ESCUELA POLITECNICA NACIONAL

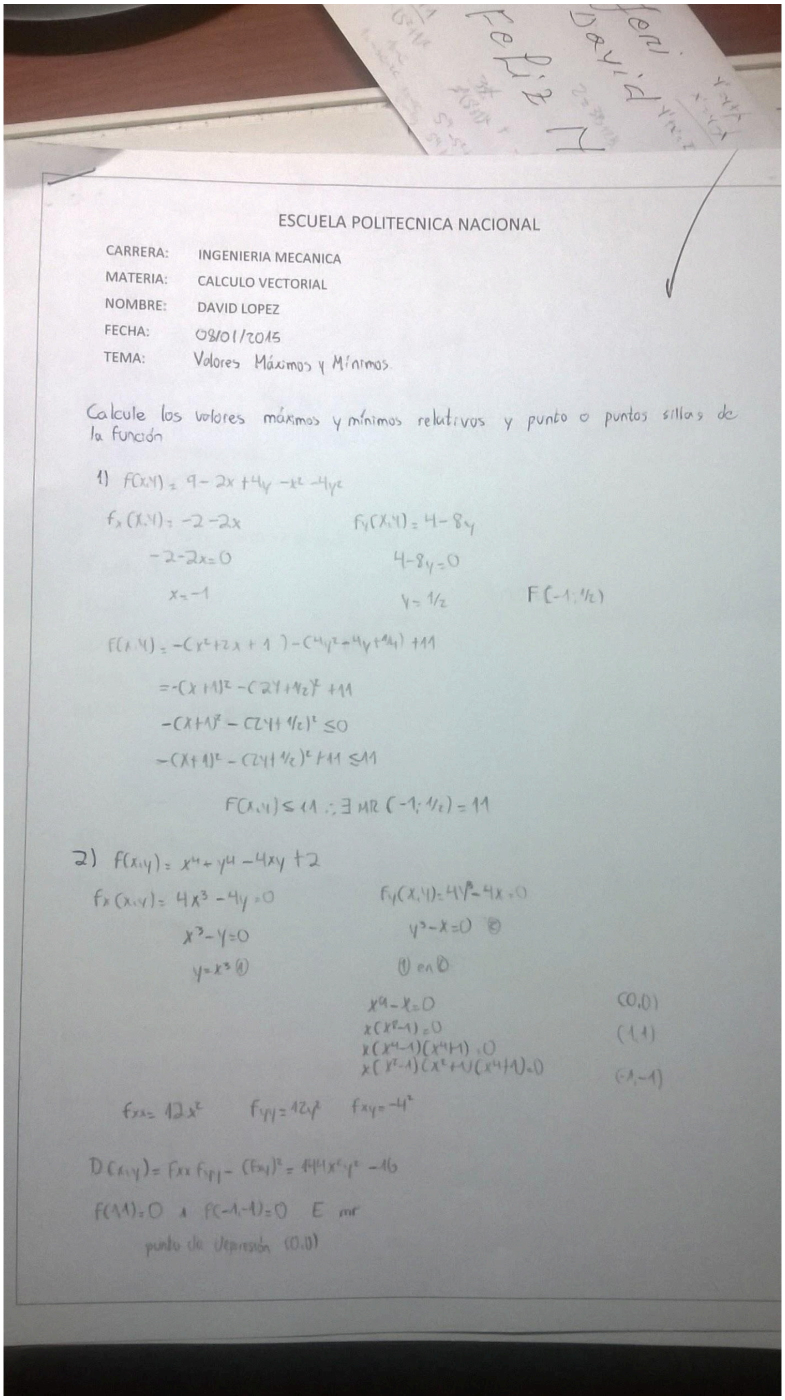
CARRERA: INGENIERIA MECANICA

MATERIA: CALCULO VECTORIAL

NOMBRE: DAVID LOPEZ

FECHA: 13/02/2015

TEMA: Deberes



ESCUELA POLITECNICA NACIONAL

CARRERA: INGENIERIA MECANICA

MATERIA: CALCULO VECTORIAL

NOMBRE: DAVID LOPEZ

FECHA: 08/01/2015

TEMA: Valores Máximos y Mínimos

Calcule los valores máximos y mínimos relativos y punto o puntos sillas de la función

