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B.Sc. (Hons.) Mathematics (Semester – 3rd)

MATHEMATICAL STATISTICS

Subject Code: BMATS1-302

Paper ID: [19131212]

Time: 03 Hours **Maximum Marks: 60**

Instruction for candidates:

1. Section A is compulsory. It consists of 10 parts of two marks each.
2. Section B consist of 5 questions of 5 marks each. The student has to attempt any 4 questions out of it.
3. Section C consist of 3 questions of 10 marks each. The student has to attempt any 2 questions.

Section – A **(2 marks each)**

Q1. Attempt the following:

a. If A and B are two independent events, then show that A and $\overline{B}$ are also independent.

b. The p.d.f. of a continuous random variable is given by

$$f(x)=\begin{cases} cx^2 & \text{for } 1\le x\le 2 \\ 0 & \text{elsewhere} \end{cases}$$

Find the value of the constant c.

c. The joint p.d.f. of two random variables X and Y is given by $f(x,y)=\dfrac{9(1+x+y)}{2(1+x)^4(1+y)^4}$; $0\le x<\infty,\ 0<y<\infty$, Find the marginal distributions of X and Y.

d. A variable X has following probability distribution:

X	-3	6	9
P(X)	$\frac{1}{6}$	$\frac{1}{2}$	$\frac{1}{3}$

Evaluate $E(X^2)$.

e. If on an average one ship out of 10 is wrecked, find the probability that out of 5 ships expected to arrive the port, at least four will arrive safely.

f. Show that in a Poisson distribution with unit mean, mean deviation about mean is $\frac{2}{e}$ times the standard deviation.

g. Which type of error is more dangerous Type-I or Type-II? Explain.

h. Explain the concept of sampling and population. Also give an example.

i. Comment on the following statement:

Mean of a binomial distribution is 3 and variance is 5.

Justify your answer.

j. Akash is giving an exam with multiple choice questions, and his probability of attempting the question with the right answer is 60%. What is the probability that Akash gives the third correct answer for the fifth attempted question?

Section – B **(5 marks each)**

Q2. In a bolt factory, machines A, B and C manufacture respectively 25%, 35% and 40% of the total bolts. Of their outputs 5%, 4% and 2% are defective bolts. A bolt is drawn at random from the product.

a. If the bolt drawn is found to be defective. What is the probability that it is manufactured by the machine B?

b. What is the probability that the bolt drawn is defective?

Q3. X and Y are two random variables having the joint density function, $f(x,y)=\frac{1}{27}(2x+y)$, where x and y can assume only the integer values 0, 1 and 2. Find the conditional distribution of Y for $X=x$.

Q4. Fit Poisson distribution to the following and calculate theoretical frequencies:

Deaths	0	1	2	3	4
Frequency	122	60	15	2	1

Q5. A certain stimulus administered to each of the 12 patients resulted in the following increase of blood pressure:

5, 2, 8, -1, 3, 0, -2, 1, 5, 0, 4 and 6

Can it be concluded that the stimulus will, in general, be accompanied by an increase in blood pressure?

Q6. In an experiment on pea breeding the following frequency of seeds were obtained:

Rounded yellow	Wrinkled and yellow	Rounded and green	Wrinkled and green	Total
315	101	108	32	556

Theory predicts that the frequencies should be in proportions 9 : 3 : 3 : 1. Examine the correspondence between theory and experiment.

Section – C **(10 marks each)**

Q7.

a. The random variable with pdf is given by $f(x)=\begin{cases} k\left(x^2+x\right) & ;\ 0\le x\le 1 \\ 0 & ;\ \text{else} \end{cases}$.

Find the cumulative distribution function. (5)

b. The distribution function of a random variable X is as follows:

$$F(x)=\begin{cases} 0 & \text{for } x\le 0 \\ \dfrac{x^2}{9} & \text{for } 0<x\le 3 \\ 1 & \text{for } x>3 \end{cases}$$

Find the probability density function of X. (5)

Q8. a. If X and Y are two random variables having joint density function:

$$f\left(x,y\right)=\begin{cases} \dfrac{1}{8}\left(6-x-y\right) & ;\ 0\le x<2,\ \ 2\le y<4 \\ 0 & ;\ \text{otherwise} \end{cases}$$

Find *(i)* $P\left(X<1\cap Y<3\right)$, *(ii)* $P\left(X+Y<3\right)$, and *(iii)* $P\left(X<1\,|\,Y<3\right)$. (5)

b. Suppose that you have a fair 4-sided die, and let X be the random variable representing the value of the number rolled.

1. Write down the moment generating function for X.
2. Use this moment generating function to compute the first and second moments of X. (5)

Q9. (a) The probability that a bulb produced by a factory will fuse after 150 days of use is 0.05. find the probability that out of 5 such bulbs

(i) None
(ii) not more than one
(iii) more than one
(iv) at least one

will fuse after 150 days of use. (5)

(b) What are the assumptions of F-test? (2)

(c) In two independent samples of sizes 8 and 10, the sum of squares of deviation of the sample values from the respective sample means were 84.4 and 102.6. Test whether the difference of variances of populations is significant or not? (3)