Contents

Discrete Cascaded Motion Control

Speed Loop:

Governing Equations

$$\frac{\overline{\Omega}_m(z)}{M(z)} = \frac{(K_a\,T_s)\,z + K_a\,T_s}{(2\,J_p)\,z^2 + (-2\,J_p)\,z}$$

Control loop:

$$\frac{(K_{\text{pv}} + K_{\text{iv}}\,T_s)\,z - K_{\text{pv}}}{z - 1}$$

Or,

$$-\frac{K_{\text{pv}} - K_{\text{piv}}\,z}{z - 1}$$

Simplifying,

$$\delta_c = \frac{K_{\text{pv}}}{K_{\text{piv}}}$$

Open loop:

$$\frac{\overline{\Omega}_m(z)}{\overline{\Omega}_{\text{err}}(z)} = -\frac{K_{\text{piv}}\,\left((K_a\,T_s)\,z + K_a\,T_s\right)\,(\delta_c - z)}{\left((2\,J_p)\,z^2 + (-2\,J_p)\,z\right)\,(z - 1)}$$

$$K_{\text{tot}} = \frac{K_a\,K_{\text{piv}}\,T_s}{2\,J_p}$$

Closed Loop:

$$\frac{\overline{\Omega}_m(z)}{\overline{\Omega}^*{}_m(z)} = \frac{K_{\text{tot}}\,z^2 + (K_{\text{tot}} - K_{\text{tot}}\,\delta_c)\,z - K_{\text{tot}}\,\delta_c}{z^3 + (K_{\text{tot}} - 2)\,z^2 + (K_{\text{tot}} - K_{\text{tot}}\,\delta_c + 1)\,z - K_{\text{tot}}\,\delta_c}$$

General approach to tuning Inner Velocity Loop:

Find initial delta_c via a sweep, assess range selected via

1. PZ plot (identify other poles (should be positive, <1)
2. Bode plot < 50 Hz (negative gain window, small phase lag)

Iterate on delta_c.

Utilize delta_c selected to solve for single pole.

Find K_tot via bode plot sweep, selecting for < 50 Hz (negative gain in window, small phase lag). Check PZ Plot to identify locations of other zeros (stable, decaying quickly).

Iterate to manually tune K_tot for a given delta_c.

Tuning

Solving for K_tot

$$K_{\text{tot}} = -\frac{e^{4\pi T_s f_d} - 2\,e^{2\pi T_s f_d} + 1}{e^{2\pi T_s f_d} + e^{4\pi T_s f_d} - \delta_c\,e^{4\pi T_s f_d} - \delta_c\,e^{6\pi T_s f_d}}$$

CE:

$$z - K_{\text{tot}}\,\delta_c + K_{\text{tot}}\,z + K_{\text{tot}}\,z^2 - 2\,z^2 + z^3 - K_{\text{tot}}\,\delta_c\,z$$

System of Equations

eq_1 =

$$\delta_c = \frac{K_{\text{pv}}}{K_{\text{pv}} + K_{\text{iv}}\,T_s}$$

eq_2 =

$$K_{\text{tot}} = \frac{K_a\,T_s\,(K_{\text{pv}} + K_{\text{iv}}\,T_s)}{2\,J_p}$$

Solve:

