HOW TO NOT BE WRONG

The Power of Mathematical Thinking

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WHEN AM I GOING TO USE THIS?

Mathematics is not just a sequence of computations to be carried out until your patience or stamina runs out…. Integrals to Mathematics is weight training and calisthenics are to soccer…. But if the drills are too much for you to take, you can still play for fun.

- Abraham Wald and The Missing Bullet Holes
- Mathematics is the Extension of Common Sense by other means
    - a + b = b + a
    - a * b = b * a
    - What doesn't seem like intuition at first, like calculus, is still **derived from common sense.**
    - Mathematics without common sense is nothing more than book keeping and following a set of rules
    - This book deals with the **Profound/Simple quadrant**. These are **Principles with applications in real life.** These will ***help you not be wrong.***

Part 1: Linearity

Less like Sweden

- **False Linearity**: The Cato Institute's Criticism of Obama's Affordable Care Act.
- **Non-Linear Thinking**: Which way you should go depends on where you already are. The optimum is somewhere in between.
    - Example of popular Non-Linear Thinking: Tax Cut in the Reagan Era and the Laffer Curve

Straight Locally, Curved Globally

- Newton argued that a parabola is just infinite straight lines, only if you zoom in close enough.
- **Method of Exhaustion**

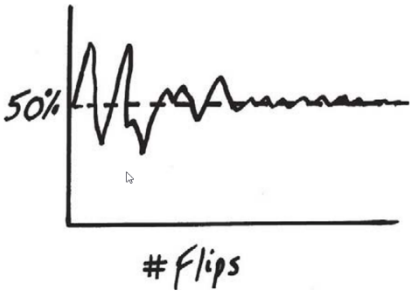
    - **Basic Rule of Mathematical Life:** If the Universe gives you a complicated problem, solve a simpler one. Hope that the simpler one is close enough so the Universe doesn't complain.
    - Archimedes was trying to find out the **area of a circle**, r = 1.
        - **Inscription** and Circumscription of polygons to find upper limits and **lower** limits of the area
- **Calculus,** within a page
    - **Where the problem lies**
        - Grandis' Series (1-1+1-1+1…. = ½ or 0 or 1?)
        - Zeno's Paradox (Can you ever reach the Ice Cream store?)
        - 0.9999 riddle
        - Cauchy's theory; Hardy's Divergent Series
    - **Solution**: The closer you get, the more linear it seems. The problem is one of definition.

Everyone is Obese

- Some article claims that, by 2048, every American will be Obese.
- **Linear Regression:** Finding the "Best Fit" line in a scatter plot
    - Fine in a small local area
    - Misleading if extrapolated indefinitely
    - The rules can be followed by the computers as well. But knowing whether the result is correct – whether the method used is the right one – needs a guiding Human hand. Enter the **math pedagogy war!**
        - Traditional vs *Reform*

How Much is That In Dead Americans?

- Examples of Lineocentrism in the media given. **Death tolls are compared,** but should they?
- Important rule of Mathematical Hygiene: If you're field testing a method, try computing in several different ways. The answers should match!
- Proportions are not always telling the truth! Getting 80% on 10 coin flips is much more likely than getting 80% on a 1000.
    - **The Law of Large Numbers.** Small sample sizes are to dominate both the upper and



lower ends of the table, simply because outliers affect the data more.
    - Proportions favour smaller sample sizes, total would obviously see the larger ones win. **What now?**
    - Ask **how many square roots away** you are from 50%
    - **Law of Averages** – A *FALSE* law
    - Why the Conflict? Just a misunderstanding. The Law of Large numbers don't balance out what's already happened; it only dilutes what's already happened.
- So how exactly do we compare atrocities over time, space and populations?
    - **Partially ordered set:** You can't compare every atrocity with another. The answer is often subjective and based on weight of different aspects.

More Pie Than Apple

- Wrong Picture of the Employment in the US painted, using percentages.
- Don't talk about percentages of numbers, when they be negative.
    - Statements created to produce a wrong impression
    - **Dividing one number by another is mere computation. Figuring out what you should divide by what is Mathematics.**

Part 2: Inference

The Baltimore Stockbroker and The Bible Code

- ELS found references to the future in version of the Torah.
- The Baltimore Stockbroker is the 10-week newsletter haox.
- The mistake is being surprised by the improbable.
- The case with the Rabbi names is the same. There was a bit of wiggle room regarding how the rabbis were named in the Torah.
- Mutual fund companies use the same wiggle room in "incubation"
- **When you're trying to draw reliable inference from improbable events, wiggle room is the enemy.**
- But the experiment with Rabbi names had passed Statistical tests. Why did we not accept them? Are the tests themselves concerning?

