Unit 8:
Graphs of
Sine & Cosine

Pre-Calculus

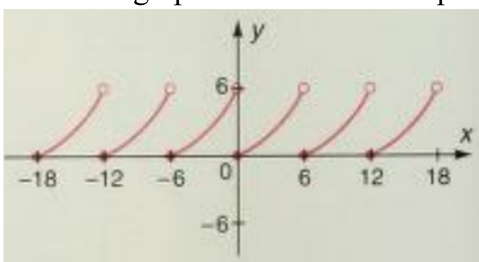
Name: ___________________________

<u>Graphs of Sine & Cosine</u>

Course Day: _______

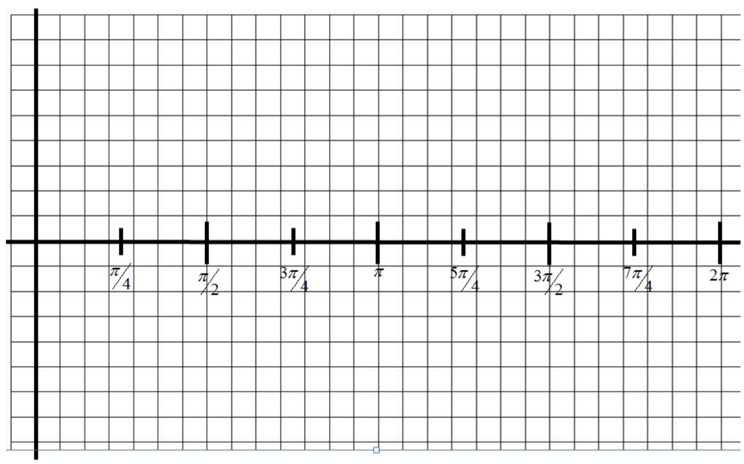
Sine and Cosine functions are what is known as periodic functions. A function f is **periodic** if there is a positive real number h such that $f(x + h) = f(x)$ for every x in the domain of f. the smallest such positive number h is called the **period** of f; one **cycle** of the graph is completed in one period.

Example 1: The graph below is an example of a periodic function. Identify the value of the period.



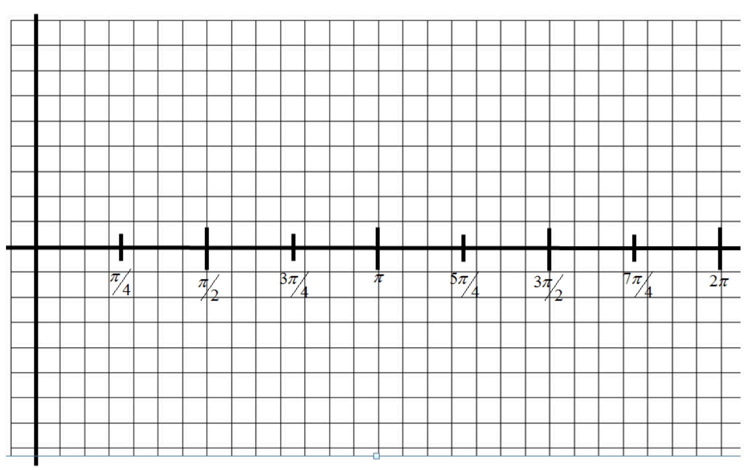
Graph $f(x) = \sin sin\ x$

x	$\sin sin\ x$
0	
$\frac{\pi}{2}$	
π	
$\frac{3\pi}{2}$	
2π	



Graph $g(x) = \cos cos\ x$

x	$\cos cos\ x$
0	
$\frac{\pi}{2}$	
π	
$\frac{3\pi}{2}$	
2π	



<u>Desmos Activity:</u>

DESMOS ACTIVITY IS GOO D- NEED MORE GUIDED PRACTICE AFTER TO MAKE SURE THAT ALL STUDENTS UNDERSTAND WHAT THEY'RE LOOKING AT.

- **MOVE THE TABLE ON PAGE 7 EARLIER**
- **DO <u>THREE</u> PRACTICE PROBLEMS TOGHETET**
  - **GRAPHING WITH AMPLITUDE CHANGE**
  - **GRAPHING WITH PERIOD CHANGE**
  - **GRAPHING WITH VERTICAL SHIFT**

(Slides 4 & 5)

What do you notice about how each of the values of A, B, C, and D, change the graph? Make notes below.

A: ____

B: ____

C: ____

D: ____

(Slides 6 – 17)

How do each of the values shift the graph/what is the proper name for each shift?

A: ____

B: ____

C: ____

D: ____

(Slide 18)

Write an equation of the graph in $y = A \sin sin\,(Bx - C) + D$ $or\ y = A \cos cos\,(Bx - C) + D$