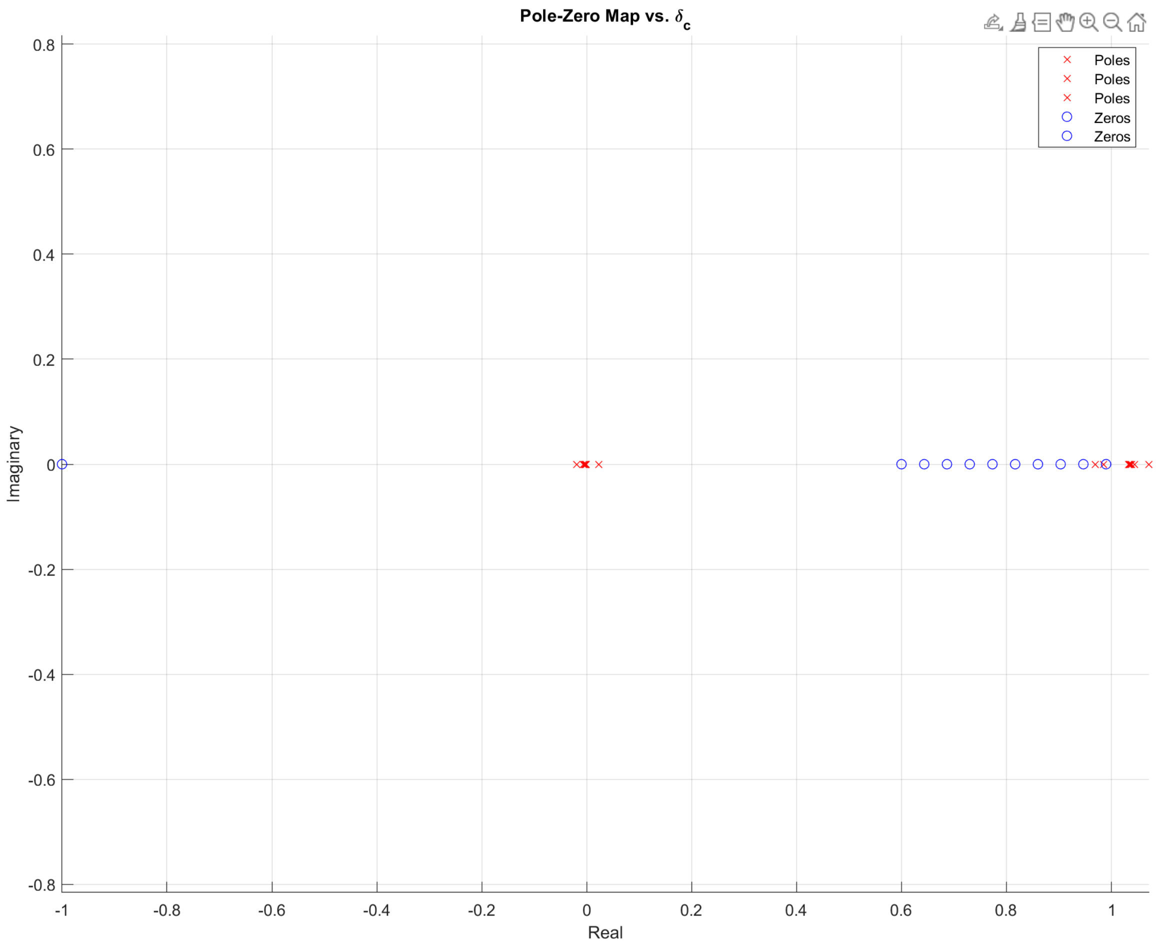
K_pv_sol =

$$\frac{2\,J_p\,K_{\text{tot}}\,\delta_c}{K_a\,T_s}$$

K_iv_sol =

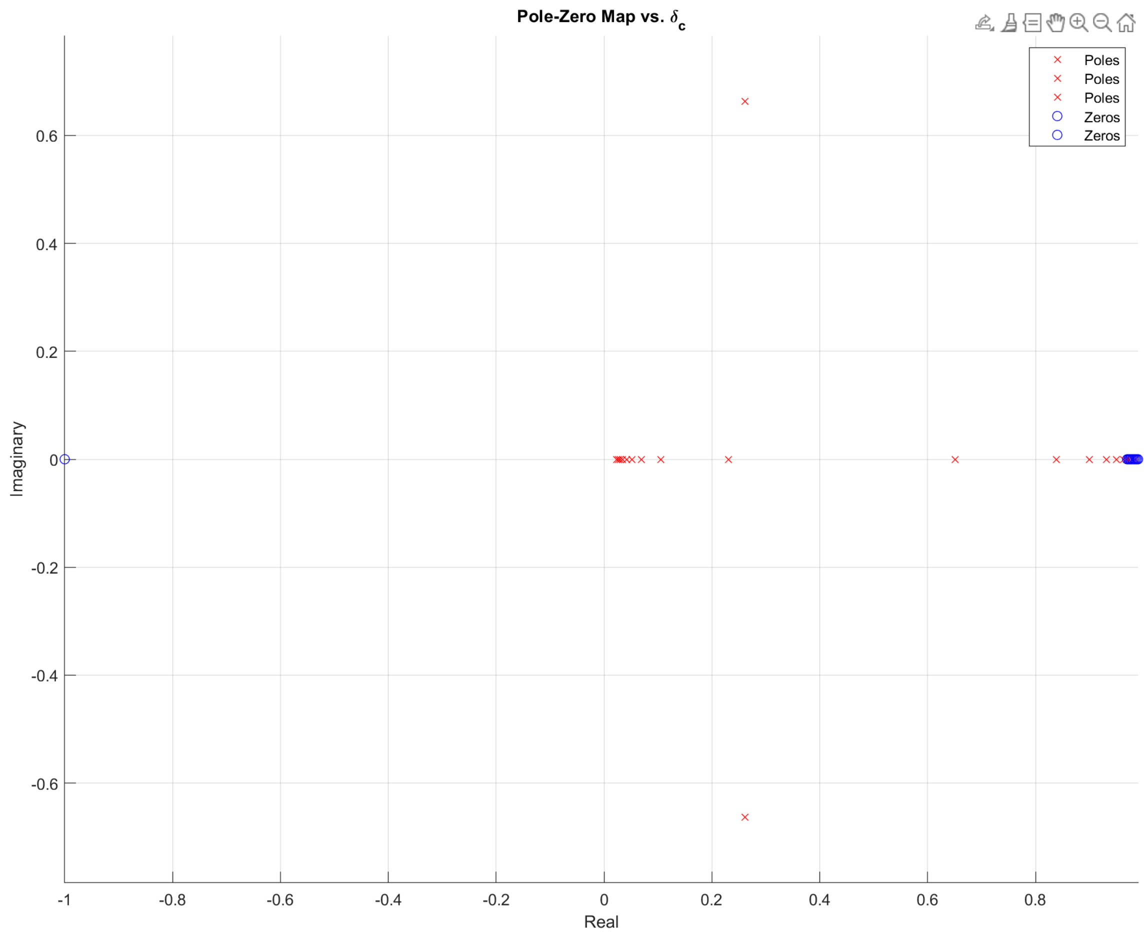
$$\frac{2\,J_p\,K_{\text{tot}} - 2\,J_p\,K_{\text{tot}}\,\delta_c}{K_a\,T_s^2}$$

Sweep delta_c for initial range [0.6, .99]



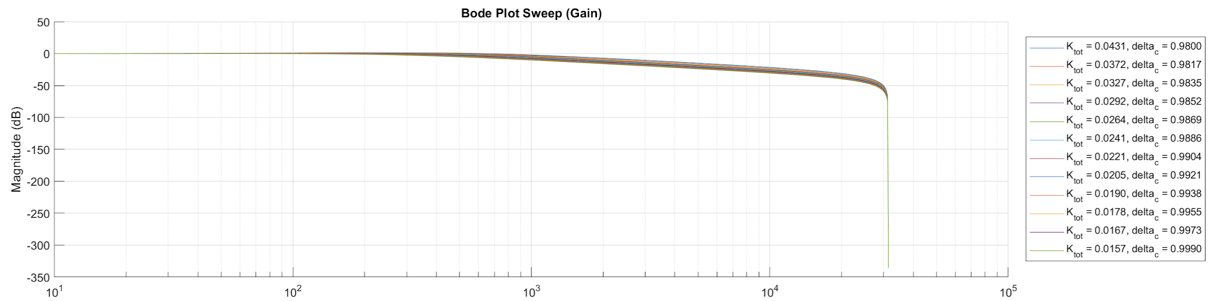

```
sol_Ktot = -0.0013
sol_Ktot = -0.0014
sol_Ktot = -0.0017
sol_Ktot = -0.0020
sol_Ktot = -0.0024
sol_Ktot = -0.0031
sol_Ktot = -0.0043
sol_Ktot = -0.0072
sol_Ktot = -0.0210
sol_Ktot = 0.0225
```

