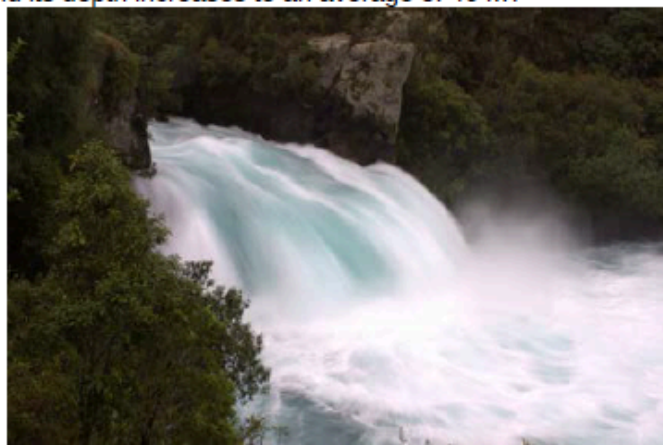


## Book Questions

**Flow rate**

1. What is the average flow rate in  $\text{cm}^3/\text{s}$  of gasoline to the engine of a car traveling at 100 km/h if it averages 10.0 km/L?
2. The heart of a resting adult pumps blood at a rate of 5.00 L/min. (a) Convert this to  $\text{cm}^3/\text{s}$ . (b) What is this rate in  $\text{m}^3/\text{s}$ ?
3. Blood is pumped from the heart at a rate of 5.0 L/min into the aorta (of radius 1.0 cm). Determine the speed of blood through the aorta.
4. Blood is flowing through an artery of radius 2 mm at a rate of 40 cm/s. Determine the flow rate and the volume that passes through the artery in a period of 30 s.
5. The Huka Falls on the Waikato River is one of New Zealand's most visited natural tourist attractions (see **Figure 12.30**). On average the river has a flow rate of about 300,000 L/s. At the gorge, the river narrows to 20 m wide and averages 20 m deep. (a) What is the average speed of the river in the gorge? (b) What is the average speed of the water in the river downstream of the falls when it widens to 60 m and its depth increases to an average of 40 m?



**Figure 12.30** The Huka Falls in Taupo, New Zealand, demonstrate flow rate. (credit: RaviGogna, Flickr)

6. A major artery with a cross-sectional area of  $1.00 \text{ cm}^2$  branches into 18 smaller arteries, each with an average cross-sectional area of  $0.400 \text{ cm}^2$ . By what factor is the average velocity of the blood reduced when it passes into these branches?

9. (a) Estimate the time it would take to fill a private swimming pool with a capacity of 80,000 L using a garden hose delivering 60 L/min. (b) How long would it take to fill if you could divert a moderate size river, flowing at  $5000 \text{ m}^3/\text{s}$ , into it?

10. The flow rate of blood through a  $2.00 \times 10^{-6}\text{-m}$  -radius capillary is  $3.80 \times 10^9 \text{ cm}^3/\text{s}$ . (a) What is the speed of the blood flow? (This small speed allows time for diffusion of materials to and from the blood.) (b) Assuming all the blood in the body passes through capillaries, how many of them must there be to carry a total flow of  $90.0 \text{ cm}^3/\text{s}$ ? (The large number obtained is an overestimate, but it is still reasonable.)

11. (a) What is the fluid speed in a fire hose with a 9.00-cm diameter carrying 80.0 L of water per second? (b) What is the flow rate in cubic meters per second? (c) Would your answers be different if salt water replaced the fresh water in the fire hose?

12. The main uptake air duct of a forced air gas heater is 0.300 m in diameter. What is the average speed of air in the duct if it carries a volume equal to that of the house's interior every 15 min? The inside volume of the house is equivalent to a rectangular solid 13.0 m wide by 20.0 m long by 2.75 m high.

13. Water is moving at a velocity of 2.00 m/s through a hose with an internal diameter of 1.60 cm. (a) What is the flow rate in liters per second? (b) The fluid velocity in this hose's nozzle is 15.0 m/s. What is the nozzle's inside diameter?

14. Prove that the speed of an incompressible fluid through a constriction, such as in a Venturi tube, increases by a factor equal to the square of the factor by which the diameter decreases. (The converse applies for flow out of a constriction into a larger-diameter region.)

15. Water emerges straight down from a faucet with a 1.80-cm diameter at a speed of 0.500 m/s. (Because of the construction of the faucet, there is no variation in speed across the stream.) (a) What is the flow rate in  $\text{cm}^3/\text{s}$ ? (b) What is the diameter of the stream 0.200 m below the faucet? Neglect any effects due to surface tension.