Is the graph a sine or cosine graph? ________

What is the amplitude? ________

What is the period? ________ What would that make your B value? ________

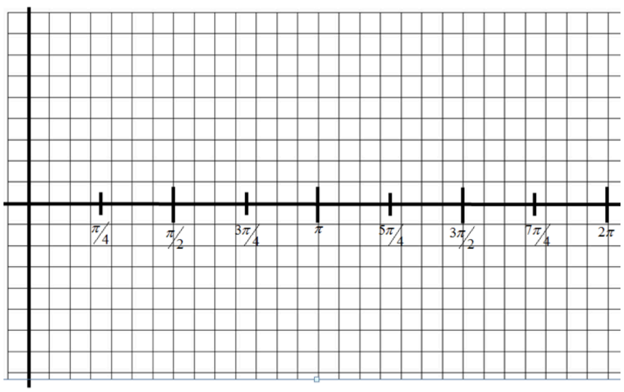
Is there a phase shift? ________

Is there a vertical shift? ________ What is it? ________

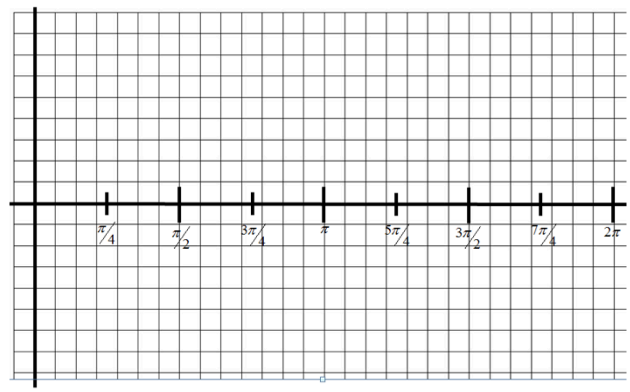
What is the equation you came up with?

Directions: Accurately graph the following trig functions between $0 \le x \le 2\pi$. Use two different colors to distinguish the graphs or use a dotted line for the first graph. Label your graphs appropriately. Include all important points (maximums, minimums, where the graph crosses the x-axis).

