

ISSUE 517: Does the axiom of non-reflexivity follow from the definition of transitivity?

Draft, CEO 26.10.2021

The potential transitive, symmetric and/or reflexive properties are the properties with identical domain and range. The task is to go through the list of these properties (32 in all) and check whether they are transitive/not transitive, symmetric/non-symmetric or reflexive/non-reflexive. To make the CRMbase document clear, I suggest we add three statements to all these properties:

This property is transitive/not transitive, symmetric/non-symmetric and reflexive/non-reflexive.

In this document for each property the current definition is examined and possible amendments are suggested.

Transitivity, symmetry, reflexivity

1. By transitivity we mean that for all x,y,z : $[P(x,y) \wedge P(y,z)] \Rightarrow P(x,z)$.
 - a. A property P is not transitive if there exists at least one triple x,y,z such that $[P(x,y) \wedge P(y,z)] \Rightarrow P(x,z)$ is not the case

A property is intransitive iff there exists no triple x,y,z that fulfils the transitivity criterion, that is, $[P(x,y) \wedge P(y,z)] \Rightarrow \neg P(x,z)$

2. By symmetry we mean that for all x,y : $P(x,y) \Rightarrow P(y,x)$.
 - a. A property P is not symmetric if there exists at least one pair x,y such that $P(x,y) \Rightarrow P(y,x)$ is not the case
 - b. A property is asymmetric iff there is no pair x, y that fulfils the symmetry criterion, that is, $P(x,y) \Rightarrow \neg P(y,x)$

3. By reflexivity we mean that for all x,y : $P(x,x)$.
 - a. A property P is not reflexive if there exists at least one x such that $P(x,x)$ is not the case
 - b. A property P is irreflexive iff there is no x that fulfils the reflexivity criterion, that is $\neg P(x,x)$

In the current CRMbase v. 7.1.2 non-symmetric and non-reflexive is used instead of asymmetric and irreflexive.

A non-symmetric property will also be non-reflexive. Thus it is redundant to state that a non-symmetric relation is non-reflexive. Similarly, a property will be reflexive. Thus, in this case it is redundant to state that the property is reflexive.

For clarity, I suggest that we add the axioms for (non-)transitivity, (non-)reflexivity and (non-)symmetry in all cases where appropriate even though we may state redundant axioms.

Some properties are declared as 'not transitive', for example *P69 has association with (is associated with)* and *P122 borders with*. It is unclear what is meant: 1.a or 1.b? The case 1.a is weaker than 1.b. If we are not sure, we can add an axiom of the form $\exists x \exists y \exists z. [P(x,y) \wedge P(y,z)] \Rightarrow \neg P(x,z)$. We can also leave it open. This will simply be an underspecification.

The potentially transitive, reflexive or symmetric properties

The list below consist of all properties with identical domain and range plus 'P165 incorporates (is incorporated in)'. The latter needs some extra comments. The text should be read together with the complete CRMbase v. 7.1.2.

The text highlighted with red needs considerations and possibly amendments in the definition of the property.

P5 consists of (forms part of)

Domain: E3 Condition State

Range: E3 Condition State

Quantification: many to many (0,n:0,n)

...This property is transitive and **non-reflexive**...

In First Order Logic:

$$P5(x,y) \Rightarrow E3(x)$$

$$P5(x,y) \Rightarrow E3(y)$$

$$[P5(x,y) \wedge P5(y,z)] \Rightarrow P5(x,z)$$

$$P5(x,y) \Rightarrow \neg P5(y,x)$$

Comment

Replace 'non-reflexive' with 'non-symmetric'. As mentioned above the non-symmetric implies non-reflexive. Add the axiom $\neg P5(x,x)$?

P9 consists of (forms part of)

Domain: E4 Period

Range: E4 Period

Subproperty of: E92 Spacetime Volume. P10i contains: E92 Spacetime Volume

Quantification: many to many (0,n:0,n)

...This property is transitive and non-symmetric...

In First Order Logic:

$$P5(x,y) \Rightarrow E3(x)$$

$$P5(x,y) \Rightarrow E3(y)$$

$$[P5(x,y) \wedge P5(y,z)] \Rightarrow P5(x,z)$$

$$P5(x,y) \Rightarrow \neg P5(y,x)$$

Comment

P10 falls within (contains)

Domain: E92 Spacetime Volume

Range: E92 Spacetime Volume

Subproperty of: E92 Spacetime Volume. P132 spatiotemporally overlaps with: E92 Spacetime Volume

Quantification: many to many, necessary, dependent (1,n:0,n)

...This property is transitive and reflexive...