## Bernoulli's equation

21. Every few years, winds in Boulder, Colorado, attain sustained speeds of 45.0 m/s (about 100 mi/h) when the jet stream descends during early spring. Approximately what is the force due to the Bernoulli effect on a roof having an area of  $220 \text{ m}^2$ ? Typical air density in Boulder is  $1.14 \text{ kg/m}^3$ , and the corresponding atmospheric pressure is  $8.89 \times 10^4 \text{ N/m}^2$ . (Bernoulli's principle as stated in the text assumes laminar flow. Using the principle here produces only an approximate result, because there is significant turbulence.)

22. (a) Calculate the approximate force on a square meter of sail, given the horizontal velocity of the wind is 6.00 m/s parallel to its front surface and 3.50 m/s along its back surface. Take the density of air to be  $1.29 \text{ kg/m}^3$ . (The calculation, based on Bernoulli's principle, is approximate due to the effects of turbulence.) (b) Discuss whether this force is great enough to be effective for propelling a sailboat.

23. (a) What is the pressure drop due to the Bernoulli effect as water goes into a 3.00-cm-diameter nozzle from a 9.00-cm-diameter fire hose while carrying a flow of 40.0 L/s? (b) To what maximum height above the nozzle can this water rise? (The actual height will be significantly smaller due to air resistance.)

24. (a) Using Bernoulli's equation, show that the measured fluid speed  $v$  for a pitot tube, like the one in **Figure 12.7(b)**,

is given by 
$$v = \left( \frac{2\rho'gh}{\rho} \right)^{1/2},$$

where  $h$  is the height of the manometer fluid,  $\rho'$  is the density of the manometer fluid,  $\rho$  is the density of the moving fluid, and  $g$  is the acceleration due to gravity. (Note that  $v$  is indeed proportional to the square root of  $h$ , as stated in the text.) (b) Calculate  $v$  for moving air if a mercury manometer's  $h$  is 0.200 m.

26. A frequently quoted rule of thumb in aircraft design is that wings should produce about 1000 N of lift per square meter of wing. (The fact that a wing has a top and bottom surface does not double its area.) (a) At takeoff, an aircraft travels at 60.0 m/s, so that the air speed relative to the bottom of the wing is 60.0 m/s. Given the sea level density of air to be

$1.29 \text{ kg/m}^3$ , how fast must it move over the upper surface

to create the ideal lift? (b) How fast must air move over the upper surface at a cruising speed of 245 m/s and at an altitude where air density is one-fourth that at sea level?

(Note that this is not all of the aircraft's lift—some comes from the body of the plane, some from engine thrust, and so on. Furthermore, Bernoulli's principle gives an approximate answer because flow over the wing creates turbulence.)

27. The left ventricle of a resting adult's heart pumps blood at a flow rate of  $83.0 \text{ cm}^3/\text{s}$ , increasing its pressure by 110 mm Hg, its speed from zero to 30.0 cm/s, and its height by 5.00 cm. (All numbers are averaged over the entire heartbeat.) Calculate the total power output of the left ventricle. Note that most of the power is used to increase blood pressure.

28. A sump pump (used to drain water from the basement of houses built below the water table) is draining a flooded basement at the rate of 0.750 L/s, with an output pressure of

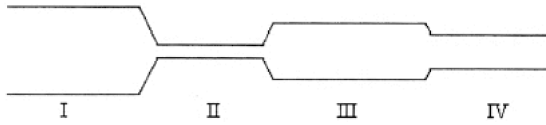
$3.00 \times 10^5 \text{ N/m}^2$ . (a) The water enters a hose with a

3.00-cm inside diameter and rises 2.50 m above the pump.

What is its pressure at this point? (b) The hose goes over the foundation wall, losing 0.500 m in height, and widens to 4.00 cm in diameter. What is the pressure now? You may neglect frictional losses in both parts of the problem.

## Workbook Questions

3. A fluid is forced through a pipe of changing cross section as shown. In which section would the pressure of the fluid be a minimum?

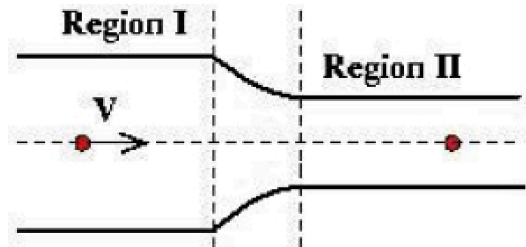


A) I B) II C) III D) IV

Water flows through a horizontal pipe. The diameter of the pipe at point B is larger than at point A. Where is the water pressure greater?