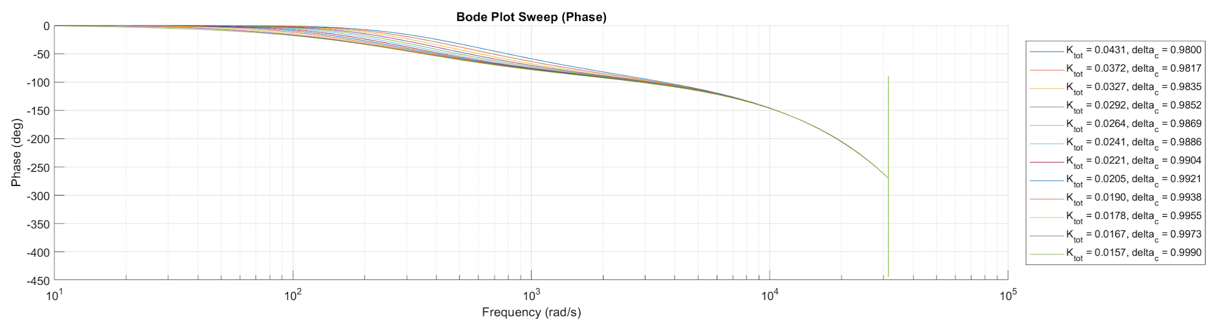
Sweep [0.97, 0.99]



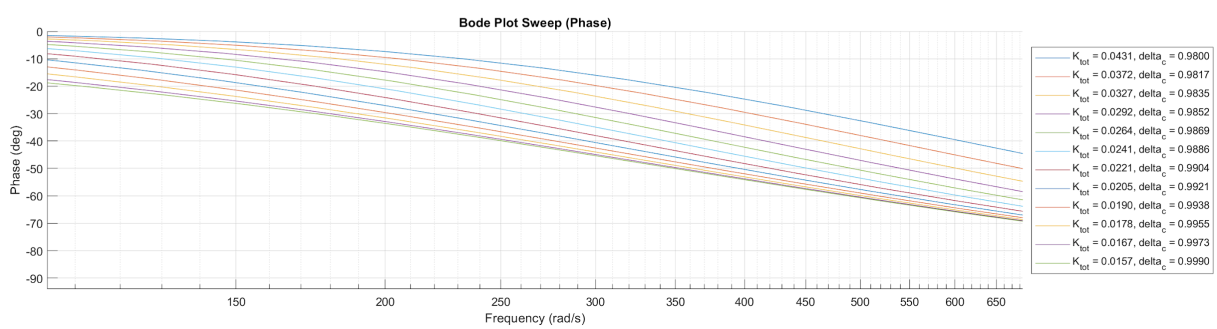

```
sol_Ktot = 0.5075
sol_Ktot = 0.1495
sol_Ktot = 0.0876
sol_Ktot = 0.0620
sol_Ktot = 0.0480
sol_Ktot = 0.0391
sol_Ktot = 0.0330
sol_Ktot = 0.0286
sol_Ktot = 0.0252
sol_Ktot = 0.0225
```

Sweep bode plot for delta_c [0.98, 0.999]





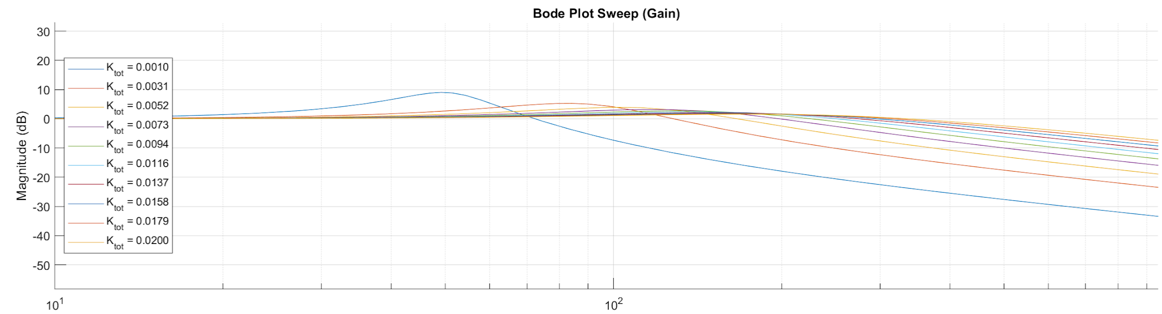
Zoom on 50 Hz:

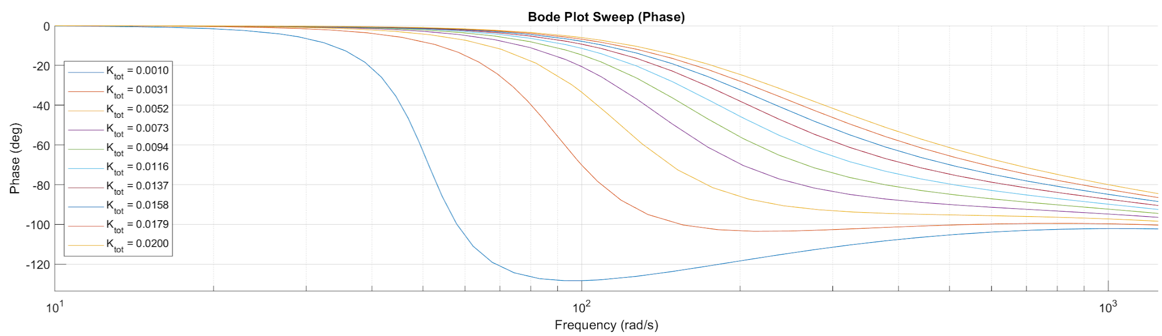


delta_c shows <0 gain for < 50 Hz, select 0.987 for zero for low phase lag.