In First Order Logic:

$$P10(x,y) \Rightarrow E92(x)$$

$$P10(x,y) \Rightarrow E92(y)$$

$$P10(x,y) \Rightarrow P132(x,y)$$

$$P10(x,y) \wedge P10(y,z) \Rightarrow P10(x,z)$$

$$P10(x,x)$$

Comment

OK

P46 is composed of (forms part of)

Domain: E18 Physical Thing

Range: E18 Physical Thing

Superproperty of: E19 Physical Object. P56 bears feature (is found on): E26 Physical Feature

Quantification: many to many (0,n:0,n)

...This property is transitive and non-reflexive...

In First Order Logic:

$$P46(x,y) \Rightarrow E18(x)$$

$$P46(x,y) \Rightarrow E18(y)$$

$$P46(x,y) \Rightarrow P132(x,y)$$

$$[P46(x,y) \wedge P46(y,z)] \Rightarrow P46(x,z)$$

$$P46(x,y) \Rightarrow (\exists uzv)[E93(u) \wedge P195i(x,u) \wedge E52(z) \wedge P164(u,z) \wedge E93(v) \wedge P195i(v,y) \wedge$$

$$P164(w,z) \wedge P10(w,u)]$$

$$\neg P46(x,x)$$

Comment

OK

P69 has association with (is associated with)

Domain: E29 Design or Procedure

Range: E29 Design or Procedure

Quantification: many to many (0,n:0,n)

...The property is not transitive...

In First Order Logic:

$$P69(x,y) \Rightarrow E29(x)$$

$$P69(x,y) \Rightarrow E29(y)$$

$$P69(x,y,z) \Rightarrow [P69(x,y) \wedge E55(z)]$$

Comment

Reflexive, non-reflexive, neither? Move the statement to the end of the scopenote, otherwise ok.

P73i has translation (is translation of)

Domain: E33 Linguistic Object

Range: E33 Linguistic Object

Subproperty of: E70 Thing. P130i features are also found on: E70 Thing

Quantification: many to many (0,n:0,n)

...This property is non-symmetric...

In First Order Logic:

$$P73(x,y) \Rightarrow E33(x)$$

$$P73(x,y) \Rightarrow E33(y)$$

$$P73(x,y) \Rightarrow P130i(x,y)$$

$$P73(x,y) \Rightarrow \neg P73(y,x)$$

Comment

Transitive or not transitive? Neither is stated in the scopenote

P86 falls within (contains)

Domain: E52 Time-Span

Range: E52 Time-Span

Quantification: many to many (0,n:0,n)

... This property is transitive...

In First Order Logic:

$$P86(x,y) \Rightarrow E52(x)$$

$$P86(x,y) \Rightarrow E52(y)$$

$$[P86(x,y) \wedge P86(y,z)] \Rightarrow P86(x,z)$$

Comment

OK, but is the property reflexive, cf. P89

P89 falls within (contains)

Domain: E53 Place

Range: E53 Place

Quantification: many to many, necessary, dependent (1,n:0,n)

...This property is transitive and reflexive...

In First Order Logic:

$$P89(x,y) \Rightarrow E53(x)$$

$$P89(x,y) \Rightarrow E53(y)$$

$$[P89(x,y) \wedge P89(y,z)] \Rightarrow P89(x,z)$$

$$P89(x,x)$$

Comment

OK

P106 is composed of (forms part of)

Domain: E90 Symbolic Object

Range: E90 Symbolic Object

Superproperty of: E73 Information Object. P165 incorporates (is incorporated in): E90 Symbolic Object

Quantification: many to many (0,n:0,n)

...This property is transitive and non-reflexive...

In First Order Logic:

$P106(x,y) \Rightarrow E90(x)$

$P106(x,y) \Rightarrow E90(y)$

$[P106(x,y) \wedge P106(y,z)] \Rightarrow P106(x,z)$

$\neg P106(x,x)$

Comment

OK

P121 overlaps with

Domain: E53 Place

Range: E53 Place

Quantification: many to many (0,n:0,n)

...This property is symmetric...

