Suggested topics for presentations

Geometry Graduate Student Seminar - Spring 2023

- Lie group homomorphisms and the covering map SU(2) → SO(3) (see Hall Section 1.4)
- Connected Lie groups, disconnected Lie groups, and the identity component (see Hall Section 1.3.2)
- Lie algebras and the matrix exponential map (see Hall 2nd edition Section 2.1 and 3.3 and Lee 2nd edition pages 516-522)
- Lie brackets and left-invariant vector fields (see Lee 2nd edition pages 185-199)
- Proof of fundamental theorem of algebra using differential topology (see Milnor, *Topology from the Differentiable Viewpoint*, page 8)
- Brouwer fixed point theorem (see Milnor, *Topology from the Differentiable Viewpoint*, Section 2, statement on page 14)
- Morse functions (see Guillemin and Pollack Chapter 1 Section 7 and Milnor, *Morse Theory*, Part 1 Section 2)
- Hyperbolic space (see Petersen 3rd ed Example 1.1.7; relate to rotationally symmetric metrics (Petersen Example 1.4.5); see also Petersen 3rd ed Section 4.4.2)
- Lie groups with biinvariant metrics (see Petersen 3rd ed Prop 4.4.2; cover examples (SO(n), SU(n)); see also Lee Intro to Riem Manifolds, 2nd ed, Cor 3.15)
- Riemannian maps:
    - first lecture: Riemannian isometries and isometry groups (see Petersen 3rd ed page 3 and Section 1.3.1)
    - second lecture: Riemannian isometries vs. immersions vs. submersions (see Petersen 3rd ed pages 3-5; cover examples (recall isometry groups of Euclidean plane, sphere, hyperbolic plane (first lecture); Petersen Examples 1.1.3 - 1.1.5; Do Carmo page 185 Exercise 8))
- Homogeneous spaces:
    - In the smooth category, G-homogeneous spaces are manifolds M that admit a smooth action by a Lie group G. Describe necessary and sufficient conditions for a G-homogeneous space M to admit a Riemannian metric that is invariant under the G-action (see Lee Intro to Riem Manifolds, 2nd ed, Thm 3.17 and Cor 3.18)
    - (see also Cheeger and Ebin, Chapter 3, Def. 3.10, Def. 3.12, parts of Prop. 3.16; see also Koda, *An introduction to the geometry of homogeneous spaces*, 2009, Sections 2.1-2.2; cover examples (sphere, Euclidean space, hyperbolic space, real projective space, complex projective space, tori, etc.); there's a lot here, so ask Lawrence for suggestions)
- Killing vector fields, which are vector fields whose flows are by isometries (see Petersen 3rd ed pages 51-52 and Section 8.1)
- Examples of geodesics (see Examples 5.2.7 - 5.2.11 in Petersen 3rd ed)

Lawrence is happy to help presenters develop a presentation that appeals to their interests. Feel free to propose topics beyond those listed above.