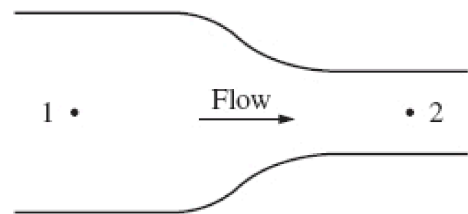
- A) Point A
- B) Point B
- C) Same at both A and B
- D) Cannot be determined from the information given.

14. The speed of an ideal fluid is marked as it moves along a horizontal streamline through a pipe, as shown in the figure. In Region I, the speed of the fluid on the streamline is  $V$ . The cylindrical, horizontal pipe narrows so that the radius of the pipe in Region II is half of what it was in Region I. What is the speed of the marked fluid when it is in Region II?



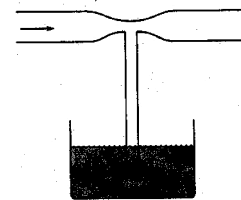
(A)  $4V$  (B)  $2V$  (C)  $V/2$  (D)  $V/4$

15. A fluid flows steadily from left to right in the pipe shown. The diameter of the pipe is less at point 2 than at point 1, and the fluid density is constant throughout the pipe. How do the velocity of flow and the pressure at points 1 and 2 compare?



- Velocity Pressure
- (A)  $v_1 < v_2$   $p_1 = p_2$
  - (B)  $v_1 < v_2$   $p_1 > p_2$
  - (C)  $v_1 = v_2$   $p_1 < p_2$
  - (D)  $v_1 > v_2$   $p_1 = p_2$

18. **Multiple Correct:** A T-shaped tube with a constriction is inserted in a vessel containing a liquid, as shown. What happens if air is blown through the tube from the left, as shown by the arrow in the diagram? Select two answers.

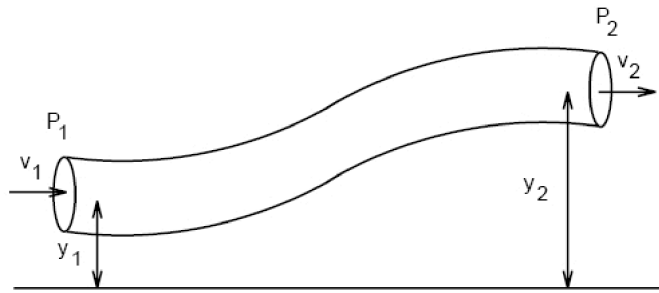


- (A) The liquid level in the tube rises to a level above the surface of the liquid in the surrounding tube
- (B) The liquid level in the tube falls below the level of the surrounding liquid
- (C) The pressure in the liquid in the constricted section increases.

(D) The pressure in the liquid in the constricted section decreases.

26. Water flows in a pipe of uniform cross-sectional area  $A$ .

The pipe changes height from  $y_1 = 2$  meters to  $y_2 = 3$  meters. Since the areas are the same, we can say  $v_1 = v_2$ . Which of the following is true?



(A)  $P_1 = P_2 + \rho g(y_2 - y_1)$

(B)  $P_1 = P_2$

(C)  $P_1 = 0$

(D)  $P_2 = 0$

# Answer key

## 12.1 FLOW RATE AND ITS RELATION TO VELOCITY

1. *What is the average flow rate in  $\text{cm}^3/\text{s}$  of gasoline to the engine of a car traveling at 100 km/h if it averages 10.0 km/L?*

Solution 
$$Q = \frac{V}{t} = \frac{\text{speed}}{\text{gas mileage}} = \frac{100 \text{ km/h}}{10.0 \text{ km/L}} \times \frac{1000 \text{ cm}^3}{1 \text{ L}} \times \frac{1 \text{ h}}{3600 \text{ s}} = \underline{2.78 \text{ cm}^3/\text{s}}$$

2. *The heart of a resting adult pumps blood at a rate of 5.00 L/min. (a) Convert this to  $\text{cm}^3/\text{s}$ . (b) What is this rate in  $\text{m}^3/\text{s}$ ?*

Solution

(a) 
$$\left( \frac{5.00 \text{ L}}{1 \text{ min}} \right) \left( \frac{1000 \text{ cm}^3}{1 \text{ L}} \right) \left( \frac{1 \text{ min}}{3600 \text{ s}} \right) = 1.389 \text{ cm}^3/\text{s} = \underline{1.39 \text{ cm}^3/\text{s}}$$