Deadfish Don't Read Minds

- We've been circling around the question: How unlikely an event should be to catch us surprised?
- **The more you let yourself be surprised, the more you will be.**
    - You will only hear about the exceptional stories. This is the definition of news.
- **Inference: Building theories from observation.**
- Improbability is relative; It depends upon the underlying assumptions we make along the course of reasoning.
- Enter **Null Hypothesis Testing.**
    - Null Hypothesis = Negation of your theory.
    - The data matching your theory is not enough. The data has to be inconsistent with the negation of your theory.
- **<u>Significance</u> == existence != important**
    - We also have to be careful about underpowering and overpowering experiments and statements.

Reductio Ad unlikely

- Separate spin on Reductio Ad Absurdum
- Random Patterns will have non random looking clusters. If it does not, there is some other factor at play.
- The author discusses Prime Numbers, Prime Clusters, Experiments and Proves found, and their implications. These all lead to the challenge: **Can we find the structure of structurelessness?**

The International Journal of Haruspicy

- By chance alone, 1/20 "false" studies will pass the arbitrarily selected p-value of 0.05.
    - Some researchers also "hack" p-values to get it slightly on the other side of 0.05.
        - They torture the data until it confesses
        - They also delete a few rows
    - The culture is the problem. Journals won't publish studies that are p > 0.05. They will also not publish replications.

- **One way to fix this is to include "Confidence Intervals" in data.**
- **But the bigger problem to solve is fixing the publication bias.**
    - ➔ **Publish EVERYTHING. Even replications.**
    - ➔ **p < 0.05 means something is interesting enough to be investigated. It does not indicate the truth.**
    - ➔ **Scientists need to stop running the stockbroker con on themselves.**

Bayesian Inference

- Predicting Behaviour using algorithms is something SV companies, and even Target has pinned to near perfection. And they keep getting better as you feed more data.
- But could you simulate something much more *chaotic,* say for example, the weather?
    - Lorenz thought the *hard limit* was 2 weeks. And we haven't broken that.
    - But is Human behaviour is as predictable as the asteroid, or is it as swaying as the weather?
- Facebook Terrorist algorithm. It will report 0.05% users as terrorist. Each of them have a 99.99% probability of NOT being a terrorist.
    - **Use Conditional Probabili**ty
- **Alternative -** Bayesian Inference: Starting off with ***a priori,*** doing the experiment, and according to the results, tweaking our existing beliefs into a ***posterior.***
- But shouldn't science be objective and free from prejudice?
    - But without prejudices, we would run the risk of being astounded by everything.
    - This also gives us an interesting insight into **conspiracy theorists**. Instead of adjusting the probability of their crazy theory's probability of being true, they invent another crazy theory explaining the adverse observations. A cycle is formed, and sustained. T+U...
- "It is an old maxim of mine that when you have excluded the impossible, whatever remains, however improbable, must be the truth." **UNLESS the truth is a hypothesis it didn't occur to you to consider.**

- The author thinks our logic, reasoning and our ability to assign probability numbers to ultimate questions, such as the one about existence of GOD, is far too limited to be addressed.

Part 3: Expectations

What to Expect When You're Expecting To Win the Lottery

- Calculate the **Expected Value -** Value you oughta receive due to The Law of Big Numbers.
- The author talks about why Lottery shouldn't be played, and how some MIT/Harvard gamed the lottery system by calculating the Threshold that would raise their Expected Value enough to make it worth taking the risk.
    - And while the cartels were closed off after restrictions were put in place following a news article unravelling the mystery, the lottery officers *knew.* **Why?**
    - Because the state had taken the money it needed. It didn't care who won. The cartels weren't taking money from the state. It was taking money from other cartels . ***They were the house.***
- Additivity : E(x+y) = E(x) + E(y)
- **Buffon's Needle Problem = Buffon's Coin Problem**
    - When we're stuck with a problem, we can do two things:
        - Make it simpler and solve it, hoping that the knowledge gained in the process smoothly translates to the real problem at hand.
        - Make it harder - this is what Barbier did to explain why p = 2/pi. Using Additivity.
        - The Author closes with a note depicting his view on practiced mathematics today: a mixture of Monastic Contemplation, and blowing obstacles with dynamite.

Miss More Planes!

- **Utils** are a non-material measure of your convenience, time, other non-quantifiable-by-money things. And ***it is purely subjective.***
- The author again addresses the question of God, this time using utils.
- St. Petersberg's paradox - 2 ducats for every throw.
    - Tells us that money can also be measured in utils. Curve that bends downward. $1 equivalent util for a poor person isn't the $1 equil for a rich person.

- Ellsberg Paradox - Red, Black, NotRed, NotBlac.
    - Questions **utility theory** as more prefereable and less preferable options mean the same thing.
    - Known unknowns are **risks**. Unknown unknowns are **uncertainties**.