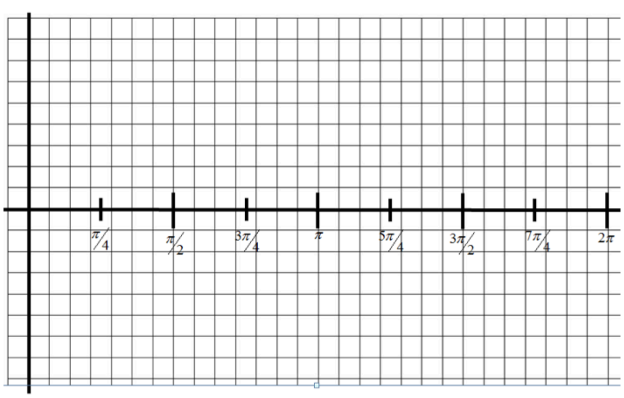
1. $y = 2 \sin sin\, x$



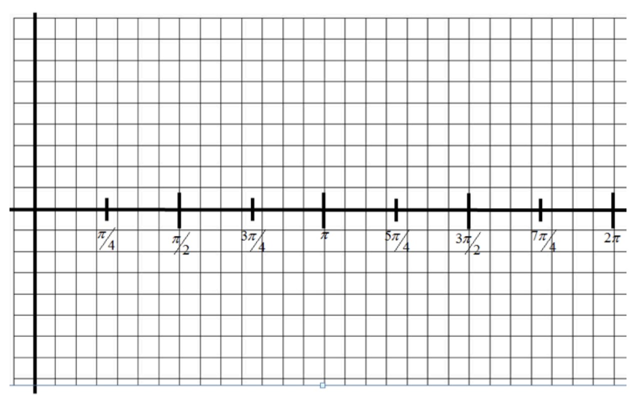
2. $y = 3 \cos cos\, x$



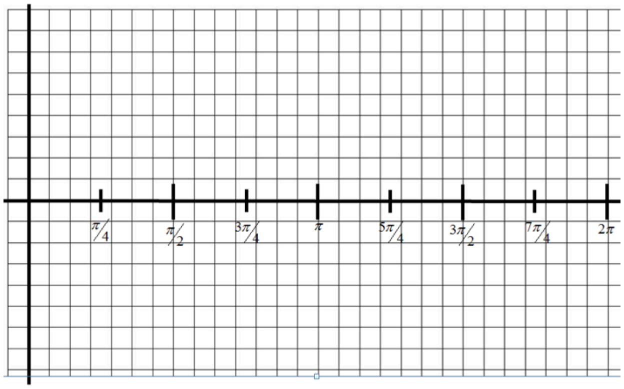
3. $y = \frac{1}{2} \sin sin\, x$



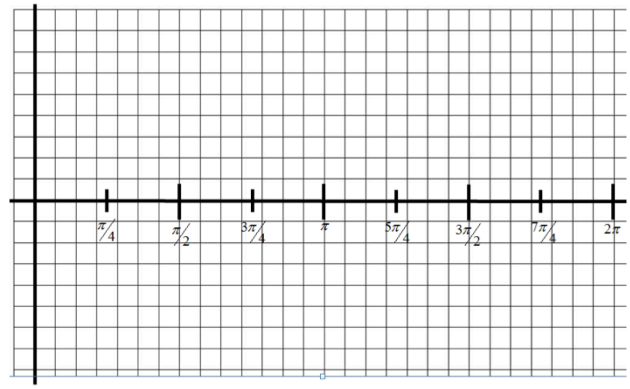
4. $y = \frac{1}{2} \cos cos\, x$



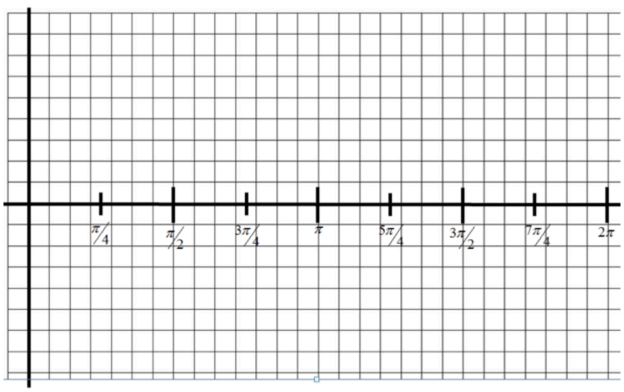
5. $y =- \sin sin\, x$



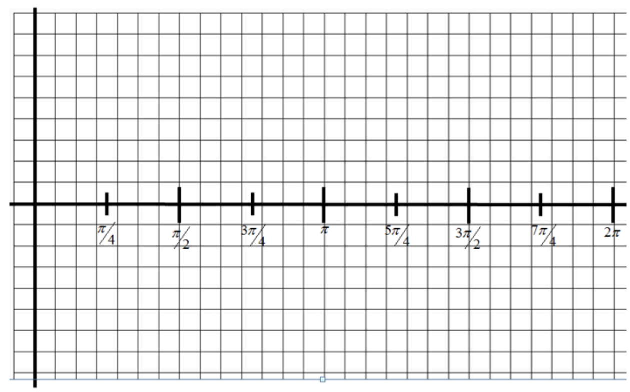
6. $y =- \cos cos\, x$



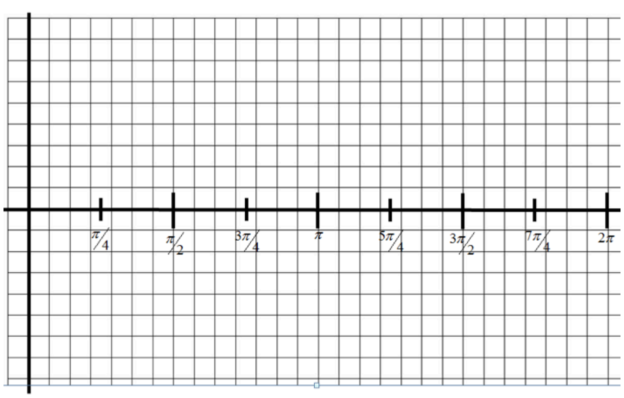
7. $y = \sin sin\, (x) + 2$



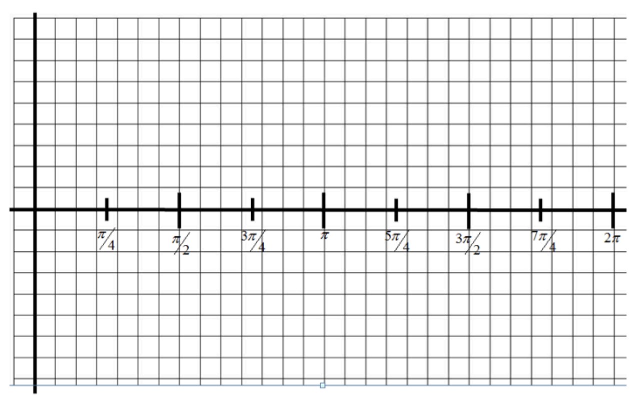
8. $y = \cos cos\, (x) - 2$



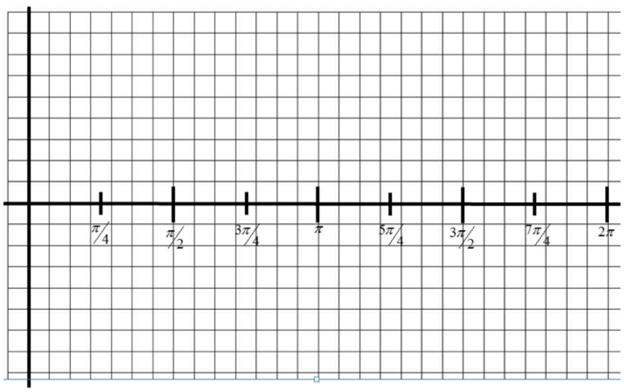
9. $y = \sin sin\, (4x)$



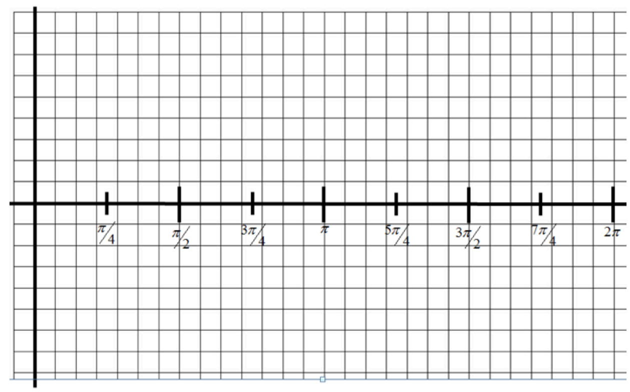
10. $y = \cos cos\, (2x)$



11. $y = \sin sin\left(\frac{1}{2}x\right)$



12. $y = \cos cos\left(\frac{1}{2}x\right)$



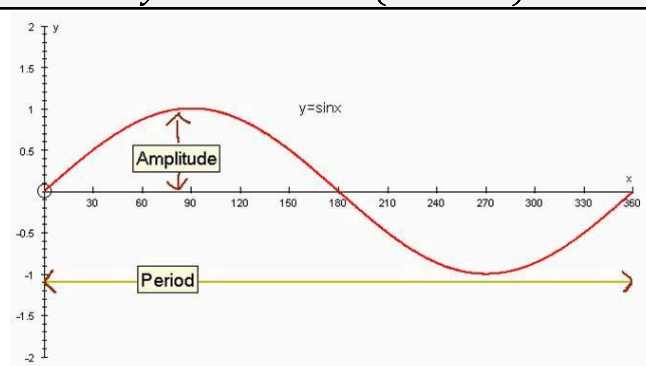
For $y = A\sin sin\,(Bx - C) + D$ or $y = A\cos cos\,(Bx - C) + D$

Amplitude = $|A|$

Period = $\frac{2\pi}{B}$

Phase Shift: $Bx - C = 0$

Vertical Shift(Midline): $y = D$



What is the amplitude of the sine graph?