(b) 
$$(1.389 \text{ cm}^3/\text{s}) \left( \frac{1 \text{ m}^3}{10^6 \text{ cm}^3} \right) = 1.389 \times 10^{-6} \text{ m}^3/\text{s} = \underline{1.39 \times 10^{-6} \text{ m}^3/\text{s}}$$

3. *Blood is pumped from the heart at a rate of 5.0 L/min into the aorta (of radius 1.0 cm). Determine the speed of blood through the aorta.*

Solution 
$$Q = Av$$
$$v = \frac{Q}{A} = \frac{5000 \text{ cm}^3/60 \text{ s}}{\pi (1.0 \text{ cm})^2} = \underline{27 \text{ cm/s}}$$

4. *Blood is flowing through an artery of radius 2 mm at a rate of 40 cm/s. Determine the flow rate and the volume that passes through the artery in a period of 30 s.*

Solution 
$$Q = Av = \pi r^2 v = \pi (0.20 \text{ cm})^2 (40 \text{ cm/s}) = \underline{5.03 \text{ cm}^3/\text{s}}$$
$$V = Qt = (5.03 \text{ cm}^3/\text{s})(30 \text{ s}) = \underline{151 \text{ cm}^3}$$

5. *The Huka Falls on the Waikato River is one of New Zealand's most visited natural tourist attractions (see [Figure 12.29](#)). On average the river has a flow rate of about 300,000 L/s. At the gorge, the river narrows to 20 m wide and averages 20 m deep.*

(a) What is the average speed of the river in the gorge? (b) What is the average speed of the water in the river downstream of the falls when it widens to 60 m and its depth increases to an average of 40 m?

Solution

$$(a) \quad Q = A\bar{v} \Rightarrow \bar{v} = \frac{Q}{A} = \frac{300 \text{ m}^3/\text{s}}{400 \text{ m}^2} = \underline{0.75 \text{ m/s}}$$

$$(b) \quad Q = A\bar{v} = wd\bar{v} \Rightarrow \bar{v} = \frac{Q}{wd} = \frac{300 \text{ m}^3/\text{s}}{(60 \text{ m})(40 \text{ m})} = \underline{0.13 \text{ m/s}}$$

6. A major artery with a cross-sectional area of  $1.00 \text{ cm}^2$  branches into 18 smaller arteries, each with an average cross-sectional area of  $0.400 \text{ cm}^2$ . By what factor is the average velocity of the blood reduced when it passes into these branches?

Solution

$$Q = A_1\bar{v}_1 = 18A_2\bar{v}_2, \text{ so that } \bar{v}_2 = \frac{A_1}{18A_2}\bar{v}_1 = \frac{(1.00 \text{ cm}^2)}{18(0.400 \text{ cm}^2)}\bar{v}_1 = \underline{0.139\bar{v}_1}$$

9. (a) Estimate the time it would take to fill a private swimming pool with a capacity of 80,000 L using a garden hose delivering 60 L/min. (b) How long would it take to fill if you could divert a moderate size river, flowing at  $5000 \text{ m}^3/\text{s}$ , into it?

Solution

$$(a) \quad t = \frac{V}{Q} = \frac{80000 \text{ L}}{60 \text{ L/min}} = 1300 \text{ min} = \underline{22 \text{ h}}$$

$$(b) \quad t = \frac{V}{Q} = \frac{80000 \text{ L}}{5000000 \text{ L/s}} = \underline{0.016 \text{ s}}$$

10. The flow rate of blood through a  $2.00 \times 10^{-6} \text{ m}$ -radius capillary is  $3.80 \times 10^{-9} \text{ cm}^3/\text{s}$ . (a) What is the speed of the blood flow? (This small speed allows time for diffusion of materials to and from the blood.) (b) Assuming all the blood in the body passes through capillaries, how many of them must there be to carry a total flow of  $90.0 \text{ cm}^3/\text{s}$ ? (The large number obtained is an overestimate, but it is still reasonable.)

