YEAR 12 - MATHEMATICS

HSC Topic 6 - Differentiation of trigonometric, exponential and logarithmic functions MA - C2.1 - Rules for Differentiation MA - C2.2

MATHEMATICS ADVANCED

LEARNING PLAN			
Learning Intentions Student is able to:	**Learning Experiences** Implications, considerations and implementations:	**Success Criteria** I can:	**Resources**
1. establish the formulae $\frac{d}{dx}(\sin x) = \cos x$ and $\frac{d}{dx}(\cos x) =- \sin x$ $\frac{d}{dx}(\tan x) = x$			
2. calculate derivatives of trigonometric functions including: $\sin f(x)$, $\cos f(x)$ and $tan(f(x))$	e.g. Differentiate: $x^2 \cos x$, $\sin(2x+3)$, $\cos^2 5x$, $\sin 3x \cos 2x$, $\frac{\sin x}{\cos x + 1}$.	- Apply the function of a function rule, product rule and quotient rule to the derivatives of expressions involving trigonometric functions.	

		- Find equations of tangents and normals to the graphs of trigonometric functions. - Apply calculus of trigonometric functions to Rates of Change questions	
3. establish and use the formula $\frac{d}{dx}(e^x) = e^x$ $\frac{d}{dx}(e^{f(x)}) = f'(x)e^{f(x)}$ $\frac{d}{dx}(a^x) = (\ln a)a^x$	Eg. $\frac{d}{dx}(2^x) = (\ln 2)2^x$ $\frac{d}{dx}(2^{3x}) = 3(\ln 2)2^{3x}$	- Apply the function of a function rule, product rule and quotient rule to the derivatives of expressions involving Exponential functions functions. - Find equations of tangents and normals to the graphs of Exponential functions. - Apply calculus of Exponential functions to Rates of Change questions including Growth and Decay.	Derivative Of Exponentials Worksheet
4. calculate the derivative of the natural logarithm function and apply the rules for logarithms to simplify first BEFORE the derivative is carried out $\frac{d}{dx}(\ln x) = \frac{1}{x}$ $\frac{d}{dx}(\ln f(x)) = \frac{f'(x)}{f(x)}$	I can apply the following log laws to simplify and solve problems. $\log_a m + \log_a n = \log_a(mn)$, $\log_a m - \log_a n = \log_a\left(\frac{m}{n}\right)$, $\log_a(m^n) = n\log_a m$, $\log_a a = 1$, $\log_a 1 = 0$, $\log_a \frac{1}{x} = -\log_a x$ Proof $y = \ln x$ $x = e^y$ $\frac{dx}{dy} = e^y$	Apply the log laws to firstly simplify a function before differentiating.	Revision of Log Laws Derivatives of Log Functions Worksheet

	$\frac{dy}{dx}=\frac{1}{e^y}$ $\frac{dy}{dx}=\frac{1}{x}$ $\frac{d\ln x}{dx}=\frac{1}{x}$ Eg. Differentiate: $y=\log_e x$, $y=\log_e(ax+b)$, etc Eg. Differentiate after first simplifying: $y=\log\left\{\frac{\sqrt{x}}{(x-1)^2}\right\}$		
5. establish and use the formula $\frac{d}{dx}\left(log_a x\right)=\ \frac{1}{a}$			
6. Use of product, quotient and chain rule to differentiate expressions containing trigonometric, exponential and logarithmic functions of the form:	Eg. Differentiate: x^2e^x, $e^{\sin x}$, $\tan 4x+e^{5x}$, $\ln(\sin x)$, $\frac{\log x}{x}$,	Apply the product, quotient and chain rule when differentiating log and exponential functions.	

$f(x)g(x)$, $\frac{f(x)}{g(x)}$ and $f(g(x))$ where $f(x)$ and $g(x)$ and for $f(ax+b)$.	$\frac{\log x}{x^2}$, e.g. Differentiate $x^2e^x, e^{\sin x}, \tan 4x + e^{5x}, \ln(\sin x), \ln(x^2), (\ln x)^2$	Apply the derivative of logs, exponentials and trig functions to solve a variety of real-world questions including motion, rates are work on functions.	
7. **Without using calculus**, sketch exponential and logarithmic functions. If sketching $y = xe^x$, students should sketch on the same diagram, $y = x$ and $y = e^x$, determine any critical points including x-intercepts, and VA (if they exist), then determine where the graph lies by considering the product/quotient of the y-values. 2. Special consideration should be taken of the behaviour of the functions at $x = 0$ and for large values of x. The calculator may be used to obtain an idea of the behaviour of the graph for these values while better pupils could consider appropriate sections of the	Eg. Sketch: $y = \frac{\log x}{x}$, $y = \frac{\log x}{x^2}$, $y = xe^x$, $y = x^2e^{-x}$ and $y = \log\{x(1-x)\}$. Sketch $y = 2\ln x, y = \ln(x+1)$ $y = \ln(x^2), y = \ln\|x\|,$ $y = \|\ln x\|, y = \ln(2-x)$ Sketch $y = e^{-x}, y = -e^x, y = -e^{-x},$ $y = e^x + 1, y = \frac{e^x}{2}, y = e^{\|x\|},$ $y = e^{x-2}, y = 1 - 2e^x$ Further graphs should include:		

graphs of $y=e^x$ $y=x^2$, $y=x$ and $y=\log x$.	$y=\frac{\ln x}{x}, y=\frac{\ln x}{x^2}, y=xe^x, y=x^2e^{-x}$, $y=\ln\{x(1-x)\}$		

Established Goals(Syllabus Outcomes): MA12-3, MA12-6, MA12-9, MA12-10

ASSESSMENT	
Performance Tasks: (Linked to Essential Questions) Past HSC Paper Exams 2017 Q8, Q26 (d), Q30 (c) · 2015 Q7, Q9, Q22, Q30 (e) · 2014 Q23, Q26 (b), Q28 (b) 2013 Q4, Q24, Q28(a) · 2012 Q4, Q10, Q20, Q29 (c), Q27 (d) · 2011 Q9, Q24 (c) 2010 Q24 (d) · 2009 Q23 (a)	**Other Evidence:**