What is the amplitude of the cosine graph?

What is the period of the sine graph?

What is the period of the cosine graph?

For each trig function below, write its amplitude, period, and vertical shift.

Trig Functions	Amplitude	Period (in radians)	Vertical Shift
$y = \sin(3x) + 2$			

$y = 2\cos(x) - 3$			
$y = -3\cos(2x)$			
$y = 5\sin\left(\frac{1}{2}x\right) + 1$			
$y = \frac{1}{3}\cos(5x) - 6$			
$y = 0.5\sin x - 1$			

Graphs of Sine & Cosine (Day 2)

Course Day: _______

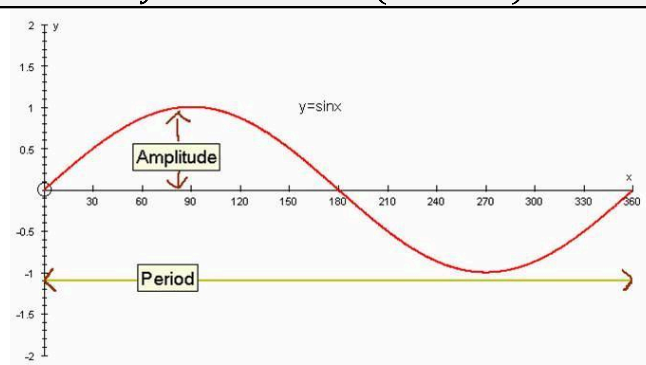
For $y = A \sin sin (Bx - C) + D$ or $y = A \cos cos (Bx - C) + D$

Amplitude $= |A|$

Period $= \frac{2\pi}{B}$

Phase Shift: $Bx - C = 0$

Vertical Shift(Midline): $y = D$



Determine the amplitude and period of each function below.