Solution

$$(a) \quad Q = A\bar{v} = \pi r^2\bar{v} \Rightarrow \bar{v} = \frac{Q}{\pi r^2} = \frac{3.80 \times 10^{-9} \text{ cm}^3/\text{s}}{\pi(2.00 \times 10^{-4} \text{ cm})^2} = \underline{3.02 \times 10^{-2} \text{ cm/s}}$$

$$(b) \text{ total flow/single capillary flow} = \frac{90.0 \text{ cm}^3/\text{s}}{3.80 \times 10^{-9} \text{ cm}^3/\text{s}} = \underline{2.37 \times 10^{10} \text{ capillaries}}$$

11. (a) What is the fluid speed in a fire hose with a 9.00-cm diameter carrying 80.0 L of water per second? (b) What is the flow rate in cubic meters per second? (c) Would your answers be different if salt water replaced the fresh water in the fire hose?



Solution

$$Q = A\bar{v} = \pi r^2 \bar{v} \Rightarrow v = \frac{Q}{\pi r^2} = \frac{(80.0 \text{ L/s})(\frac{1 \text{ m}^3}{1000 \text{ L}})}{\pi (4.50 \times 10^{-2} \text{ m})^2} = \underline{12.6 \text{ m/s}}$$

(a)

$$Q = (80.0 \text{ L/s}) \times \frac{1 \text{ m}^3}{1000 \text{ L}} = \underline{0.0800 \text{ m}^3/\text{s}}$$

(b)

(c) No, the flow rate and the velocity are independent of the density of the fluid.

12. The main uptake air duct of a forced air gas heater is 0.300 m in diameter. What is the average speed of air in the duct if it carries a volume equal to that of the house's interior every 15 min? The inside volume of the house is equivalent to a rectangular solid 13.0 m wide by 20.0 m long by 2.75 m high.

Solution

$$Q = A\bar{v} \Rightarrow \bar{v} = \frac{Q}{A} = \frac{V/t}{\pi r^2} = \frac{13.0 \text{ m} \times 20.0 \text{ m} \times 2.75 \text{ m}}{(15.0 \text{ min})(60.0 \text{ s/1 min})\pi (0.150 \text{ m})^2} = \underline{11.2 \text{ m/s}}$$

13. Water is moving at a velocity of 2.00 m/s through a hose with an internal diameter of 1.60 cm. (a) What is the flow rate in liters per second? (b) The fluid velocity in this hose's nozzle is 15.0 m/s. What is the nozzle's inside diameter?

Solution

$$(a) Q = A\bar{v} = \pi r^2 \bar{v} = \pi (0.800 \times 10^{-2} \text{ m})^2 (2.00 \text{ m/s}) = \underline{0.402 \text{ L/s}}$$

$$Q = A_1 \bar{v}_1 = A_2 \bar{v}_2, \text{ so that } \left( \frac{1}{4} \pi d_1^2 \right) \bar{v}_1 = \left( \frac{1}{4} \pi d_2^2 \right) \bar{v}_2, \text{ giving}$$

$$(b) d_2 = \left[ d_1^2 \left( \frac{\bar{v}_1}{\bar{v}_2} \right) \right]^{1/2} = d_1 \left( \frac{\bar{v}_1}{\bar{v}_2} \right)^{1/2} = (1.60 \text{ cm}) \left( \frac{2.00 \text{ m/s}}{15.0 \text{ m/s}} \right)^{1/2} = \underline{0.584 \text{ cm}}$$

14. Prove that the speed of an incompressible fluid through a constriction, such as in a Venturi tube, increases by a factor equal to the square of the factor by which the diameter decreases. (The converse applies for flow out of a constriction into a larger-diameter region.)

Solution If the fluid is incompressible, the flow rate through both sides will be equal:

$$Q = A_1 \bar{v}_1 = A_2 \bar{v}_2, \text{ or } \pi \frac{d_1^2}{4} \bar{v}_1 = \pi \frac{d_2^2}{4} \bar{v}_2 \Rightarrow \bar{v}_2 = \bar{v}_1 (d_1^2 / d_2^2) = \underline{\bar{v}_1 (d_1 / d_2)^2}$$

15. Water emerges straight down from a faucet with a 1.80-cm diameter at a speed of 0.500 m/s. (Because of the construction of the faucet, there is no variation in speed across the stream.) (a) What is the flow rate in  $\text{cm}^3/\text{s}$ ? (b) What is the diameter of the stream 0.200 m below the faucet? Neglect any effects due to surface tension.