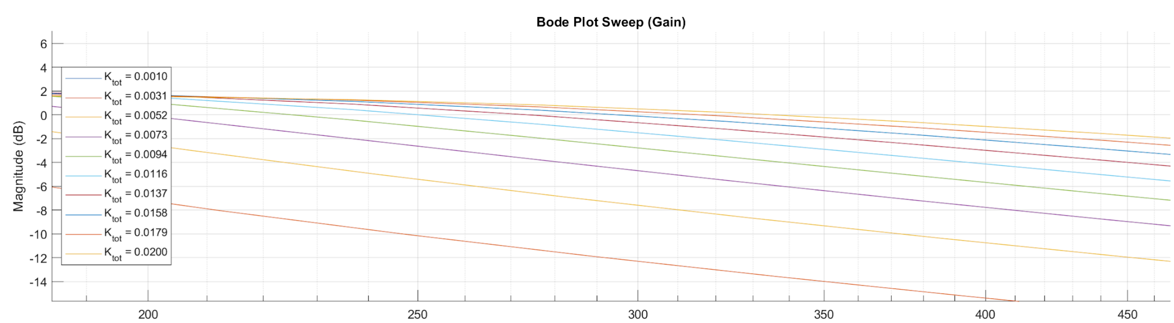
At **delta_c = 0.987**, K_tot = 0.263.

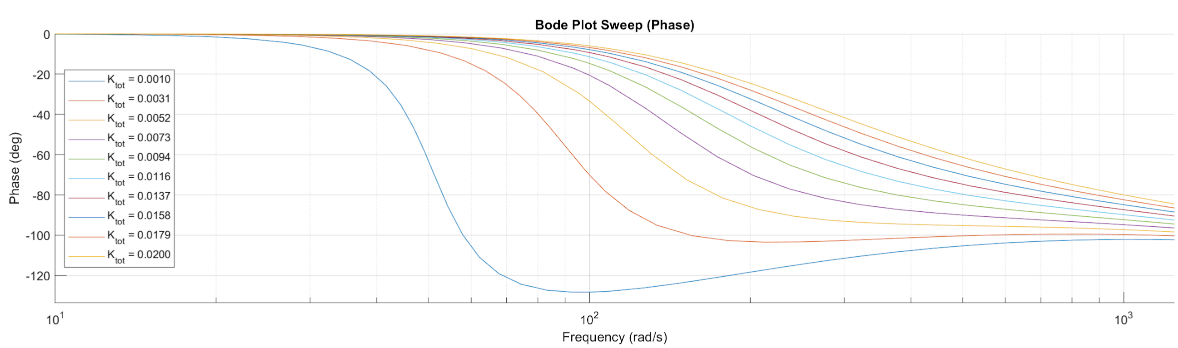
Sweep K_tot from [10% to 2x] this value





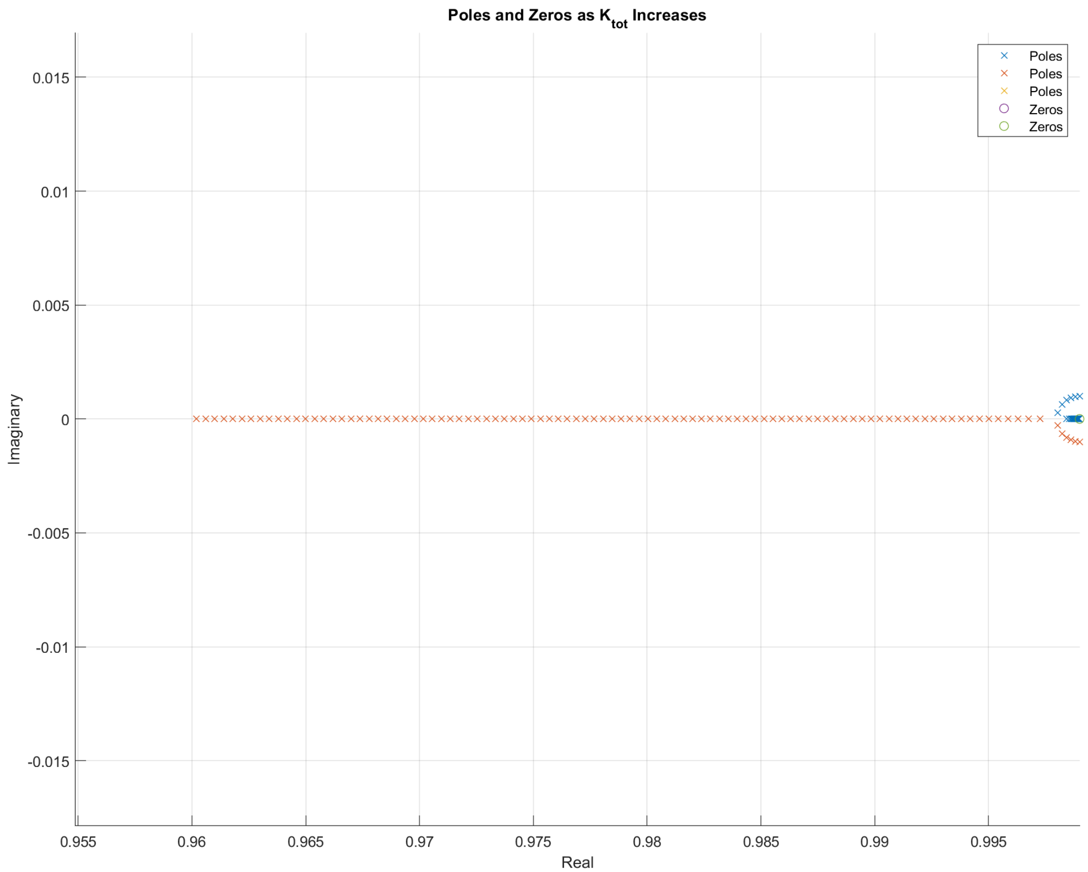
Low K_tot shows high phase lag, >0 gain.