a. $y = sin(4x)$

Amplitude: ________

Period: ________

b. $y = \cos cos (5x)$

Amplitude: ________

Period: ________

c. $y = 3 \sin sin x$

Amplitude: ________

Period: ________

d. $y = 4 \cos cos \left(\frac{1}{2}x\right)$

Amplitude: ________

Period: ________

e. $y =- 2 \sin sin (2x)$

Amplitude: ________

Period: ________

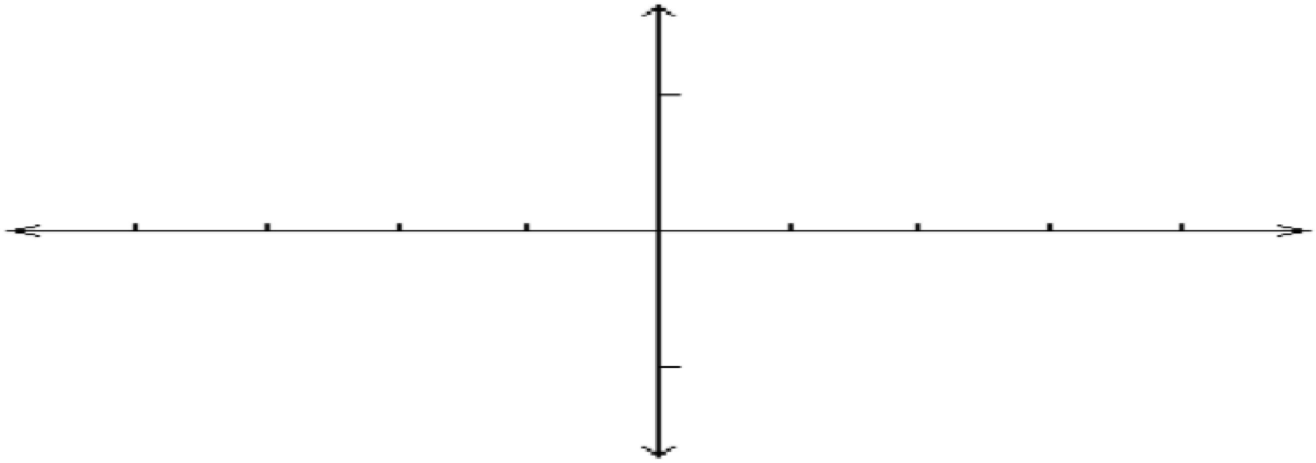
f. $y = 2sin\left(\frac{1}{4}x\right)$

Amplitude: ________

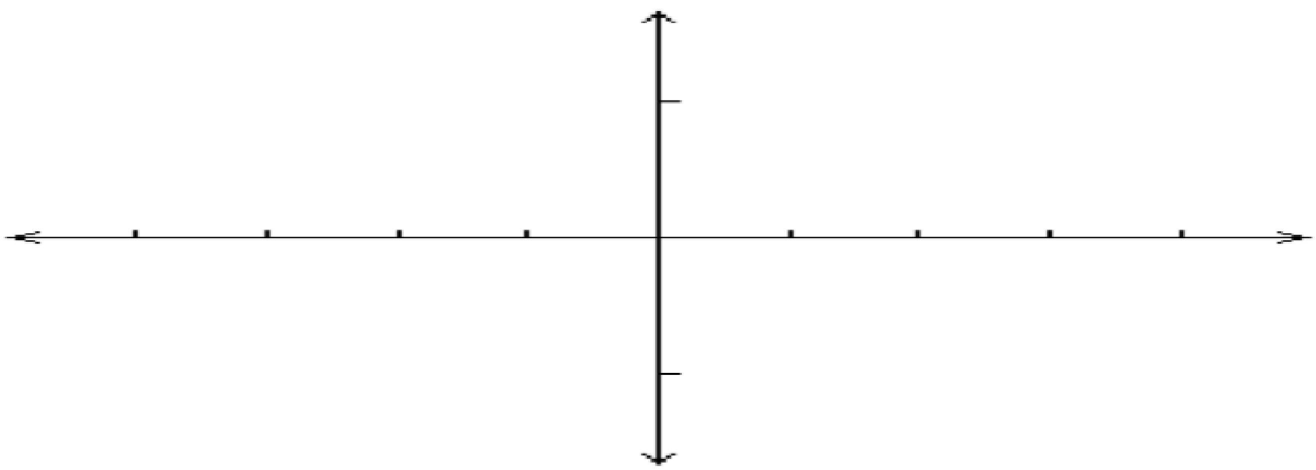
Period: ________

Graph the following functions over at least two periods, one in the positive direction and one in the negative direction. Label the axes appropriately. (Graphs may vary depending on how you label axis)