Solution

$$Q = A\bar{v} = \pi r^2 \bar{v}$$

$$(a) = \pi (9.00 \times 10^{-3} \text{ m})^2 (0.500 \text{ m/s}) = 1.27 \times 10^{-4} \text{ m}^3/\text{s} = \underline{127 \text{ cm}^3/\text{s}}$$

$$v^2 = v_0^2 + 2gx \Rightarrow v = [v_0^2 + 2gx]^{1/2}$$

$$= [(0.500 \text{ m/s})^2 + 2(9.80 \text{ m/s}^2)(0.200 \text{ m})]^{1/2} = 2.042 \text{ m/s}$$

$$Q = \pi r^2 v \Rightarrow$$

$$(b) \quad r = \left( \frac{Q}{\pi v} \right)^{1/2} = \left[ \frac{1.27 \times 10^{-4} \text{ m}^3/\text{s}}{\pi (2.042 \text{ m/s})} \right]^{1/2} = 4.45 \times 10^{-3} \text{ m, or } d = 2r = \underline{0.890 \text{ cm}}$$

21. *Every few years, winds in Boulder, Colorado, attain sustained speeds of 45.0 m/s (about 100 mi/h) when the jet stream descends during early spring. Approximately what is the force due to the Bernoulli effect on a roof having an area of  $220 \text{ m}^2$ ? Typical air density in Boulder is  $1.14 \text{ kg/m}^3$ , and the corresponding atmospheric pressure is  $8.89 \times 10^4 \text{ N/m}^2$ . (Bernoulli's principle as stated in the text assumes laminar flow. Using the principle here produces only an approximate result, because there is significant turbulence.)*

Solution

$$P_1 + \frac{1}{2} \rho v_1^2 + \rho g h_1 = P_2 + \frac{1}{2} \rho v_2^2 + \rho g h_2, \quad \text{so that} \quad P_1 - P_2 = \frac{1}{2} \rho v_2^2. \quad \text{Now,}$$

$$F = (P_1 - P_2)A = \frac{1}{2} \rho (v_2^2 - v_1^2)A \quad \text{so that}$$

$$F = \frac{1}{2} (1.14 \text{ kg/m}^3) [(45 \text{ m/s})^2 - (0.0 \text{ m/s})^2] (220 \text{ m}^2) = \underline{2.54 \times 10^5 \text{ N}}$$

22. (a) *Calculate the approximate force on a square meter of sail, given the horizontal velocity of the wind is 6.00 m/s parallel to its front surface and 3.50 m/s along its back surface. Take the density of air to be  $1.29 \text{ kg/m}^3$ . (The calculation, based on Bernoulli's principle, is approximate due to the effects of turbulence.)* (b) *Discuss whether this force is great enough to be effective for propelling a sailboat.*

Solution

$$P_1 + \frac{1}{2} \rho v_1^2 = P_2 + \frac{1}{2} \rho v_2^2$$

$$F = (P_1 - P_2)A = \frac{1}{2} \rho (v_2^2 - v_1^2)A$$

$$(a) \quad = \frac{1}{2} (1.29 \text{ kg/m}^3) [(6.0 \text{ m/s})^2 - (3.5 \text{ m/s})^2] (1.00 \text{ m}^2) = \underline{15.3 \text{ N}}$$

(b) This force is small, but when the sails are large, the forces can be great enough to propel a sailboat. For larger sailboats, sometimes more than one sail is used to increase the surface areas thereby increasing the force applied.

23. (a) What is the pressure drop due to the Bernoulli effect as water goes into a 3.00-cm-diameter nozzle from a 9.00-cm-diameter fire hose while carrying a flow of 40.0 L/s? (b) To what maximum height above the nozzle can this water rise? (The actual height will be significantly smaller due to air resistance.)

Solution

$$v_1 = \frac{Q}{A_1} = \frac{40.0 \times 10^{-3} \text{ m}^3/\text{s}}{\pi (0.0450 \text{ m})^2} = 6.29 \text{ m/s, and}$$

$$v_2 = \frac{Q}{A_2} = \frac{40.0 \times 10^{-3} \text{ m}^3/\text{s}}{\pi (0.0150 \text{ m})^2} = 56.6 \text{ m/s. So that}$$

$$P_1 - P_2 = \frac{1}{2} \rho (v_2^2 - v_1^2)$$

$$(a) \quad = 0.5 (1.00 \times 10^3 \text{ kg/m}^3) [(56.6 \text{ m/s})^2 - (6.29 \text{ m/s})^2] = \underline{1.58 \times 10^6 \text{ N/m}^2}$$

$$(b) \quad h = \frac{v_0^2}{2g} = \frac{(56.6 \text{ m/s})^2}{2(9.80 \text{ m/s}^2)} = \underline{163 \text{ m}}$$

24. (a) Using Bernoulli's equation, show that the measured fluid speed  $v$  for a pitot tube,

like the one in [Figure 12.7\(b\)](#), is given by  $v = \left( \frac{2\rho'gh}{\rho} \right)^{1/2}$ , where  $h$  is the height of the manometer fluid,  $\rho'$  is the density of the manometer fluid,  $\rho$  is the density of the moving fluid, and  $g$  is the acceleration due to gravity. (Note that  $v$  is indeed proportional to the square root of  $h$ , as stated in the text.) (b) Calculate  $v$  for moving air if a mercury manometer's  $h$  is 0.200 m.

