Name:

APPLICATION: Recursive Formulas for Loan Balances

Level 1

Paying Down an Auto Loan

Morgan recently bought a used car. They took out an auto loan for $12,000 at a fixed 4.8% APR.

The table below represents their auto loan balance over time. Each month, they accrue interest at the same percentage rate and make the same monthly payment.

n # of months	**f(n)** Loan balance ($)
0	12000
1	$12000(1 + \frac{0.048}{12}) - 225 = 11823$
2	$11823(1 + \frac{0.048}{12}) - 225 = 11645$
3	$11645(1 + \frac{0.048}{12}) - 225 = 11467$
4	$11466(1 + \frac{0.048}{12}) - 225 = 11288$

1. **Describe the pattern of how the loan balance changes.**

2. **How do you know the sequence in the table is recursive?**

3. **How much does Morgan pay towards their auto loan each month?**

4. **Morgan's <u>annual</u> interest rate is 4.8%. What is their <u>monthly</u> interest rate?**

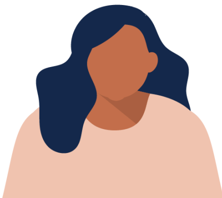
5. **Write the recursive formula to find their balance, f(n), after n months.**

6. **How much will Morgan owe after 5 months?**

7. **Alan reviewed Morgan's loan information and said "I think it will take Morgan 53 months to repay their auto loan. I found that estimate by dividing the loan amount by the monthly payment. 12000 ÷ 225 = 53." Is Alan correct? Why or why not?**

Level 2

Paying Off Student Loans



Jada recently graduated from college with $34,000 in federal student loans at a fixed 3.73% annual interest rate, compounded monthly. She makes a monthly payment of $340 with the goal of paying her loans off in ten years.

1. **What is the <u>monthly</u> interest rate on Jada's student loans? Round to the nearest thousandth of a percent.**

2. **Complete the table with Jada's loan balance over the first three months of repayment.**

n # of months	**f(n)** Loan balance ($)
0	34,000
1	
2	
3	

3. **Describe how you could use the third month's balance to find the fourth month's balance.**

4. **Write the recursive formula to find her balance, f(n), after n months.**

5. Open the Loan Repayment Calculator and enter Jada's loan information: $34,000 in student loans with a 10-year term and 3.73% APR.

a. What is the total cost of Jada's student loans?

b. Click "Show Amortization Schedule". What is Jada's loan balance after 5 years?

c. Why is a calculator more useful than your formula for answering questions a and b?

Level 3

Comparing Repayment Schedules

Haruto recently bought his first home, a small two-bedroom house in Kansas City for $160,500. He put down a $25,000 down payment and financed the remaining cost with a $135,500 30-year mortgage at a fixed 5.5% APR, compounded monthly. His monthly mortgage payment is $769.

Part I: Repayment Options

1. **Write a recursive formula to represent Haruto's loan balance, f(n) after n months.**

2. **Haruto is considering increasing his monthly payment to $900 to pay off his mortgage sooner. Write a recursive formula to represent his balance, g(n), after n months.**

3. **Use the Loan Repayment Calculator to complete the table below. First, enter Haruto's information with the minimum $769 monthly payment and complete the first column. Then, click "add extra payments" to complete the second column with the additional $131 payment.**

	Option 1: $769 monthly payment (minimum payment only)	**Option 2: $900 monthly payment** ($250 minimum + $150 additional)
Total interest paid		
Total cost of the loan		
Time to repayment		

Part II: Investment Value

Haruto wants to invest any additional money that he isn't paying towards his mortgage. Remember that we can use the following formula to estimate the future value of a periodic investment.

Future Value of a Period Deposit Formula

$$B = \frac{D\left(\left(1+\frac{r}{n}\right)^{n\cdot t}-1\right)}{\frac{r}{n}}$$

B = balance at the end of the investment period
D = periodic deposit amount
r = annual interest rate or rate of return
n = number of times interest is compounded annually
t = length of investment in years

4. Haruto wants to know if he should invest the additional $131 each month instead of making additional payment towards his mortgage. Use the Future Value of a Periodic Deposit Formula to calculate his investment's predicted value after 30 years, assuming he earns an 8% annual rate of return, compounded monthly.

5. However, if Haruto pays the additional $131 towards his mortgage, he'll repay the loan sooner and can start investing the full $900.

   a. Haruto wants to compare both scenarios on a 30-year timeline. How many months will he be investing $900 if he only starts AFTER he's paid off his mortgage? Refer back to Question 4.

   b. Use the Future Value of a Periodic Deposit Formula to calculate his investment's predicted value, assuming the same 8% annual rate of return, compounded monthly.

6. Should Haruto make additional mortgage payments or invest the additional money? Justify your reasoning. If you're unsure, what additional information would you need to decide?

7. We assumed Haruto's investments would earn an 8% annual rate of return; however, investment returns aren't guaranteed and can be unpredictable. What is the minimum rate of return that Haruto would need to earn to make it worth investing additional money rather than paying it towards his mortgage? Justify your reasoning.

Challenge: Explicit Formulas

Recursive and Explicit Equations
In this lesson, you learned about recursive formulas that give the value of a term based on the previous term. Many functions can be represented by BOTH a recursive formula and an explicit formula.

An explicit formula defines each term in a sequence directly. It does NOT rely on knowing the value of the previous terms. You've already seen many explicit formulas, including linear equations like y = 325x - 127.50 and exponential equations like $f(x) = 1500(1.09)^x$.

1. **Match the recursive formula with the corresponding explicit formula.**

A. f(1) = 500
f(n) = f(n-1) + 400

B. f(0) = 400
f(n) = 1.004 · f(n-1)

C. f(1) = 500
f(n) = 1.25 · f(n-1)

D. f(2) = 312
f(n) = f(n-1) + 94

E. f(0) = 312
f(n) = 1.0025 · f(n-1)

F. f(2) = 400
f(n) = 2 · f(n-1)

G. f(2) = 100
f(n) = ½ · f(n-1)

I. $y = 400(1.25)^x$

II. $y = 94x + 500$

III. $y = \frac{400}{2^x}$

IV. $y = 400\left(1 + \frac{0.048}{12}\right)^x$

V. $y = 100 \cdot 2^x$

VI. $y = 400x + 100$

VII. $y = 312\left(1 + \frac{0.03}{12}\right)^x$

2. **The table and graph represent the balance of a Roth IRA investment account with a recurring $6000 annual deposit, assuming an 8% annual rate of return.**

x year	**f(x)** balance ($)
0	6000
1	12480
2	19478.40
3	27036.67
4	35199.61

a. Write a <u>recursive formula</u> for the account balance, f(x), after x years.

b. David writes the following non-recursive equation to represent the balance in the Roth IRA after x years: $f(x) = 6000(1.08)^x + 6000x$

Why is David's equation NOT an accurate representation of the account balance?

c. Write an <u>explicit formula</u> for the account balance $f(x)$ after x months.

d. What is the advantage of having an explicit equation to model the account's growth, rather than only a recursive formula?

3. Write a recursive formula that would be challenging to write as an explicit formula. Explain your reasoning.