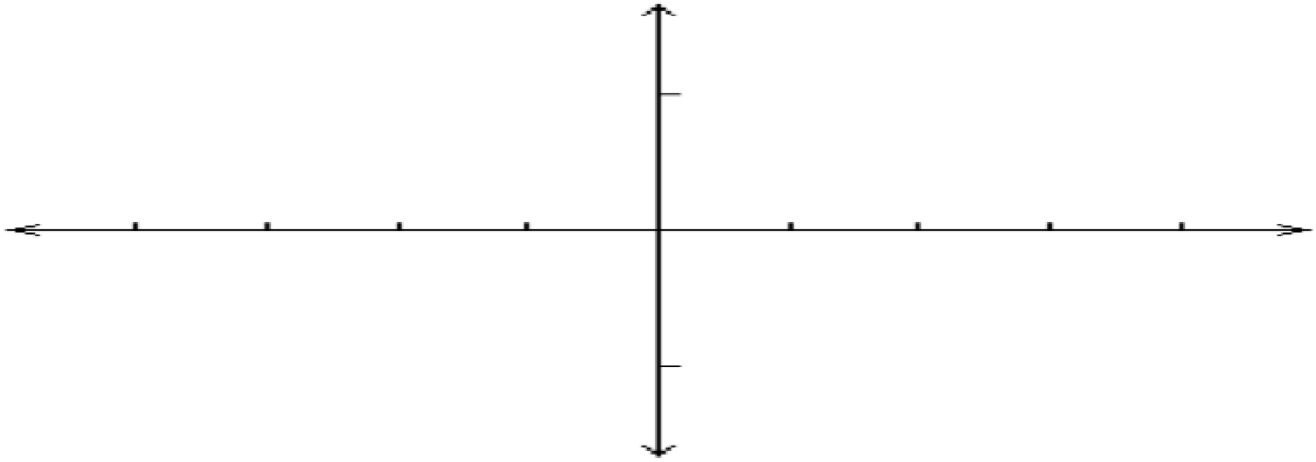
a. $y =- \ 3 \cos cos\ x$



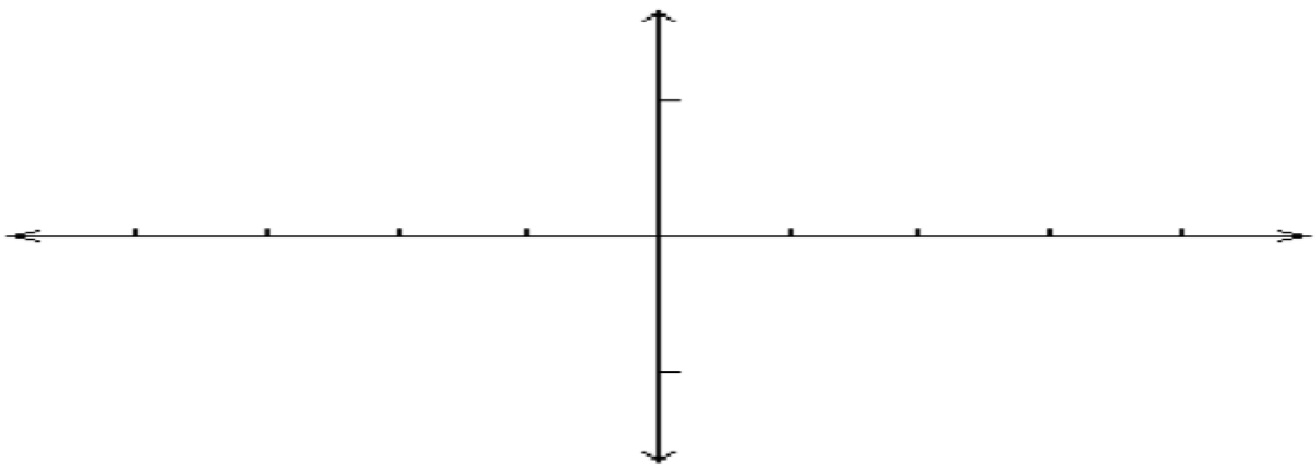
b. $y \ = \ \sin sin\ (4x)$



c. $y \ = \ 2 \cos cos \left(\frac{1}{4}x\right)$

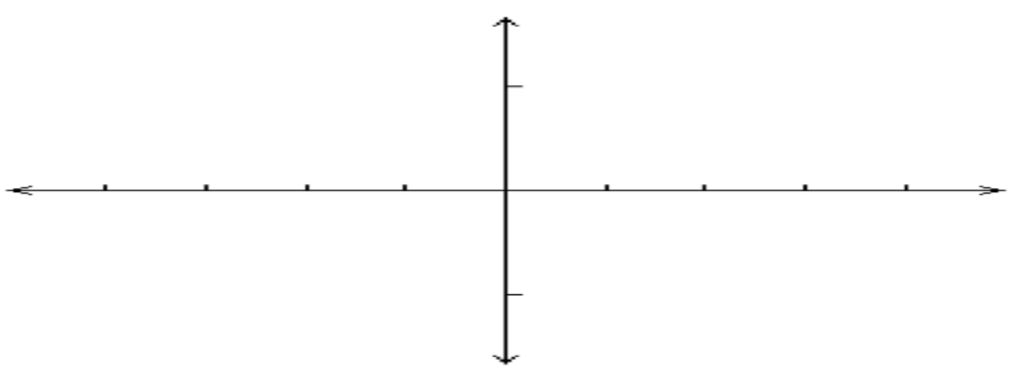


d. $y = -\ 3 \sin sin\,(2x) +\ 1$

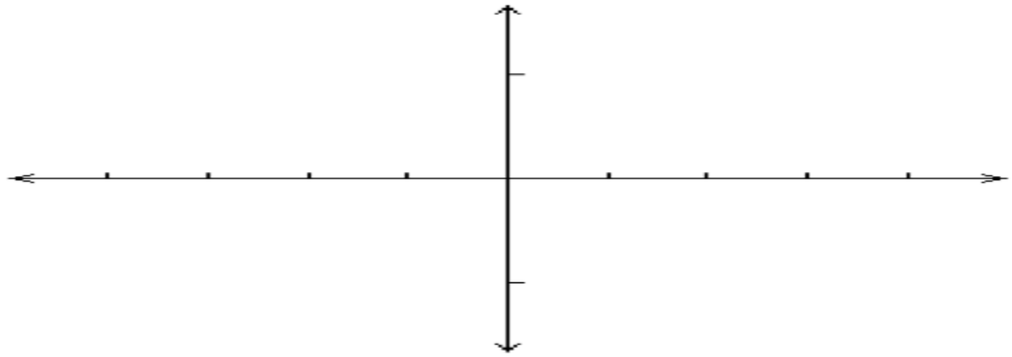


Graphing with Phase Shifts

Graph $f(x) =\ 3\ sin\left(2x\ + \frac{\pi}{4}\right)$

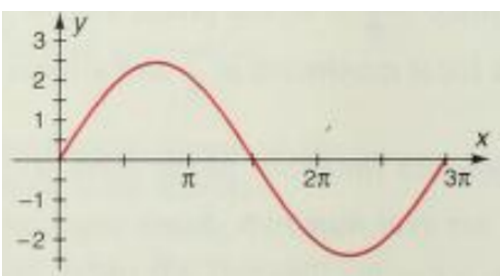


Graph $f(x) = \frac{1}{2} cos\,(4x\, -\, \pi) +\, 1$



Determine the sinusoidal function with the characteristics: a sine function with amplitude 6, period 6π, phase shift $\frac{\pi}{2}$, and translated 1 unit up.

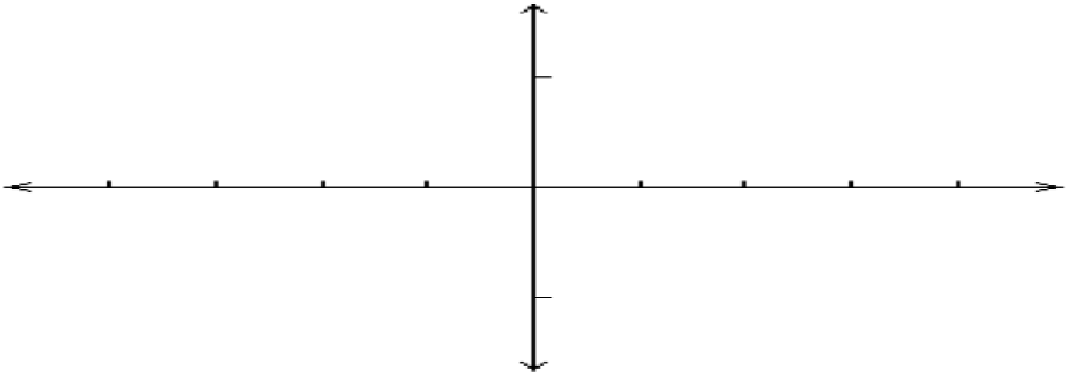
Determine a sinusoidal function for the graph below.



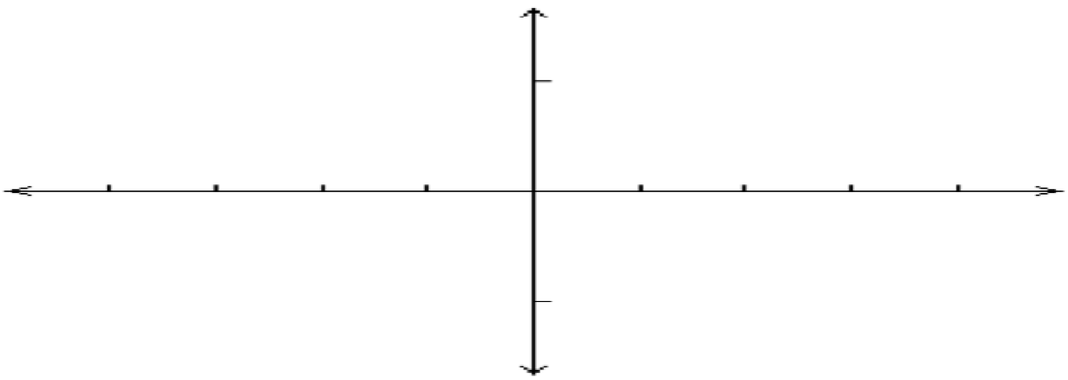
<u>On Your Own:</u>

Graph each of the following trigonometric functions.

