac{u}{\ln 2} - \ln u \right)$ $= \left(\frac{1}{2} \right) \left(\frac{u}{\ln 2} - \ln u \right)$ $= \left(\frac{1}{2} \right) \left(\frac{u}{\ln 2} - \ln u \right)$ $= \left(\frac{1}{2} \right) \left(\frac{u}{\ln 2} - \ln u \right)$ $= \left(\frac{1}{2} \right) \left(\frac{u}{\ln 2} - \ln u \right)$ $= \left(\frac{1}{2} \right) \left(\frac{u}{\ln 2} - \ln u \right)$ $= \left(\frac{1}{2} \right) \left(\frac{u}{\ln 2} - \ln u \right)$ $= \left(\frac{1}{2} \right) \left(\frac{u}{\ln 2} - \ln u \right)$ $= \left(\frac{1}{2} \right) \left(\frac{u}{\ln 2} - \ln u \right)$ $= \left(\frac{1}{2} \right) \left(\frac{u}{\ln 2} - \ln u \right)$ $= \left(\frac{1}{2} \right) \left(\frac{u}{\ln 2} - \ln u \right)$ $= \left(\frac{1}{2} \right) \left(\frac{u}{\ln 2} - \ln u \right)$ $= \left(\frac{1}{2} \right) \left(\frac{u}{\ln 2} - \ln u \right)$ $= \left(\frac{1}{2} \right) \left(\frac{u}{\ln 2} - \ln u \right)$ $= \left(\frac{1}{2} \right) \left(\frac{u}{\ln 2} - \ln u \right)$ $= \left(\frac{1}{2} \right) \left(\frac{u}{\ln 2} - \ln u \right)$ $= \left(\frac{1}{2} \right) \left(\frac{u}{\ln 2} - \ln u \right)$ $= \left(\frac{1}{2} \right) \left(\frac{u}{\ln 2} - \ln u \right)$ $= \left(\frac{1}{2} \right) \left(\frac{u}{\ln 2} - \ln u \right)$ $= \left(\frac{1}{2} \right) \left(\frac{u}{\ln 2} - \ln u \right)$ $= \left(\frac{1}{2} \right) \left(\frac{u}{\ln 2} - \ln u \right)$ $= \left(\frac{1}{2} \right) \left(\frac{u}{\ln 2} - \ln u \right)$ $= \left(\frac{1}{2} \right) \left(\frac{u}{\ln 2} - \ln u \right)$ $= \left(\frac{1}{2} \right) \left(\frac{u}{\ln 2} - \ln u \right)$ $= \left(\frac{1}{2} \right) \left(\frac{u}{\ln 2} - \ln u \right)$ $= \left(\frac{1}{2} \right) \left(\frac{u}{\ln 2} - \ln u \right)$ $= \left(\frac{1$		