In First Order Logic:

$P121(x,y) \Rightarrow E53(x)$

$P121(x,y) \Rightarrow E53(y)$

$P121(x,y) \Rightarrow P121(y,x)$

Comment

Missing: This property is not transitive, reflexive, non-reflexive, neither?

Add reflexive in the scope note and in the FOL

P122 borders with

Domain: E53 Place

Range: E53 Place

Quantification: many to many (0,n:0,n)

...This property is not transitive. This property is symmetric...

In First Order Logic:

$P122(x,y) \Rightarrow E53(x)$

$P122(x,y) \Rightarrow E53(y)$

$P122(x,y) \Rightarrow P122(y,x)$

Comment

OK, reflexive, non-reflexive, neither?

P127 has broader term (has narrower term)

Domain: E55 Type

Range: E55 Type

Quantification: many to many (0,n:0,n)

...This property is transitive...

In First Order Logic:

$P127(x,y) \Rightarrow E55(x)$

$P127(x,y) \Rightarrow E55(y)$

$[P127(x,y) \wedge P127(y,z)] \Rightarrow P127(x,z)$

Comment

OK, reflexive, non-reflexive, neither?

P130 shows features of (features are also found on)

Domain: E70 Thing

Range: E70 Thing

Superproperty of: E33 Linguistic Object. P73i is translation of: E33 Linguistic Object

E18 Physical Thing. P128 carries (is carried by): E90 Symbolic Object

Quantification: many to many (0,n:0,n)

...This property is not transitive...

In First Order Logic:

$P130(x,y) \Rightarrow E70(x)$

$P130(x,y) \Rightarrow E70(y)$

$P130(x,y,z) \Rightarrow [P130(x,y) \wedge E55(z)]$

Comment

OK, reflexive, non-reflexive, neither? Symmetric, non-symmetric?

P132 spatiotemporally overlaps with

Domain: E92 Spacetime Volume

Range: E92 Spacetime Volume

Superproperty of: E4 Period. P9 consists of (forms part of): E4 Period

E92 Spacetime Volume. P10 falls within (contains): E92 Spacetime Volume

Quantification: many to many (0,n:0,n)

...This property is symmetric...

In First Order Logic:

$P132(x,y) \Rightarrow E92(x)$

$P132(x,y) \Rightarrow E92(y)$

$P132(x,y) \Rightarrow P132(y,x)$

$P132(x,y) \Rightarrow P133(x,y)$

Comment

Missing: This property is not transitive

P133 is spatiotemporally separated from

Domain: E92 Spacetime Volume

Range: E92 Spacetime Volume

Quantification: many to many (0,n:0,n)

...This property is not transitive. This property is symmetric.

In First Order Logic:

$$P133(x,y) \Rightarrow E92(x)$$

$$P133(x,y) \Rightarrow E92(y)$$

$$P133(x,y) \Rightarrow P133(y,x)$$

$$P133(x,y) \Rightarrow \neg P132(x,y)$$

Comment

Missing: non-reflexive

P134 continued (was continued by)

Domain: E7 Activity

Range: E7 Activity

Subproperty of: E7 Activity. P15 was influenced by (influenced): E1 CRM Entity

E2 Temporal Entity. P176i starts before the start of (starts after the start of): E2 Temporal Entity

Quantification: many to many (0,n:0,n)

...This property is not transitive...

In First Order Logic:

$$P134(x,y) \Rightarrow E7(x)$$

$$P134(x,y) \Rightarrow E7(y)$$

$$P134(x,y) \Rightarrow P15(x,y)$$

$$P134(x,y) \Rightarrow P176i(x,y)$$

Comment

Missing non-reflexive

P139 has alternative form

Domain: E41 Appellation

Range: E41 Appellation

Quantification: many to many (0,n:0,n)

... It is an asymmetric relationship, where the range expresses the derivative, if such a direction can be established. Otherwise, the relationship is symmetric. The relationship is not transitive...

In First Order Logic:

$$P139(x,y) \Rightarrow E41(x)$$

$$P139(x,y) \Rightarrow E41(y)$$

$$P139(x,y,z) \Rightarrow [P139(x,y) \wedge E55(z)]$$

$$P139(x,y) \Rightarrow P139(y,x)$$

$\neg P139(x,x)$

Comment

Reflexive? It is neither symmetric nor non-symmetric, see the beginning of this document (case 2 a).

P148 has component (is component of)

Domain: E89 Propositional Object

Range: E89 Propositional Object

Quantification: many to many (0:n,0:n)

...This property is transitive...

