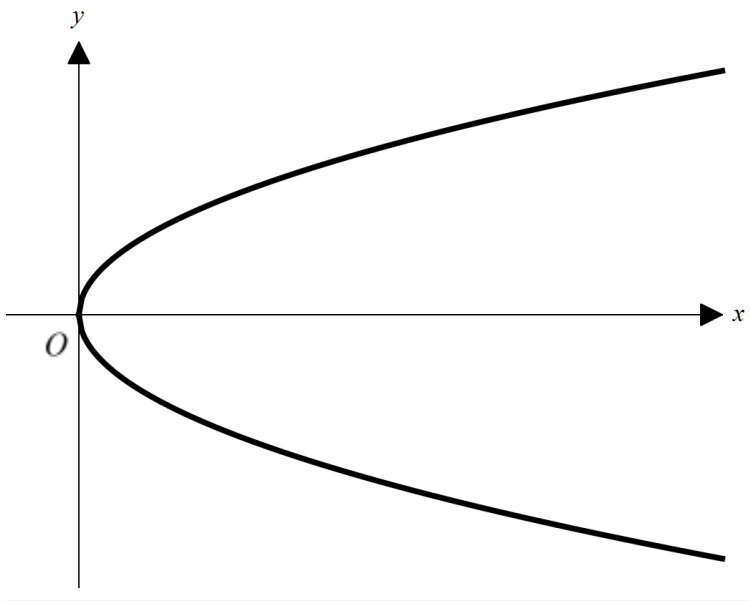
	Solutions
1	$\cot 75° = \dfrac{1-\tan 45° \tan 30°}{\tan 45° + \tan 30°}$ $= \dfrac{1-1\times\frac{\sqrt{3}}{3}}{1+\frac{\sqrt{3}}{3}}$ $= \dfrac{3-\sqrt{3}}{3+\sqrt{3}} \times \dfrac{3-\sqrt{3}}{3-\sqrt{3}}$ $= 2-\sqrt{3}$ $\cos ec^2 75° = 1+\left(2-\sqrt{3}\right)^2$ $= 1+4+3-4\sqrt{3}$ $= 8-4\sqrt{3}$
	Alternative Solution 1: $\sin 75° = \sin 45° \cos 30° + \cos 45° \sin 30°$ $= \dfrac{\sqrt{2}}{2}\times\dfrac{\sqrt{3}}{2} + \dfrac{\sqrt{2}}{2}\times\dfrac{1}{2}$ $= \dfrac{\sqrt{6}+\sqrt{2}}{4}$ $\cos ec^2 75° = \dfrac{16}{\left(\sqrt{6}+\sqrt{2}\right)^2}$ $= \dfrac{16}{8+4\sqrt{3}}$ $= \dfrac{4}{2+\sqrt{3}}\times\dfrac{2-\sqrt{3}}{2-\sqrt{3}}$ $= 8-4\sqrt{3}$
	Alternative Solution 2:

$$\cos ec^2 75° = \frac{1}{\sin^2 75°}$$
$$= \frac{2}{1-\cos 150°}$$
$$= \frac{2}{1+\frac{\sqrt{3}}{2}}$$
$$= \frac{4}{2+\sqrt{3}} \times \frac{2-\sqrt{3}}{2-\sqrt{3}}$$
$$= 8-4\sqrt{3}$$

Total Marks: 4m

2 **(i)**



(ii)

$y^2 = 9x$ ----(1)

$y = kx^{-\frac{3}{2}}$ ----(2)

Equating equations (1) and (2) gives,

$$9x = \left(kx^{-\frac{3}{2}}\right)^2$$

Since the curves meet at a point $x = 4$.

$36 = k^2 \times 4^{-3}$

$k = \pm 48$

Alternatively,

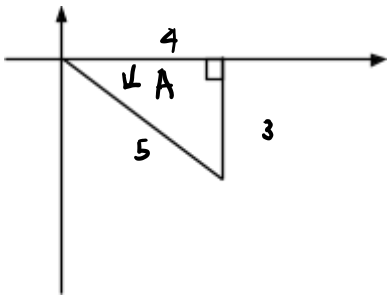
From (1), $y = \pm 3x^{\frac{1}{2}}$

$3\sqrt{x} = kx^{-\frac{3}{2}}$ or $-3\sqrt{x} = kx^{-\frac{3}{2}}$

		Since the curves meet at a point $x = 4$. $12 = k \times 4^{-\frac{3}{2}}$ $\quad -12 = k \times 4^{-\frac{3}{2}}$ $k = 48$ or $k = -48$
		Total Marks: 4m