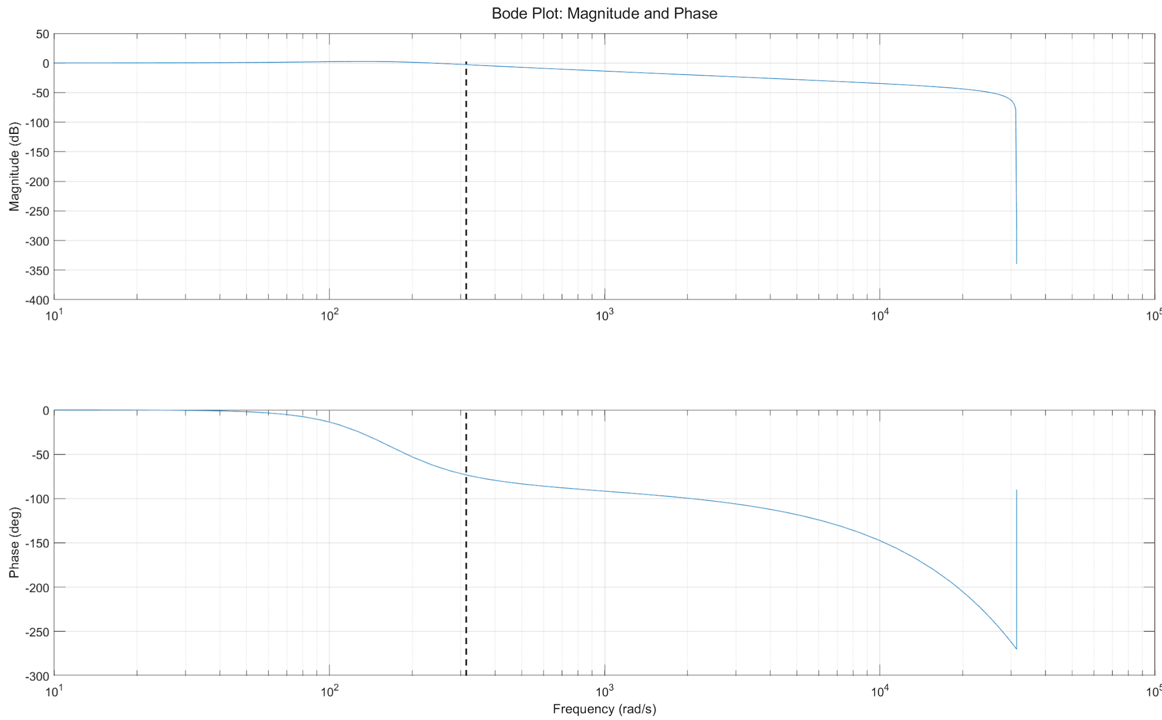
At 50Hz (~310 rad/s), K_tot = 0.0094 results in ~-3 dB gain, though large (90deg) lag.

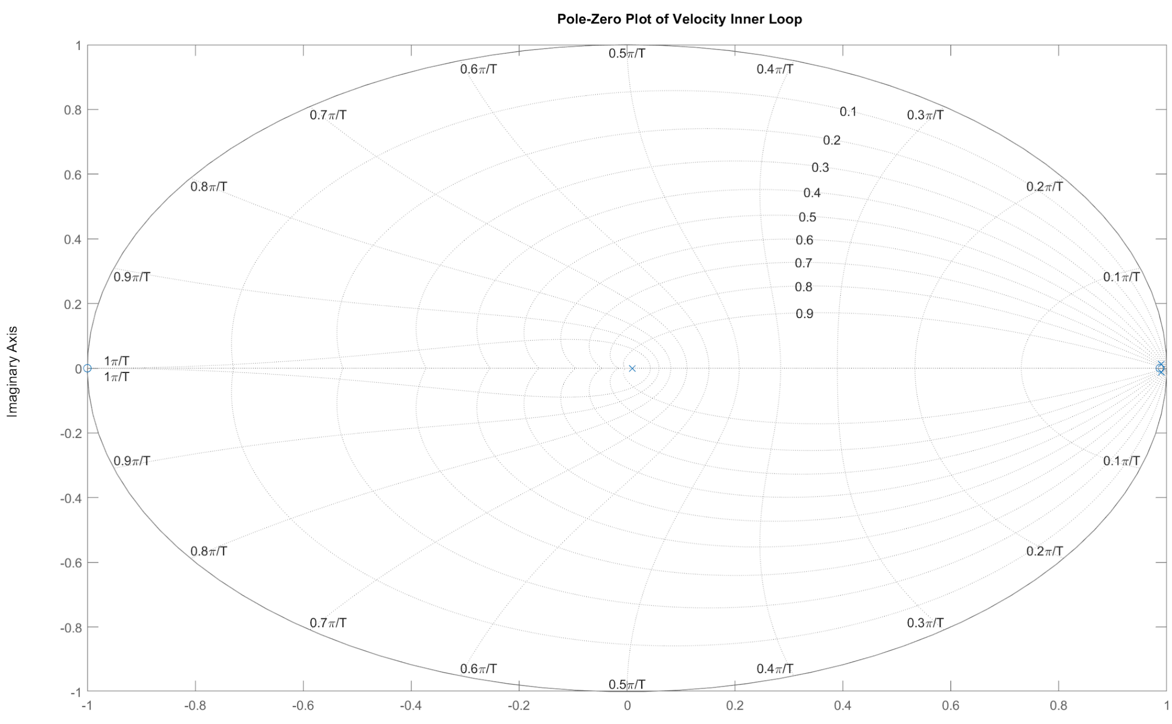
Sweep K_tot for poles to rule out forced oscillations, instability for larger K_tot:



Select K_tot = 0.01

Validate response:





Accepting small, forced oscillations at higher Hz

Solve for gains

sol_Kpv = 0.0079

sol_Kiv = 1.0400

Speed Loop Response

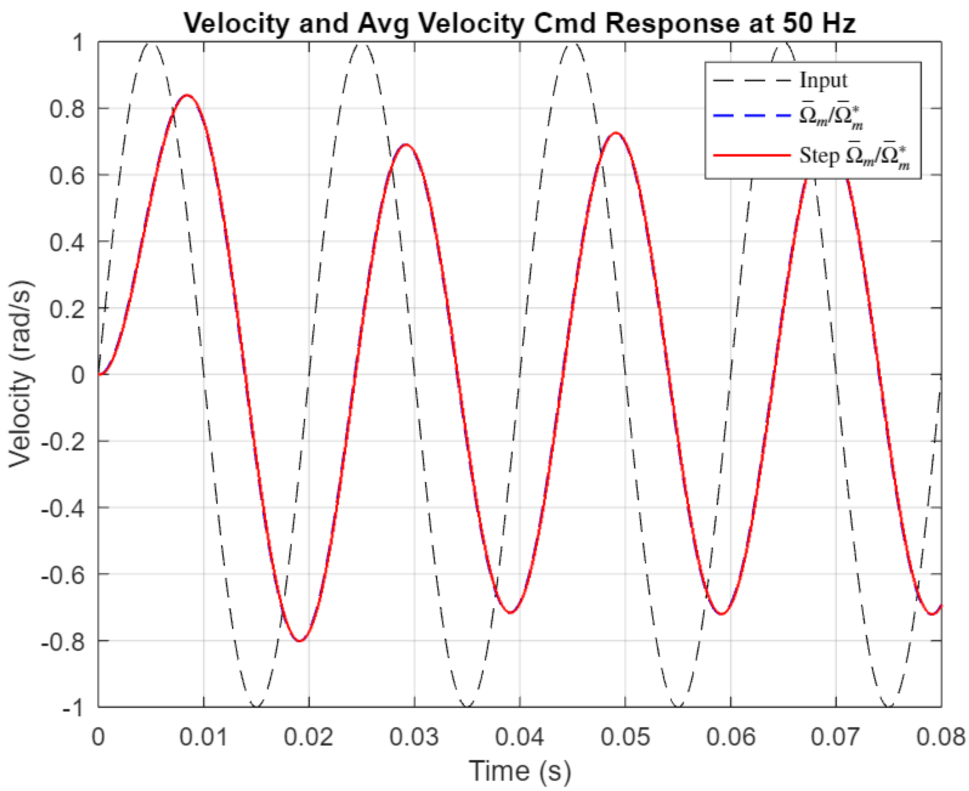
G_omega =

```
          1000 z^2 + 13 z - 987
  ----------------------------------------
  100000 z^3 - 199000 z^2 + 100013 z - 987
```

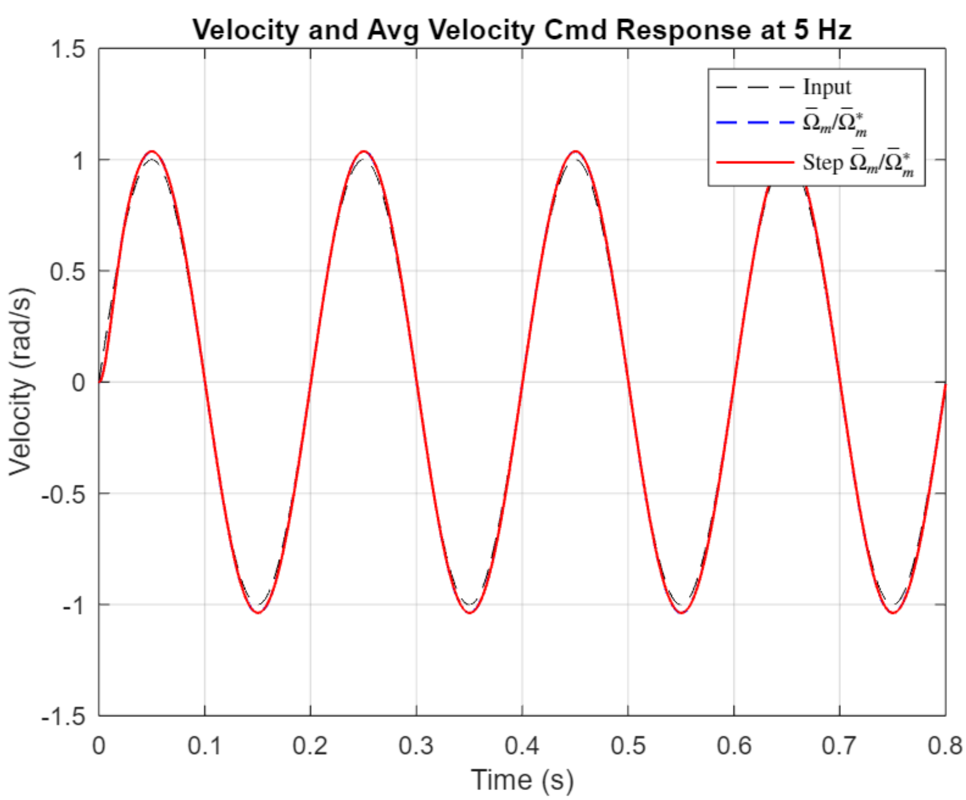
Sample time: 0.0001 seconds

Discrete-time transfer function.

