About the logical expressions used in the CIDOC CRM

The present CIDOC CRM specifications are annotated with logical axioms, providing an additional formal expression of the CIDOC CRM ontology. This section briefly introduces the assumptions that are at the basis of the logical expression of the CIDOC CRM (for a fully detailed account of the logical expression of semantic data modelling, see Reiter (1984)[1]).

The CIDOC CRM is expressed in terms of the primitives of semantic data modelling. As such, it consists of:

- *classes,* which represent general notions in the domain of discourse, such as the CIDOC CRM class *E21 Person* which represents the notion of person;
- *properties,* which represent the binary relations that link the individuals in the domain of discourse, such as the CIDOC CRM property *P152 has parent* linking a person to one of the person's parent.

Classes and properties are used to express ontological knowledge by means of various kinds of constraints, such as sub-class/sub-property links, e.g., *E21 Person* is a sub-class of *E20 Biological Object*, or domain/range constraints, e.g., the domain of *P152 has parent* is class *E21 Person.*

In contrast, first-order logic-based knowledge representation relies on a language for formally encoding an ontology. This language can be directly put in correspondence with semantic data modelling in a straightforward way:

- classes are named by *unary predicate symbols*; conventionally, we use `E21` as the unary predicate symbol corresponding to class *E21 Person*;
- properties are named by *binary predicate symbols*; conventionally, we use `P152` as the binary predicate symbol corresponding to property *P152 has parent.*

Ontology is expressed in logic by means of *logical axioms*, which correspond to the constraints of semantic modelling. In the definition of classes and properties of the CIDOC CRM the axioms are placed under the heading 'In first order logic'. There are several options for writing statements in first order logic. In this document we use a standard compact notation widely used in text books and scientific papers. The definition is given in the table below.

Symbol	Name	reads	Truth value
Operators			
$\wedge$	conjunction	and	$(\varphi \wedge \psi)$ is true if and only if both φ and ψ are true
$\vee$	disjunction	or	$(\varphi \vee \psi)$ is true if and only if at least one of either φ or ψ is true
$\neg$	negation	not	$\neg\varphi$ is true if and only if φ is false
$\rightarrow$	implication	implies, if … then ..	$(\varphi \rightarrow \psi)$ is true if and only if it is not the case that φ is true and ψ is false

[1] [should be put into the reference list] R. Reiter (1984). Towards a logical reconstruction of relational database theory. In Brodie, M. L., Mylopoulos, J., and Schmidt, J. W., editors, On Conceptual Modelling, pages 191–233. Springer Verlag, New York, NY

$\leftrightarrow$	equivalence	is equivalent to, if … and only if …	$\varphi \leftrightarrow \psi$ is true if and only if both φ and ψ are true or both φ and ψ are false
Quantifiers			
$\exists$	existential quantifier	exists, there exists at least one	
$\forall$	Universal quantifier	forall, for all	

For instance, the above sub-class link between *E21 Person* and *E20 Biological Object* can be formulated in first order logic as the axiom:

$$(\forall x)\ [E21(x) \rightarrow E20(x)]$$

(reading: for all individuals x, if x is a E21 then x is an E20).

In the definitions of classes and properties in this document the universal quantifier(s) are omitted for simplicity, so the above axiom is simply written:

$$E21(x) \rightarrow E20(x)$$

Likewise, the above domain constraint on property *P152 has parent* can be formulated in first order logic as the axiom:

$$P152(x,y) \rightarrow E21(x)$$

(reading: for all individuals x and y, if x is a P152 of y, then x is an E21).

These basic considerations should be used by the reader to understand the logical axioms that are used into the definition of the classes and properties. Further information about the first order formulation of CIDOC CRM can be found in Meghini & Doerr (2018)][2].

[2] [should be put into the reference list] C. Meghini and M. Doerr (2016). A first-order logic expression of the CIDOC Conceptual Reference Model.