3

$$\tan A = -\frac{3}{4}$$



A is in the 4th quadrant.

$$\sin\left(A-\frac{\pi}{2}\right) = \sin\left[-\left(\frac{\pi}{2}-A\right)\right]$$
$$= -\sin\left(\frac{\pi}{2}-A\right)$$
$$= -\cos A$$
$$= -\frac{4}{5}$$

Alternatively,

$$\sin\left(A-\frac{\pi}{2}\right) = \sin\left[-\left(\frac{\pi}{2}-A\right)\right]$$
$$= -\sin\left(\frac{\pi}{2}-A\right)$$
$$= -\left(\sin\frac{\pi}{2}\cos A - \cos\frac{\pi}{2}\sin A\right)$$
$$= -\left(1\times\frac{4}{5} - 0\times-\frac{3}{5}\right)$$
$$= -\frac{4}{5}$$

Alternatively,

Total Marks: 3m

4

For the equation $x^2-(k+1)x+1=0$ to have 2 distinct and real roots p and q,

$$(k+1)^2 - 4\times1 > 0$$

$$k^2+2k-3>0$$

$$(k+3)(k-1)>0$$



$$k<-3 \text{ or } k>1$$

Total Marks: 4m

5 **(i)**

$f'(x) = \frac{1}{x-2} + e^{-2x}$ for $x > a$

$\int \frac{1}{x-2} + e^{-2x}\, dx = \ln(x-2) - \frac{e^{-2x}}{2} + c$, where c is an arbitrary constant

At $\left(3, -\frac{1}{e^6}\right)$,

$-e^{-6} = \ln(3-2) - \frac{e^{-2\times 3}}{2} + c$

$-e^{-6} = -\frac{e^{-6}}{2} + c$

$c = -\frac{e^{-6}}{2}$ or -0.00124

$\therefore f(x) = \ln(x-2) - \frac{1}{2e^{2x}} - \frac{1}{2e^6}$ or

$\therefore f(x) = \ln(x-2) - \frac{1}{2e^{2x}} - 0.00124$

(ii)

$f'(x) = \frac{1}{x-2} + e^{-2x}$ for $x > a$

$f''(x) = -\frac{1}{(x-2)^2} - 2e^{-2x}$

Since $(x-2)^2 > 0$ and $e^{-2x} > 0$ for $x > 2$

$f''(x) = -\frac{1}{(x-2)^2} - 2e^{-2x} < 0$

Thus, the function $f'(x) = \frac{1}{x-2} + e^{-2x}$ is decreasing.

Total Marks: 7m

6

For the curve $y = mx^2 + (m+n)x + n$ to lie entirely above the x-axis, it would not have any x-intersections.

$b^2 - 4ac = (m+n)^2 - 4mn$

$= m^2 + 2mn + n^2 - 4mn$

$= m^2 - 2mn + n^2$

$= (m-n)^2$

	Since $(m-n)^2 \geq 0$ for all values of m and n, the curve would intersect with the x-axis. The curve will not lie completely above the x-axis.	
		Total Marks: 3m
7	**(a)**	Let $f(x) = 2x^3 - 3x^2 - 3x + 4$ $f(1) = 2(1)^3 - 3(1)^2 - 3(1) + 4 = 0$ By Factor Theorem, $(x-1)$ is a factor of $f(x)$. Let $2x^3 - 3x^2 - 3x + 4 = (x-1)(2x^2 + bx - 4)$ Compare the coefficient of x $-3 = -4 - b$ $b = -1$ $f(x) = (x-1)(2x^2 - x - 4)$ $(x-1)(2x^2 - x - 4) = 0$ $x = 1,$ or $x = \frac{1 \pm \sqrt{1+32}}{4}$ $\therefore x = 1, x = 1.69$ or -1.19
7	**(b)**	By Factor Theorem, $p(5) + 1 = 0$ $p(5) = -1$ By Remainder Theorem, $g(5) = 2(5)^3 - p(5) + 5$ $=256$ **Alternatively,** $g(x) = 2(x)^3 + 6 - p(x) - 1$ $g(5) = 2(5)^3 + 6 - p(5) - 1$ $=256$
		Total Marks: 6m