Model Properties



Velocity and Avg Velocity Cmd Response at 50 Hz
Input
$\bar{\Omega}_m/\bar{\Omega}_m^*$
Step $\bar{\Omega}_m/\bar{\Omega}_m^*$
Velocity (rad/s)
Time (s)



Velocity and Avg Velocity Cmd Response at 5 Hz
Input
$\bar{\Omega}_m/\bar{\Omega}_m^*$
Step $\bar{\Omega}_m/\bar{\Omega}_m^*$
Velocity (rad/s)
Time (s)

Accepting minor gain for low phase lag at 5Hz

Position Loop:

Governing Equations

Open Loop:

$$\frac{\Theta_m(z)}{\Theta_{\mathrm{err}}(z)} = \frac{(K_{\mathrm{pp}}\,T_s)\,z}{z-1}$$

Closed Loop:

$$\frac{\Theta_m(z)}{\Theta^*{}_m(z)} = \frac{(K_{\mathrm{pp}}\,T_s)\,z}{(K_{\mathrm{pp}}\,T_s+1)\,z-1}$$

Characteristic Equation:

$$z + K_{\mathrm{pp}}\,T_s\,z - 1$$

General Approach to Tuning Position Loop

Initially, solve CE for K_pp at 5Hz.

Sweep K_pp across bode plots to identify gain and phase margins. Iterate and manually select K_pp.

Tuning

Initial Solve of K_pp from CE

sol_Kpp_eq =

$$\frac{\mathrm{e}^{2\pi T_s f_{d,p}} - 1}{T_s}$$

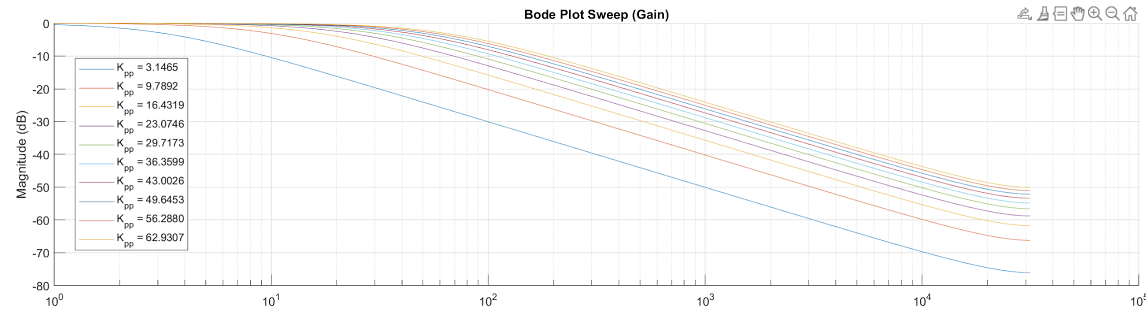
z_d_p =

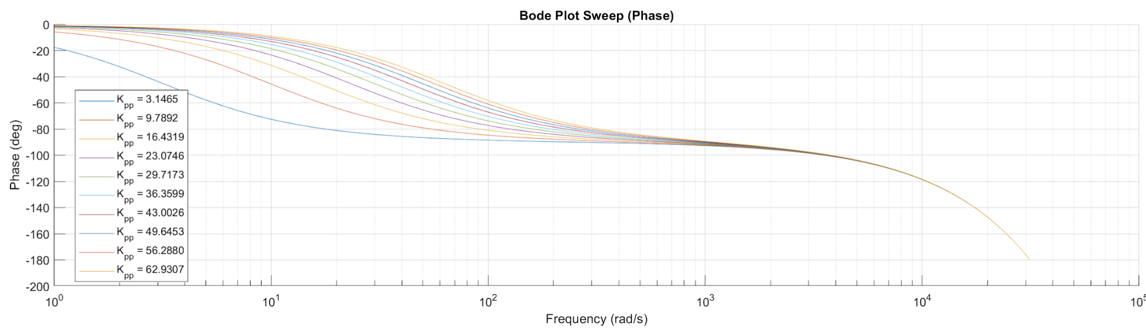
$$\mathrm{e}^{-2\pi T_s f_{d,p}}$$

Where f_d = 5Hz

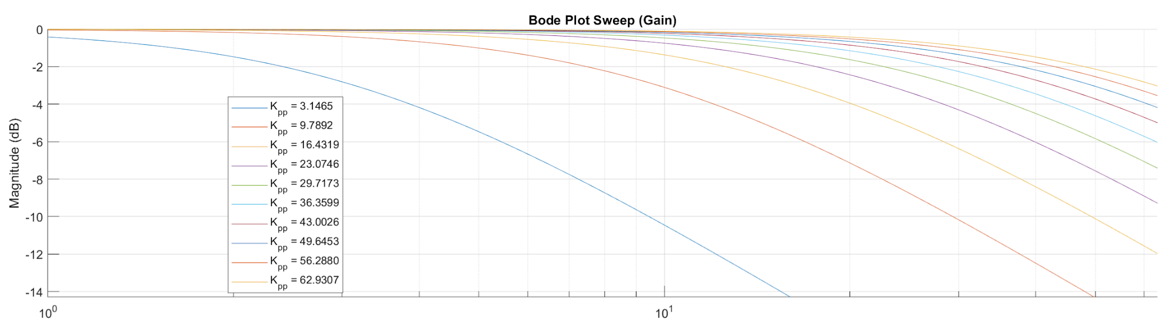
sol_Kpp = 31.4653

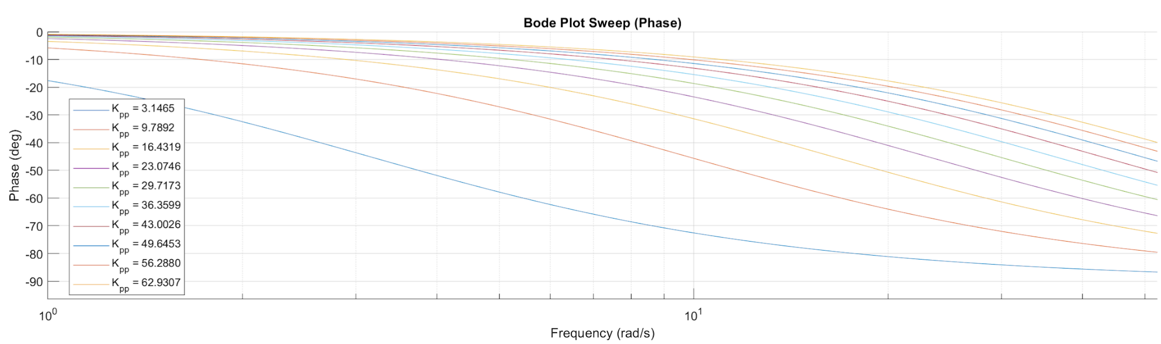
Sweep K_pp [10%, 2x]