Solution

$$(a) \quad P_1 + \frac{1}{2} \rho v_1^2 + \rho gh_1 = P_2 + \frac{1}{2} \rho v_2^2 + \rho gh_2. \text{ Set } h_1 \text{ and } v_1 = 0.$$

Because  $h_1 \approx h_2$ , the  $\rho gh_1$  and  $\rho gh_2$  terms cancel.

$P_1 - P_2 = \text{manometer pressure} = \rho' gh$ , where  $\rho'$  is the density of the fluid in the

$$\rho' gh = \frac{1}{2} \rho v^2 \text{ and } v = \left( \frac{2\rho' gh}{\rho} \right)^{1/2}$$

pitot tube. Thus,

$$(b) \quad v = \sqrt{\frac{2(13.6 \times 10^3 \text{ kg/m}^3)(9.80 \text{ m/s}^2)(0.200 \text{ m})}{(1.29 \text{ kg/m}^3)}} = \underline{144 \text{ m/s}}$$

### 12.3 THE MOST GENERAL APPLICATIONS OF BERNOULLI'S EQUATION

25. Hoover Dam on the Colorado River is the highest dam in the United States at 221 m, with an output of 1300 MW. The dam generates electricity with water taken from a depth of 150 m and an average flow rate of  $650 \text{ m}^3/\text{s}$ . (a) Calculate the power in this flow. (b) What is the ratio of this power to the facility's average of 680 MW?

Solution

$$\text{power} = (\rho gh)Q$$

$$(a) \quad = (1.00 \times 10^3 \text{ kg/m}^3)(9.80 \text{ m/s}^2)(150 \text{ m})(650 \text{ m}^3/\text{s}) = \underline{9.56 \times 10^8 \text{ W}}$$

$$(b) \quad \frac{956 \text{ MW}}{450 \text{ MW}} = \underline{2.12}$$

26. A frequently quoted rule of thumb in aircraft design is that wings should produce about 1000 N of lift per square meter of wing. (The fact that a wing has a top and bottom surface does not double its area.) (a) At takeoff, an aircraft travels at 60.0 m/s, so that the air speed relative to the bottom of the wing is 60.0 m/s. Given the sea level density of air to be  $1.29 \text{ kg/m}^3$ , how fast must it move over the upper surface to create the ideal lift? (b) How fast must air move over the upper surface at a cruising speed of 245 m/s and at an altitude where air density is one-fourth that at sea level? (Note that this is not all of the aircraft's lift—some comes from the body of the plane, some from engine thrust, and so on. Furthermore, Bernoulli's principle gives an approximate answer because flow over the wing creates turbulence.)

Solution

$$(a) \quad P_1 + \frac{1}{2} \rho v_1^2 = P_2 + \frac{1}{2} \rho v_2^2, \text{ so that } P_1 - P_2 = \Delta P = \frac{1}{2} \rho v_2^2 - \frac{1}{2} \rho v_1^2$$

$$v_2 = \left( \frac{2\Delta P}{\rho} + v_1^2 \right)^{1/2} = \left[ \frac{2(1000 \text{ N/m}^2)}{(1.29 \text{ kg/m}^3)} + (60.0 \text{ m/s})^2 \right]^{1/2} = \underline{71.8 \text{ m/s}}$$

$$(b) \quad v_2 = \left[ \frac{2(1000 \text{ N/m}^2)}{(1.29 \text{ kg/m}^3/4)} + (245 \text{ m/s})^2 \right]^{1/2} = \underline{257 \text{ m/s}}$$

27. The left ventricle of a resting adult's heart pumps blood at a flow rate of  $83.0 \text{ cm}^3/\text{s}$ , increasing its pressure by 110 mm Hg, its speed from zero to 30.0 cm/s, and its height by 5.00 cm. (All numbers are averaged over the entire heartbeat.) Calculate the total

power output of the left ventricle. Note that most of the power is used to increase blood pressure.