8

$\frac{1}{\alpha}\times\frac{1}{\beta}=\frac{1}{4}---(1)$

$\frac{1}{\alpha}+\frac{1}{\beta}=\frac{1}{2}$

$\frac{\alpha+\beta}{\alpha\beta}=\frac{1}{2}$

From(1), $\alpha\beta=4$

$\frac{1}{\alpha}+\frac{1}{\beta}=\frac{1}{2}$

$\frac{\alpha+\beta}{\alpha\beta}=\frac{1}{2}$

$\alpha+\beta=2$

Sum of roots is

$\frac{\beta}{\alpha^2}+\frac{\alpha}{\beta^2}$

$=\frac{\alpha^3+\beta^3}{(\alpha\beta)^2}$

$=\frac{(\alpha+\beta)\left[(\alpha+\beta)^2-3\alpha\beta\right]}{(\alpha\beta)^2}$

$=\frac{2\left[(2)^2-3(4)\right]}{4^2}$

$=-1$

Product of roots is

$\frac{\beta}{\alpha^2}\times\frac{\alpha}{\beta^2}=\frac{1}{\alpha\beta}$

$=\frac{1}{4}$

The quadratic equation $x^2+x+\frac{1}{4}=0$ or $4x^2+4x+1=0$

Total Marks: 5m

9	**(i)**	$\frac{d}{dx}\left(\frac{2\ln x}{x^3}\right)=\frac{x^3\left(\frac{2}{x}\right)-2\ln x\left(3x^2\right)}{x^6}$ $=\frac{2x^2-6x^2\ln x}{x^6}$ $=\frac{2x^2}{x^6}-\frac{6x^2\ln x}{x^6}$ $=\frac{2}{x^4}-\frac{6\ln x}{x^4}$
	(ii)	$\int\frac{2}{x^4}-\frac{6\ln x}{x^4}dx=\frac{2\ln x}{x^3}+c$ where c is an arbitrary constant $\int\frac{2}{x^4}\,dx-6\int\frac{\ln x}{x^4}dx=\frac{2\ln x}{x^3}+c$ $6\int\frac{\ln x}{x^4}dx=\int\frac{2}{x^4}\,dx-\frac{2\ln x}{x^3}+c_1$ where c_1 is an arbitrary constant $=-\frac{2}{3x^3}-\frac{2\ln x}{x^3}+c_2,$ where c_2 is an arbitrary constant $\int\frac{\ln x}{x^4}dx=-\frac{1}{9x^3}-\frac{\ln x}{3x^3}+c_3,$ where c_3 is an arbitrary constant or $=-\frac{1+3\ln x}{9x^3}+c_3$
		Total Marks: 6m

10 **(i)**

$$y = \frac{32}{(4-x)^2} - 8$$

$$\frac{dy}{dx} = \frac{64}{(4-x)^3}$$

when $x = 8$, $\frac{dy}{dx} = -1 \Rightarrow$ gradient $= -1$

When $x = 8$,

$$y = \frac{32}{(4-8)^2} - 8 = -6$$

$\therefore (8, -6)$

Equation of tangent

$$y + 6 = -(x - 8)$$

$$y = -x + 2$$

(ii)

When $y = 0$,

$$0 = \frac{32}{(4-x)^2} - 8$$

$x = 2$ or 6

$\therefore A(2,0)$ and $B(6,0)$

Area of shaded region

$$= \frac{1}{2} \times (8-2) \times 6 - \left| \int_6^8 \frac{32}{(4-x)^2} - 8 \, dx \right|$$

$$= 18 - \left| \left[\frac{32}{(4-x)} - 8x \right]_6^8 \right|$$

$$= 18 - \left| \left(\frac{32}{(4-8)} - 64 \right) - \left(\frac{32}{(4-6)} - 48 \right) \right|$$

$$= 18 - \left| -72 + 64 \right| - (2) + 2\frac{5}{8}$$

$$= 10 \text{ unit}^2$$

Total Marks: 9m

11 **(i)**

x	1	1.22	1.29	1.32	1.34
y	1	2	3	4	5
Y = x^2y	1	3.00	4.99	6.97	8.98
X=y	1	2	3	4	5

(ii)

$$y = \frac{1}{ax^2 + b}$$

$$\left(ax^2 + b\right)y = 1$$

$$ax^2y + by = 1$$

$$x^2y = \frac{1}{a} - \frac{b}{a}y$$

$$\frac{1}{a} = -1 \pm 0.1$$

$a = -1$ (accept -1.11 to -0.909)

$$-\frac{b}{a} = \text{Gradient}$$

$$= \frac{8-2}{4.5-1.5}$$

$$= 2 \pm 0.1$$

$$b = a \times gradient$$

(iii)