In First Order Logic:

$P148(x,y) \Rightarrow E89(x)$

$P148(x,y) \Rightarrow E89(y)$

$[P148(x,y) \wedge P148(y,z)] \Rightarrow P148(x,z)$

Comment

Non-symmetric?

P150 defines typical parts of (defines typical wholes for)

Domain: E55 Type

Range: E55 Type

Quantification: many to many (0,n:0,n)

...This property is not transitive...

In First Order Logic:

$P150(x,y) \Rightarrow E55(x)$

$P150(x,y) \Rightarrow E55(y)$

Comment

Non-symmetric?

P152 has parent (is parent of)

Domain: E21 Person

Range: E21 Person

Quantification: many to many, necessary (2,n:0,n)

...This property is not transitive...

In First Order Logic:

$P152(x,y) \Rightarrow E21(x)$

$P152(x,y) \Rightarrow E21(y)$

$P152(x,y) \Leftarrow (\exists z) [E67(z) \wedge P98i(x,z) \wedge P96(z,y)]$

$P152(x,y) \Leftarrow (\exists z) [E67(z) \wedge P98i(x,z) \wedge P97(z,y)]$

Comment

(and of course not reflexive or symmetric) to be added to the FOL

P165 incorporates (is incorporated in)

Domain: E73 Information Object

Range: E90 Symbolic Object

Subproperty of: E90 Symbolic Object. P106 is composed of (forms part of): E90 Symbolic Object

Quantification: many to many (0,n:0,n)

...When restricted to information objects, that is, seen as a property with E73 Information Object as domain and range the property is transitive. ...

In First Order Logic:

$P165(x,y) \Rightarrow E73(x)$

$P165(x,y) \Rightarrow E90(y)$

$P165(x,y) \Rightarrow P106(x,y)$

Comment

Reflexive?

P173 starts before or with the end of (ends after or with the start of)

Domain: E2 Temporal Entity

Range: E2 Temporal Entity

Superproperty of: E2 Temporal Entity. P174 starts before the end of (ends after the start of): E2 Temporal Entity

Quantification: many to many (0,n:0,n)

In First Order Logic:

$P173(x,y) \Rightarrow E2(x)$

$P173(x,y) \Rightarrow E2(y)$

Comment

The property is not transitive Transitive in a non fuzzy model(?)

P174 starts before the end of (ends after the start of)

Domain: E2 Temporal Entity

Range: E2 Temporal Entity

Subproperty of: E2 Temporal Entity. P173 starts before or at the end of (ends after or with the start of): E2 Temporal Entity

Superproperty of: E2 Temporal Entity. P175 starts before or with the start of (starts after or with the start of): E2 Temporal Entity

E2 Temporal Entity. P184 ends before or with the end of (ends with or after the end of): E2 Temporal Entity

Quantification: many to many (0,n:0,n)

... This property is not transitive...

In First Order Logic:

$P174(x,y) \Rightarrow E2(x)$

$P174(x,y) \Rightarrow E2(y)$

$P174(x,y) \Rightarrow P173(x,y)$

Comment

Non-reflexive?

P175 starts before or with the start of (starts after or with the start of)

Domain: E2 Temporal Entity

Range: E2 Temporal Entity

Subproperty of: E2 Temporal Entity. P174 starts before the end of (ends after the start of): E2 Temporal Entity

Superproperty of: E2 Temporal Entity. P176 starts before the start of (starts after the start of): E2 Temporal Entity

Quantification: many to many (0,n:0,n)

...In a model with fuzzy borders, this property will not be transitive. ...

In First Order Logic:

$$P175(x,y) \Rightarrow E2(x)$$

$$P175(x,y) \Rightarrow E2(y)$$

$$P175(x,y) \Rightarrow P174(x,y)$$

Comment

Non-reflexive?

P176 starts before the start of (starts after the start of)

Domain: E2 Temporal Entity

Range: E2 Temporal Entity

Subproperty of: E2 Temporal Entity. P175 starts before or with the start of (starts after or with the start of): E2 Temporal Entity

Superproperty of: E7 Activity. P134i was continued by: E7 Activity

E2 Temporal Entity. P182 ends before or at the start of (starts after or with the end of): E2 Temporal Entity

Quantification: many to many (0,n:0,n)

... This property is transitive. ...