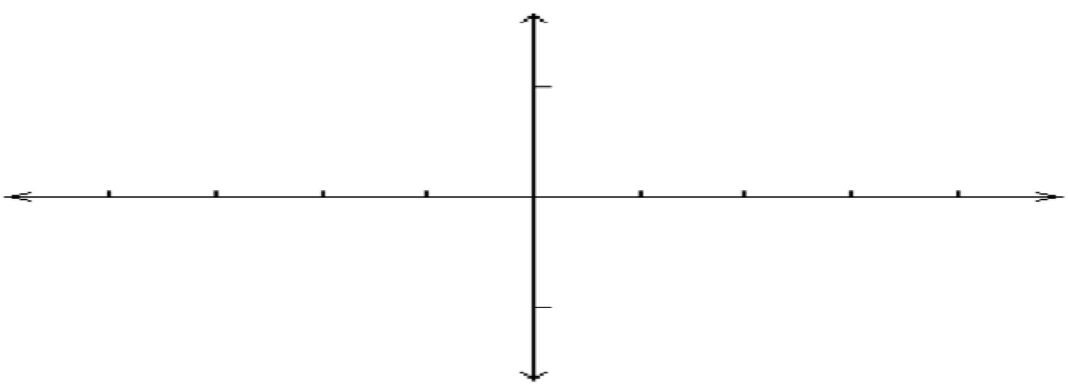
1. $y\, =\, 3\, \sin sin \left(x\, +\, \frac{\pi}{3}\right)$



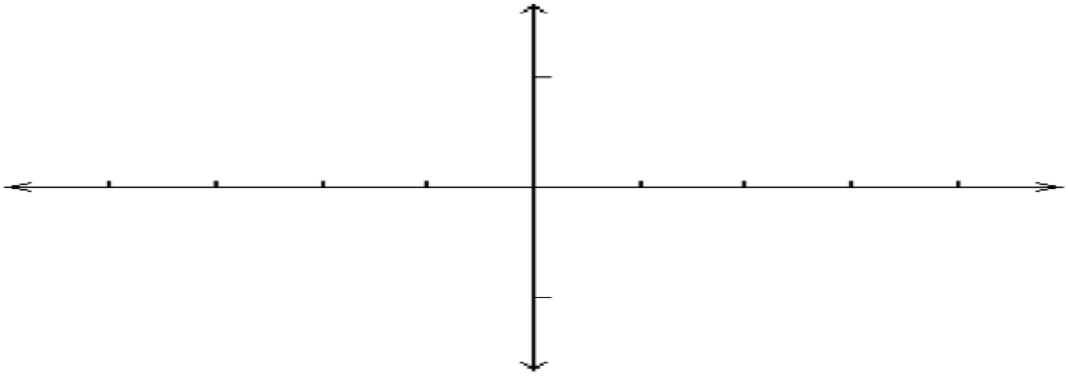
2. $y = \cos cos\left(2x - \frac{\pi}{2}\right)$



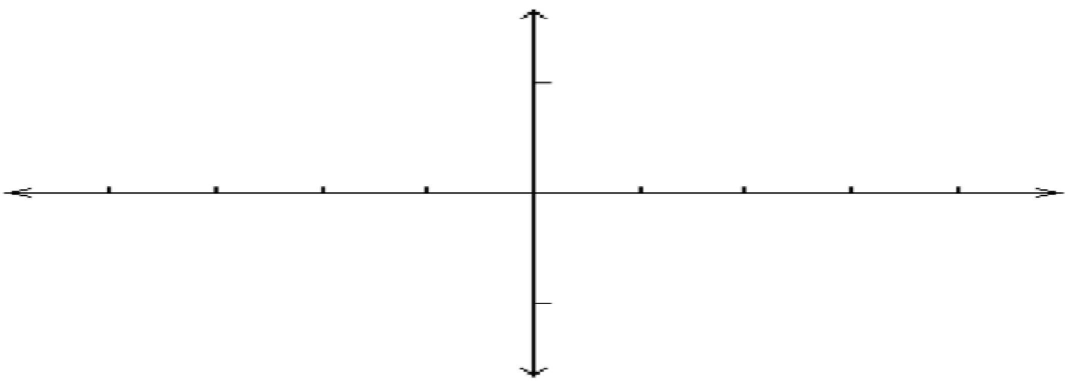
3. $y = 2\sin\left(x - \frac{\pi}{2}\right) + 1$



4. $y = 3\cos cos\left(\frac{1}{2}x + \frac{\pi}{8}\right) - 1$



5. $y = 2\cos cos\,(x + \pi)$



Applications

Course Day: _______

A tsunami (a.k.a. tidal wave) is a fast-moving ocean wave caused by an underwater earthquake. The water first goes down from its normal level, then rises an equal distance above its normal level, and finally returns to its normal level. The period is about 15 minutes.