$$2.5 = \frac{1}{ax^2 + b}$$

Draw the vertical line $X = 2.5$

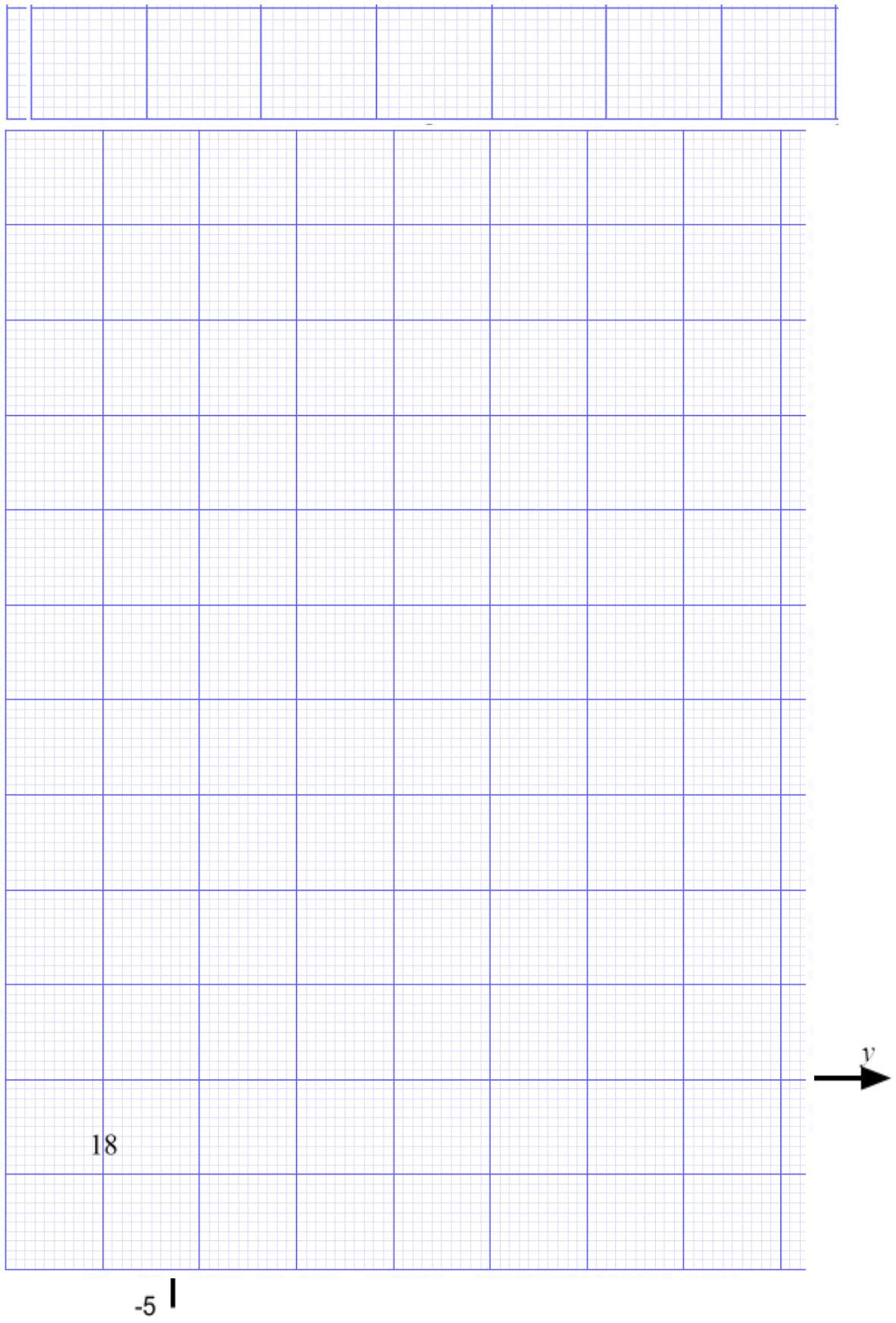
$$x^2y = 4$$

$$x = \sqrt{\frac{4}{2.5}} \quad (x > 0)$$

$$= 1.26$$

(accept from 1.24 to 1.28)

Total Marks: 8m


v
18
-5

12 **(i)**

$x^2 + y^2 - 20x + 10y + 100 = 0$

$x^2 - 20x + 10^2 + y^2 + 10y + 5^2 = -100 + 10^2 + 5^2$

$(x-10)^2 + (y+5)^2 = 25$

Radius of the circle =5 units

Centre of the cirlce: is $(10, -5)$

(ii)

