AMM Math

Consider an AMM

U = USD in AMM
B = Bitcoin in AMM

You sell x BTC to AMM in exchange for a quantity y of dollars.

Result 1: $y = (U * x) / (B+x)$.

After the transaction, there is B+x Bitcoin in the AMM and U-y USD
The product must stay the same so,

$(B+x)(U-y) = B*U$

$U-y = B*U / (B+x)$
therefore

$y = U - B*U / (B+x)$

$y = (U * x) / (B+x)$

QED

Example: U = 200,000, B = 10, x=2

$y = (2 * 200,000) / 12 = 33,333.$

Result 2: The price transacted is $U / (B+x)$

Proof. The price transacted PT is y / x.

$= (U * x / (B+x)) x = U / (B+x)$

Example as before U = 200,000, B=10, x = 2

$P2 = 200,000 / 12 = 16,666$

Result 3: The new price is $B*U / ((B+x)*(B+x))$

Proof. The new price is $(U-y)/(B+x)$

$= B*U / ((B+x)*(B+x))$

QED

Example as before $U = 200,000$, $B=10$, $x = 2$

$P2 = 10* 200,000 / (12*12) = 13,889$

Result 4: The slippage is x / B

Proof:

Old Price = U / B

Transacted Price = $U / (B+x)$ (from Result 2)

Therefore Transacted Price / Old Price = $(B+ x) / B = 1 + x / B$

QED

Intuition:

If I sell 10% of the pool in BTC, My price will move by 10%.

Result 5: If you want to sell to bring the new price to a market price PM, you need To sell Sqrt ((B*U)/PM)) - B Bitcoin

$PM = B*U / (B+x)^2$ (by result 3)

Therefore $x = \mathrm{Sqrt}(B*U/PM) - B$.

QED

Result 6: To arb the market you will get slippage of Sqrt (B*U / PM) / B - 1 + FTX + 0.001

To move to new price we need to sell

X = Sqrt (B * U / PM) - B

And we now slippage is

X / B

So, replacing

Slippage = Sqrt (B * U / PM) / B - 1

We also need to add slippage of FTX and Fees

Net Slippage = Sqrt (B*U / PM) / B - 1 + FTX + 0.001

Result 7. You need to send Y = U*X / (B+x) USD to equilibrate

If you sell X BTC, the price transacted is U / (B+x)

So you are exchanging U*X / (B+x) USD

Result 8 (with fees). If the fees are f you need to solve the quadratic equation ax^2+bx+x=0 with

a = pm*(1+f)
b = 2 B * pm * (1+f) - U * f
c = (1+f) * B * (pm*B - U)

Proof

With f

$y = (u*x) / (B+x) (1+f)$

$PM = (u - y)/(b+x)$
$= ((1+f)(B+x)*U-Ux) / (B+x)^2*(1+ff)$
$= u * (B+fB=Fx) / (B+x)^2*(1+ff)$

Therefore $pm (B+x)^2(1+f) = u B * (1+f) +U f x$

Bringing all the terms together gives the quadratic formula

QED