1) $f(x,y) = 9 - 2x + 4y - x^2 - 4y^2$

$f_x(x,y) = -2-2x$ $\quad$ $f_y(x,y) = 4-8y$

$-2-2x=0$ $\quad$ $4-8y=0$

$x=-1$ $\quad$ $y = 1/2$ $\quad$ $F(-1; 1/2)$

$f(x,y) = -(x^2+2x+1) - (4y^2-4y+1/4) + 11$

$= -(x+1)^2 - (2y+1/2)^2 + 11$

$-(x+1)^2 - (2y+1/2)^2 \leq 0$

$-(x+1)^2 - (2y+1/2)^2 + 11 \leq 11$

$F(x,y) \leq 11 \therefore \exists$ MR $(-1; 1/2) = 11$

2) $f(x,y) = x^4 + y^4 - 4xy + 2$

$f_x(x,y) = 4x^3 - 4y = 0$ $\quad$ $f_y(x,y) = 4y^3 - 4x = 0$

$x^3 - y = 0$ $\quad$ $y^3 - x = 0$ ②

$y = x^3$ ① $\quad$ ① en ②

$x^9 - x = 0$ $\quad$ $(0,0)$

$x(x^8-1) = 0$ $\quad$ $(1,1)$

$x(x^4-1)(x^4+1) = 0$

$x(x^2-1)(x^2+1)(x^4+1) = 0$ $\quad$ $(-1,-1)$

$f_{xx} = 12x^2$ $\quad$ $f_{yy} = 12y^2$ $\quad$ $f_{xy} = -4^2$

$D(x,y) = f_{xx}f_{yy} - (f_{xy})^2 = 144x^2y^2 - 16$

$f(1,1) = 0 \wedge f(-1,-1) = 0$ E mr

punto de depresión $(0,0)$

$F(x,y) = x^3y + 12x^2 - 8y$

$F_x(x,y) = 3x^2y + 24x = 0$ $\qquad$ $F_y(x,y) = x^3 - 8 = 0$

$x = 2$

$3(2)^2y + 24(2) = 0$

$y = -4$ $\qquad$ $P(2;-4)$

$F_{xx} = 24xy + 24$

$F_{xy} = 3x^2$

$F_{yy} = 0$

$D(x,y) = (24xy+24)(0) - (3x^2)^2$

$D(x,y) = -9x^4$

$D<0$

$F_{xx}(2;-4) = -168$

$F_{xx} < 0$

$\exists$ MR $(2;-4)$

4) $f(x,y) = (1+xy)(x+y) \rightarrow x+y+x^2y+xy^2$

$F_x(x,y) = 1 + 2xy + y^2$ $\qquad$ $F_y(x,y) = 1 + x^2 + 2xy$

$F_{xx} = 2y$ $\qquad$ $F_{yy} = 2x$ $\qquad$ $F_{xy} = 2x + 2y$

$D(x,y) = (2y)(2x) - (2x+2y)^2$

$D(x,y) = 4xy - 4x^2 - 8xy - 4y^2$

$D(x,y) = -4x^2 - 4xy - 4y^2$

5) $f(x,y) = 2x^3 + xy^2 + 5x^2 + y^2$

$F_x(x,y) = 6x^2 + y^2 + 10x = 0$ $\qquad$ $F_y(x,y) = 2yx + 2y = 0$

$6x^2 + 10x = 0$ $\qquad$ $y = 0$

$x = 0 \ \wedge \ x = -\frac{5}{3}$

$F_{xx} = 12x + 10$ $\qquad$ $F_{yy} = 2x + 2$ $\qquad$ $F_{xy} = 2y$

$D(x,y) = (12x+10)(2x+2) - (2y)^2$

$D(x,y) = 24x^2 + 24x + 20x + 20 - 4y^2$

$D(x,y) = 24x^2 + 44x + 20 - 4y^2$

$D(-5/3, 0) = \frac{40}{3}$ $\qquad$ $F_{xx}(-5/3, 0) = 12\left(\frac{-5}{3}\right) + 10 = -10$