Let the equation(s) of the tangent be $y = mx$

$x^2 + (mx)^2 - 20x + 10(mx) + 100 = 0$

$(m^2+1)x^2 + (10m-20)x + 100 = 0$

$b^2 - 4ac = 0$

$(10m-20)^2 - 4(m^2+1) \times 100 = 0$

$-3m^2 - 4m = 0$

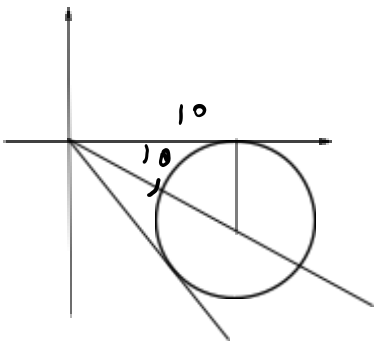
$m(-3m-4) = 0$

$m = 0$ or $m = -\frac{4}{3}$

Hence, equations of tangent are $y = 0$ or $y = -\frac{4}{3}x$.

Alternatively,

Equation of the tangent $y = 0$.



$\tan\theta = \frac{1}{2}$

$\tan 2\theta = \frac{1}{1-\frac{1}{4}}$

$= \frac{4}{3}$ (using double angle formula)

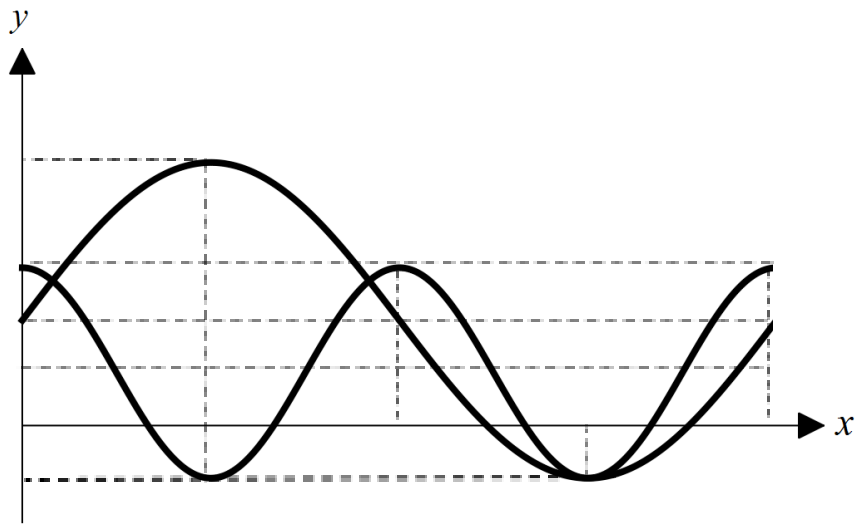
Hence the equation of the other tangent is $y = -\frac{4}{3}x$.

(iii)

Since the tangent $y = 0$ is x-axis, and tangent $\perp$ radius, the point required is

$(10, 0)$ For the other tangent line $y = -\frac{4}{3}x$, let the point required be $\left(a, -\frac{4}{3}a\right)$

		$\frac{-5-\left(-\frac{4}{3}a\right)}{10-a}=\frac{3}{4}$ $a=6$ Hence, the point required is $(6,-8)$.
		Total Marks: 11m

13	**(i)**	$2\cos 2A - 3\sin A - 1 = 0$ $2(1-2\sin^2 A) - 3\sin A - 1 = 0$ $-4\sin^2 A - 3\sin A + 1 = 0$ $(\sin A + 1)(-4\sin A + 1) = 0$ $\sin A = -1$ or $4\sin A + 1 = 0$ $A = \frac{3\pi}{2}$ or $\sin A = \frac{1}{4}$ basic $\angle$=0.25268 $A = 0.253,\ 2.89$ $\therefore A = \frac{3\pi}{2},\ 0.253$ or 2.89
	(ii)	
	(iii)	$2 + 3\sin\left(\frac{x}{2}\right) = 2\cos(x) + 1$ $2\cos(x) - 3\sin\left(\frac{x}{2}\right) - 1 = 0$ sub x = 2A $2\cos 2A - 3\sin A - 1 = 0$ From Part(i), $A = \frac{3\pi}{2},\ 0.253$ or 2.89 $x = 3\pi, 0.506, 5.78$
		Total Marks: 10m