Suppose that a tsunami with amplitude of 10 meters approaches the pier at Honolulu, where the normal depth of the water is 9 meters. First <u>sketch the graph</u> and <u>write the equation</u>, then answer the following questions.

a. Assuming that the depth of the water varies sinusoidally with time as the tsunami passes, predict the depth of the water at the following times after the tsunami first reaches the pier:
   i. 2 minutes
   ii. 4 minutes
   iii. 12 minutes

b. According to your model, what will the minimum depth of the water be? How do you interpret this answer in terms of what will happen in the real world?

c. The "crest" refers to the highest point of a wave. The "wavelength" is the distance a crest of the wave travels in one period. It is also equal to the distance between two adjacent crests. If a tsunami travels at 745 miles per hour, what is its wavelength?

The Ferris Wheel Problem

You have probably noticed that as you ride a Ferris wheel, your distance from the ground varies sinusoidally with time. When the last seat is filled and the Ferris wheel starts, your seat is at the position shown in the figure to the right. Let *t* be the number of seconds that have elapsed since the Ferris wheel started. You find that it takes you 3 seconds to reach the top, 43 feet above the ground, and that the wheel makes a revolution once every 8 seconds. The diameter of the wheel is 40 feet. Your mission, should you choose to accept it, is to do the following:

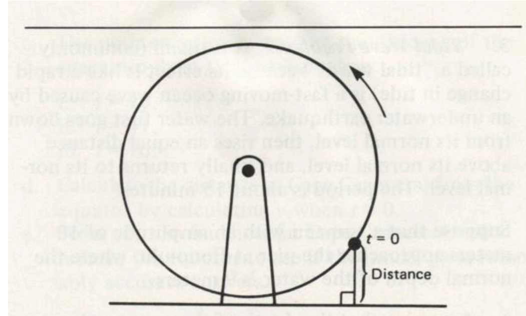


Figure 13-8d.

a. Sketch a graph of this sinusoid.

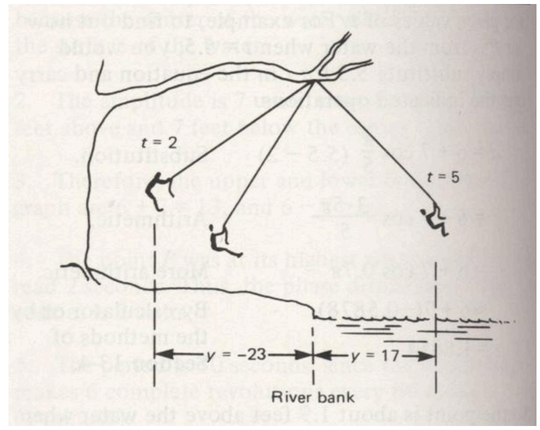
b. What is the lowest you go as the Ferris wheel turns, and why is this number greater than zero?

c. Write the particular equation of this sinusoid in radians.

d. Predict your height above the ground when:

    i. $t = 6$
    ii. $t = 4\frac{1}{3}$
    iii. $t = 9$
    iv. $t = 0$

The Tarzan Problem



Tarzan is swinging back and forth on his grapevine. As he swings, he goes back and forth across the riverbank, going alternately over land and water (see the figure). Jane decides to mathematically model his motion and starts her stopwatch. Let *t* be the number of seconds the stopwatch reads and let *y* be the number of meters Tarzan is from the riverbank. Assume that *y* varies sinusoidally with *t*, and that *y* is positive when Tarzan is over the water and negative when he is over land.

Jane finds that when $t = 2$, Tarzan is at one end of his swing, where $y = -23$. She finds that when $t = 5$ he reaches the other end of his swing and $y = 17$. Please do the following:

a. Sketch the graph of this sinusoidal function.

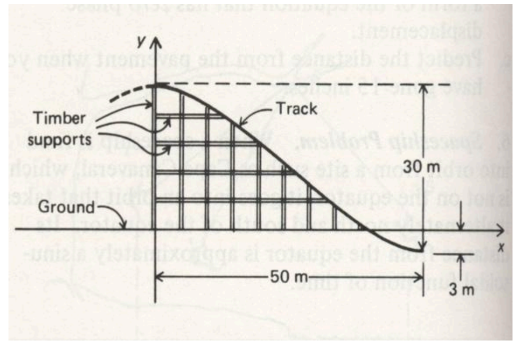
b. Write the equation expressing Tarzan's distance from the riverbank in terms of t.

c. Predict y when:
   i. $t = 2.8$
   ii. $t = 6.3$
   iii. $t = 15$

d. Where was Tarzan when Jane started her stopwatch?

Roller Coaster Problem

A portion of a roller coaster track is to be built in the shape of a sinusoid (see the figure). You have been hired to calculate the lengths of the vertical timber supports to be used.



a. The high and low points on the track are separated by 50 meters horizontally and by 30 meters vertically. The low point is 3 meters below the ground. Letting *y* be the number of meters the track is above ground and *x* be the number of meters horizontally from the high point, write the equation expressing *y* in terms of *x*. (i.e. y = _______)

b. How long is the vertical timber at the high point? At x = 4 meters? At x = 32 meters?

Pebble-in-the-Tire Problem

As you stop your car at a traffic light, a pebble becomes wedged between the tire treads. When you start off, the distance of the pebble from the pavement varies sinusoidally with the distance you have traveled. The period is the circumference of the wheel. Assume that the diameter of the wheel is 24 inches.

a. Sketch a graph of this function.

b. Write the equation as a cosine function.

c. Write the equation as a sine function.

d. Predict the distance of the pebble to the pavement when you have gone 15 inches.