Solution

$$\text{power} = \left( P + \frac{1}{2} \rho v^2 + \rho gh \right) Q, \text{ where}$$

$$P = 110 \text{ mm Hg} \times \frac{133 \text{ N/m}^2}{1.0 \text{ mm Hg}} = 1.463 \times 10^4 \text{ N/m}^2,$$

$$\frac{1}{2} \rho v^2 = \frac{1}{2} (1.05 \times 10^3 \text{ kg/m}^3) (0.300 \text{ m/s})^2 = 47.25 \text{ N/m}^2, \text{ and}$$

$$\rho gh = (1.05 \times 10^3 \text{ kg/m}^3) (9.80 \text{ m/s}^2) (0.0500 \text{ m}) = 514.5 \text{ N/m}^2, \text{ giving :}$$

$$\begin{aligned} \text{power} &= (1.463 \times 10^4 \text{ N/m}^2 + 47.25 \text{ N/m}^2 + 514.5 \text{ N/m}^2) (83.0 \text{ cm}^3/\text{s}) \frac{10^{-6} \text{ m}^3}{\text{cm}^3} \\ &= \underline{1.26 \text{ W}} \end{aligned}$$

28. A sump pump (used to drain water from the basement of houses built below the water table) is draining a flooded basement at the rate of 0.750 L/s, with an output pressure of  $3.00 \times 10^5 \text{ N/m}^2$ . (a) The water enters a hose with a 3.00-cm inside diameter and rises 2.50 m above the pump. What is its pressure at this point? (b) The hose goes over the foundation wall, losing 0.500 m in height, and widens to 4.00 cm in diameter. What is the pressure now? You may neglect frictional losses in both parts of the problem.

Solution

$$P_1 + \rho gh_1 = P_2 + \rho gh_2$$

$$P_2 = P_1 + \rho g(h_1 - h_2)$$

$$= 3.00 \times 10^5 \text{ N/m}^2 - (1.00 \times 10^3 \text{ kg/m}^3) (9.80 \text{ m/s}^2) (2.50 \text{ m})$$

$$(a) \quad = \underline{2.76 \times 10^5 \text{ N/m}^2}$$

$$v_1 = \frac{Q}{A_1} = \frac{0.750 \times 10^{-3} \text{ m}^3/\text{s}}{\pi (0.0150 \text{ m})^2} = 1.06 \text{ m/s, and}$$

$$(b) \quad v_2 = \frac{Q}{A_2} = \frac{0.750 \times 10^{-3} \text{ m}^3/\text{s}}{\pi (0.0200 \text{ m})^2} = 0.597 \text{ m/s}$$

$$P_1 + \frac{1}{2} \rho v_1^2 + \rho gh_1 = P_2 + \frac{1}{2} \rho v_2^2 + \rho gh_2, \text{ so that}$$

$$P_2 = P_1 + \rho[g(h_1 - h_2) + \frac{1}{2}(v_1^2 - v_2^2)]$$

$$P_2 = 3.00 \times 10^5 \frac{\text{N}}{\text{m}^2}$$

$$+ (1.00 \times 10^3 \frac{\text{kg}}{\text{m}^3}) \left[ (9.80 \frac{\text{m}}{\text{s}^2})(-2.00 \text{ m}) + \frac{((1.06)^2 - (0.597)^2) \left( \frac{\text{m}^2}{\text{s}^2} \right)}{2} \right]$$

$$P_2 = \underline{2.81 \times 10^5 \text{ N/m}^2}$$

## Workbook Answer key

3. This is based on two principles. 1 – Bernoulli's principle says that when speed increases pressure drops. Second, continuity says more area means less speed based on  $A_1 v_1 = A_2 v_2$ . So the smallest area would have the largest speed and therefore most pressure drop. B
14. Using fluid continuity.  $A_1 v_1 = A_2 v_2$   $\pi R^2 v_1 = \pi (2R)^2 v_2$   $v_1 = 4 v_2$  A
15. Based on the continuity principle, less area means more speed and based on Bernoulli's principle, more speed means less pressure. B
18. As the fluid flows into the smaller area constriction, its speed increases and therefore the pressure drops. Since the pressure in the constriction is less than that outside at the water surface, fluid is forced up into the lower tube. A,D
26. Apply Bernoulli's equation.  $P_1 + \rho g y_1 + \frac{1}{2} \rho v_1^2 = P_2 + \rho g y_2 + \frac{1}{2} \rho v_2^2$  A  
 $P_1 = P_2 + \rho g (y_2 - y_1)$