In First Order Logic:

$$P176(x,y) \Rightarrow E2(x)$$

$$P176(x,y) \Rightarrow E2(y)$$

$$P176(x,y) \Rightarrow P175(x,y)$$

$$[P176(x,y) \wedge P176(y,z)] \Rightarrow P176(x,z)$$

Comment

Non-Reflexive?

P182 ends before or with the start of (starts after or with the end of)

Domain: E2 Temporal Entity

Range: E2 Temporal Entity

Subproperty of: E2 Temporal Entity. P176 starts before the start of (starts after the start of): E2 Temporal Entity (transitive)
E2 Temporal Entity. P185 ends before the end of (ends after the end of): E2 Temporal Entity (transitive)

Superproperty of: E2 Temporal Entity. P183 ends before the start of (starts after the end of): E2 Temporal Entity

Quantification: many to many (0,n:0,n)

In First Order Logic:

$P182(x,y) \Rightarrow E2(x)$
 $P182(x,y) \Rightarrow E2(y)$
 $P182(x,y) \Rightarrow P176(x,y)$
 $P182(x,y) \Rightarrow P185(x,y)$

Comment

This property is transitive(?)

P183 ends before the start of (starts after the end of)

Domain: E2 Temporal Entity

Range: E2 Temporal Entity

Subproperty of: E2 Temporal Entity. P182 ends before or at the start of (starts after or with the end of): E2 Temporal Entity

Quantification: many to many (0,n:0,n)

In First Order Logic:

$P183(x,y) \Rightarrow E2(x)$
 $P183(x,y) \Rightarrow E2(y)$
 $P183(x,y) \Rightarrow P182(x,y)$
 $[P183(x,y) \wedge P183(y,z)] \Rightarrow P183(x,z)$

Comment

'This property is transitive' should be added to the scopenote. Reflexive?

P184 ends before or with the end of (ends with or after the end of)

Domain: E2 Temporal Entity

Range: E2 Temporal Entity

Subproperty of: E2 Temporal Entity. P174 starts before the end of (ends after the start of): E2 Temporal Entity (not transitive)

Superproperty of: E2 Temporal Entity. P185 ends before the end of (ends after the end of): E2 Temporal Entity (transitive)

Quantification: many to many (0,n:0,n)

In First Order Logic:

$P184(x,y) \Rightarrow E2(x)$
 $P184(x,y) \Rightarrow E2(y)$
 $P184(x,y) \Rightarrow P174(x,y)$

Comment

Transitive in a non fuzzy model(?)

P185 ends before the end of (ends after the end of)

Domain: E2 Temporal Entity

Range: E2 Temporal Entity

Subproperty of: E2 Temporal Entity. P184 ends before or with the end of (ends with or after the end of):
E2 Temporal Entity

Superproperty of: E2 Temporal Entity. P182 ends before or at the start of (starts after or with the end
of): E2 Temporal Entity

Quantification: many to many (0,n:0,n)

... This property is transitive. ...

In First Order Logic:

$$P185(x,y) \Rightarrow E2(x)$$

$$P185(x,y) \Rightarrow E2(y)$$

$$P185(x,y) \Rightarrow P184(x,y)$$

$$[P185(x,y) \wedge P185(y,z)] \Rightarrow P185(x,z)$$

Comment

Non-reflexive?

P189 approximates (is approximated by)

Domain: E53 Place

Range: E53 Place

Quantification: many to many (0,n:0,n)

... This property is not transitive. ...

In First Order Logic:

$$P189(x,y) \Rightarrow E53(x)$$

$$P189(x,y) \Rightarrow E53(y)$$

$$P189(x,y,z) \Rightarrow [P189(x,y) \wedge E53(z)]$$

Comment

Non/Reflexive?

P198 holds or supports (is held or supported by)

Domain: E18 Physical Thing

Range: E18 Physical Thing

Quantification: one to many (0,n:0,n)

... This property is transitive. ...

In First Order Logic:

$$P198(x,y) \Rightarrow E18(x)$$

$$P198(x,y) \Rightarrow E18(y)$$

$$[P198(x,y) \wedge P198(y,z)] \Rightarrow P198(x,z)$$

$$[P198(x,y) \wedge P198(z,y)] \Rightarrow [P198(x,z) \vee P198(z,x)]$$
$$P198(x,y) \Leftarrow (\exists z) [E53(z) \wedge P59(x,z) \wedge P53i(z,y)]$$

Comment

Non reflexive?