Where the Train Tracks Meet

- Quic Pics and hand-filled tickets both had the same expected value in the lottery wars. But hand-filled ones had a more certain return. **Humans like certainty.**
    - **Humans dislike *variance* when talking about profits.**
    - **Humans like *variance when talking about losses.***
    - Handpicking numbers made it certain that he'd either win the jackpot, or have a least number of deuces.
        - He could write a program to find the number sequences with the lowest variance. But the complication lay in the **combinatorial explosion.**
        - **Parallel lines may *appear* to meet.** This realization soon fuelled the invention of countless other alternatives to the Euclidian Plane and a different kind of geometry.
        - But how did he handpick?
- If you have negative utility, it's better to do nothing at all.
- The author now rattles off about the *Mathematical Theory of Communication.*
    - He also warmly embraces skepticism about formalism of aspects of nature that we *think* can't be described by Math.
    - Error Correcting Code had initially a ratio of 3:1
    - The Hamming Code was one of many much more efficient code-correction system with better ratio. This had to to ensure that the *Hamming Distance* was little enough for the system to detect and correct corrupted transmissions.
    - In mathematical terms, the Hamming Code problem = we ask that no two words centred have points in common. In other words, the Sphere Packing Problem from Geometry.
        - In discussion, the author addresses the discomfort between mathematicians and computers, as depicted by the hurt feelings of young Hales. Hales is now working to create a mathematics built again from the ground up, where all the assumptions are cross checked. Otherwise

the scaffolding of proofs and assumptions are just too fragile.
        - The author on the other hand, is not concerned. He thinks that if current mathematics is taken over by our AI overlords, new Math will be whatever they can not think of.
    - Shannon showed that *almost all* sets of code have error correcting property.
- While decision-theory, game theory and utilities told people how they **should** be making decisions based on rationality and maximizing returns, Kahneman and Tversky found **Behavioral Economics** on the **"Prospect Theory"**
    - And this finally explains why people gamble in the lottery. For Fun!
    - The author also draws parallels between entrepreneurship and gambling.

Part 4: Regression

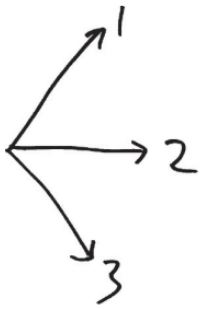
The Triumph of Mediocrity

- Regression to the Mean : Horace Secrist's study of businesses over a decade
    - Horace's ideas stem from the ones by Francis Galton.
        - He was working on *Hereditary Genius;* and later on regression to the mean in heights of children on tall/short parents.
    - "On pace for": A story about linear regression inn baseball.
    - Every kind of scientist started to think something of their field contributed to the regression. But it was mathematics, as Hotteling said so.
    - **Take a randomized sample to get rid of the effect.**

Galton's Ellipse

- Now Galton wanted to see how strong those forces were.
    - He invented the **Scatterplot**
    - If the variables are completely unrelated, we'd have circles
    - If the variables are completely determined by each other, we'll have a straight line.

    - If they are just **correlated**, we'll have ***ellipses***.
    - Galton invented this, and was convinced that conic sections were at play in other areas as well.
- Galton tackled the problem of Bertillonage. Was this the most efficient way? Because Shouldn't long legs be correlated with long hands?
    - He later went on to invent dactyloscopy - fingerprinting.
- Bertillon would store a suspect as a sequence of numbers
- The Parson's code is similar, but used for music. ***ruudddurr**
- The Author now explains **Pearson's Linear correlation test** with geometry.
    - In Pearson's formula, each data row is represented as a **vector.** The lesser the angle between the two points, the more correlated they are.
        - acute angle = positively correlated
        - obtuse angle = negatively correlated
        - perpendicular = zero correlation



- One important thing to note is that **if A is correlated to B, and B is correlated to C, it does not mean A is correlated to C.**
- Also, Just because two things are correlated, doesn't always mean that you can just increase one to increase the other.
- ALSO (again!) Just because it's uncorrelated, it doesn't mean it's unrelated. Maybe it's just not related in the way detectable by correlation.

Does Lung Cancer Make You Smoke More Cigarettes

- Correlation and Causation is a slippery subject because your intuition grasps it quite strongly in some cases, and loses balance in the others.
- **Surrogate Endpoint Problem -** There is sometimes a mystery factor in between of correlations. Back to the HDL problem!
- R.A Fisher was actually very skeptic of the tobacco-cancer relationship when the studies were against smoking.
    - Fisher was later proved wrong. Here the author makes a good point that it is not always wrong to be wrong.

- In order to know something with absolute certainty, you'd have to conduct experiments that were theoretically perfect, but practically impossible.
    - We go back to the use of expected values.
    - However, it's often very tough to assign values to real-life conundrums.
    - BUT, **if you never give advice until you're sure it's right, you're not giving enough advice.**