$D > 0$ $\qquad$ $F_{xx} < 0$

6) $f(x,y) = x^3 - 12xy + 6y^3$

$F_x(x,y) = 3x^2 - 12y = 0$

$y = \frac{1}{4}x^2$

$y = \frac{1}{4}(2)^2$

$y = 1$

$F_y(x,y) = -12x + 24y^2 = 0$

$-12x + 24\left(\frac{1}{4}x^2\right)^2 = 0$

$\frac{3}{2}x^4 = 12x$

$x = 2$

$P(2,1)$

$F_{xx} = 6x$ $\quad F_{yy} = 48y$ $\quad F_{xy} = -12$

$D(x,y) = (6x)(48y) - (-12)^2$

$D(x,y) = 288xy - 144$

$D(2,1) = 432$ $\quad F_{xx} = 6(2) = 12$ $\quad$ Existe un minimo en $(2,1)$

$D > 0$ $\quad F_{xx} > 0$

7) $f(x,y) = y\cos x$

$F_x(x,y) = -y\,\mathrm{sen}\,x = 0$

$-y\,\mathrm{sen}\,90 = 0$

$y = 0$

$F_y(x,y) = \cos x = 0$

$x = 90°$

$P(90°, 0)$

$F_{xx} = -y\cos x$ $\quad F_{yy} = 0$ $\quad F_{xy} = -\mathrm{sen}\,x$

$D(x,y) = (-y\cos x)(0) - (-\mathrm{sen}\,x)^2$

$D(x,y) = -\mathrm{sen}^2 x$

$D(90°,0) = -\mathrm{sen}^2(90) = -1$ $\quad F_{xx}(90°,0) = -0(\cos(90)) = 0$

$D \leq 0$ $\quad D = 0$

8) $F(x,y) = xy + \frac{1}{x} + \frac{1}{y}$

$F_x(x,y) = y - \frac{1}{x^2}$

$y = \frac{1}{x^2}$

$y = 1$

$F_y(x,y) = x - \frac{1}{y^2}$

$x - x^4 = 0$

$x^3 = 1 \rightarrow x = 1$

$P(1,1)$

$F_{xx} = \frac{2}{x^3}$ $\quad F_{yy} = \frac{2}{y^3}$ $\quad F_{xy} = 1$

$D(x,y) = \left(\frac{2}{x^3}\right)\left(\frac{2}{y^3}\right) - 1^2$

$D(x,y) = \frac{4}{x^3y^3} - 1$

$D(1,1) = 3$ $\qquad$ $F_{xx}(1,1) = \frac{2}{1^3} = 2$ $\qquad$ Existe un mínimo en $(1,1)$

$D \geq 0$ $\qquad$ $D \geq 0$

4) $f(x,y) = 1 + 4x - 5y$, D es la región triangular cerrada con vertices $(0,0)$, $(2,0)$ y $(0,3)$.

$F_x(x,y) = 4$ $\qquad$ $F_y(x,y) = -5$ $\qquad$ $x = (0,0)$

$f(2,0) = 1 + 4(2) - 5(0)$

$f(2,0) = 9$

$f(0,3) = 1 + 4(0) - 5(3)$

$f(0,3) = -14$

$F(0,0) = 1$

$1 < 9$

$f(x,y) < f(a,b)$

∴ Existe un máximo relativo en $(2,0) = 9$

$1 > -14$

$F(x,y) > f(a,b)$

∴ Existe un mínimo relativo en $(0,3) = -14$

10) $F(x,y) = 3 + xy - x - 2y$, D es la región triangular cerrada con vertices $(1,0)$, $(5,0)$ y $(1,4)$

$F_x(x,y) = y - 1 = 0$ $\qquad$ $F_y(x,y) = x - 2 = 0$

$y = 1$ $\qquad$ $x = 2$ $\qquad$ $P(2,1)$

$F(2,1) = 3 + (2)(1) - (2) - (2)(1)$

$f(2,1) = 1$

$F(1,0) = 3 + (1)(0) - 1 - 2(0)$ $\qquad$ $f(5,0) = 3 + (5)(0) - (5) - 2(0)$ $\qquad$ $F(1,4) = 3 + (1)(4) - (1) - 2(4)$

$F(1,0) = 2$ $\qquad$ $F(5,0) = -2$ $\qquad$ $F(1,4) = -2$

$1 < 2$

$F(x,y) < F(1,0)$

∴ Existe un máximo relativo en $(1,0) = 2$

$1 > -2$

$F(x,y) > f(a,b)$

∴ Existe un mínimo relativo en $(5,0), (1,4) = -2$