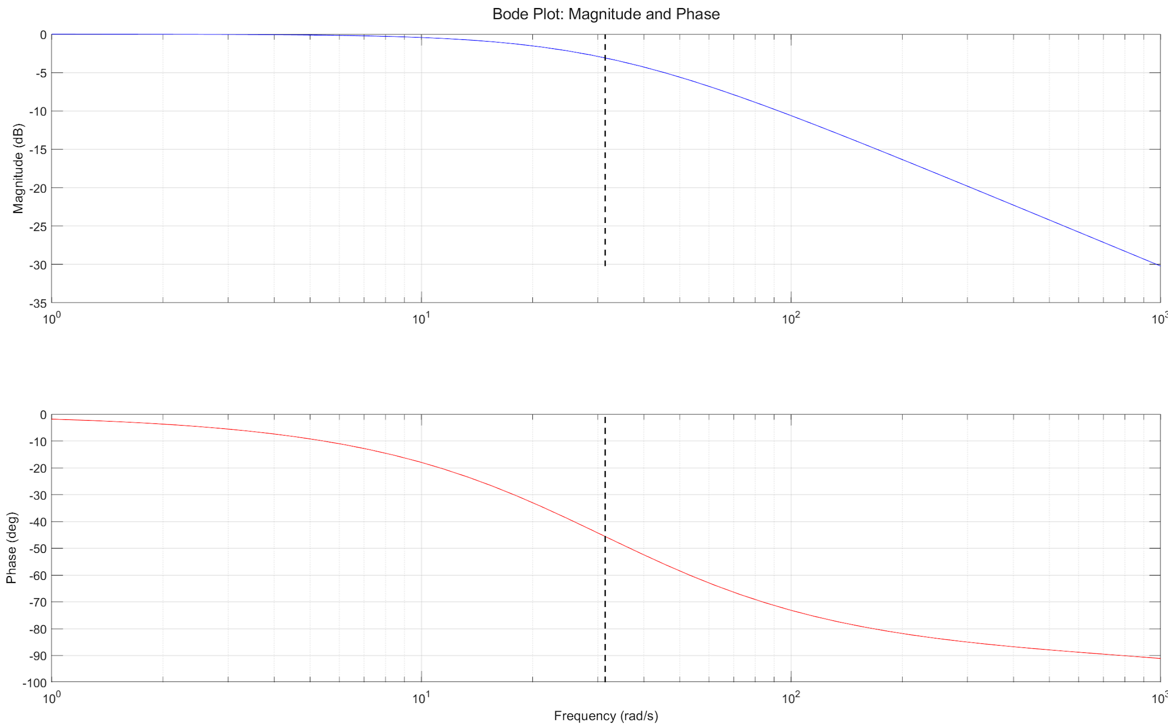
Zoomed:



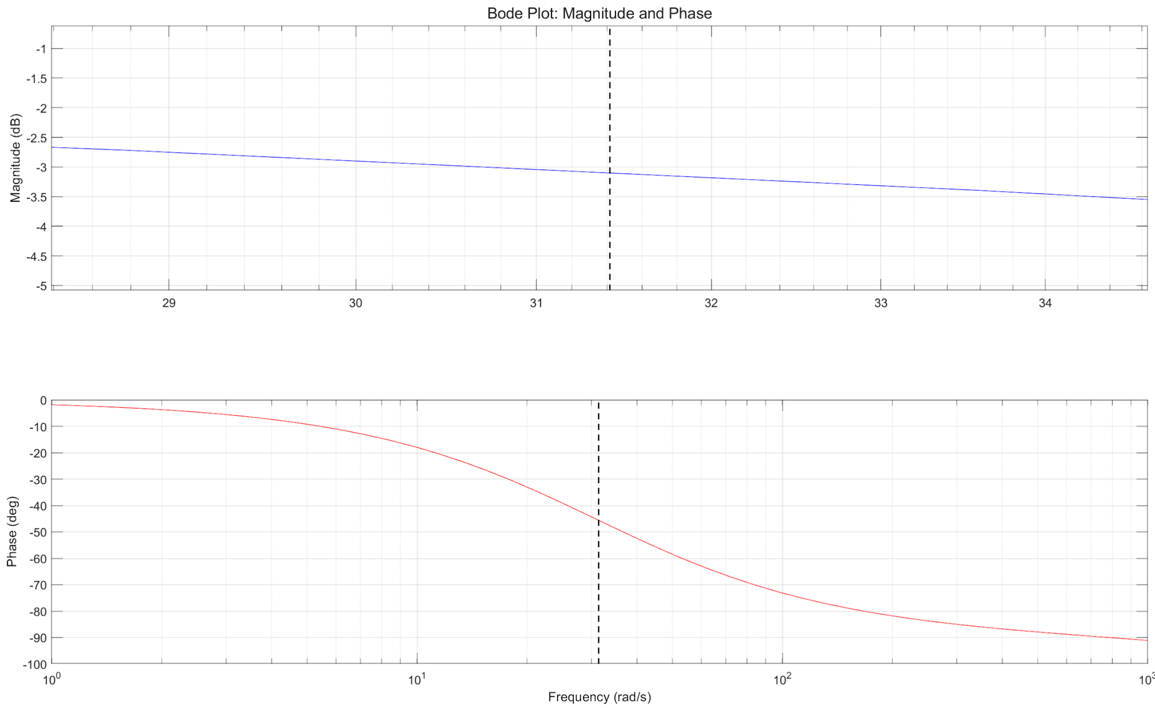


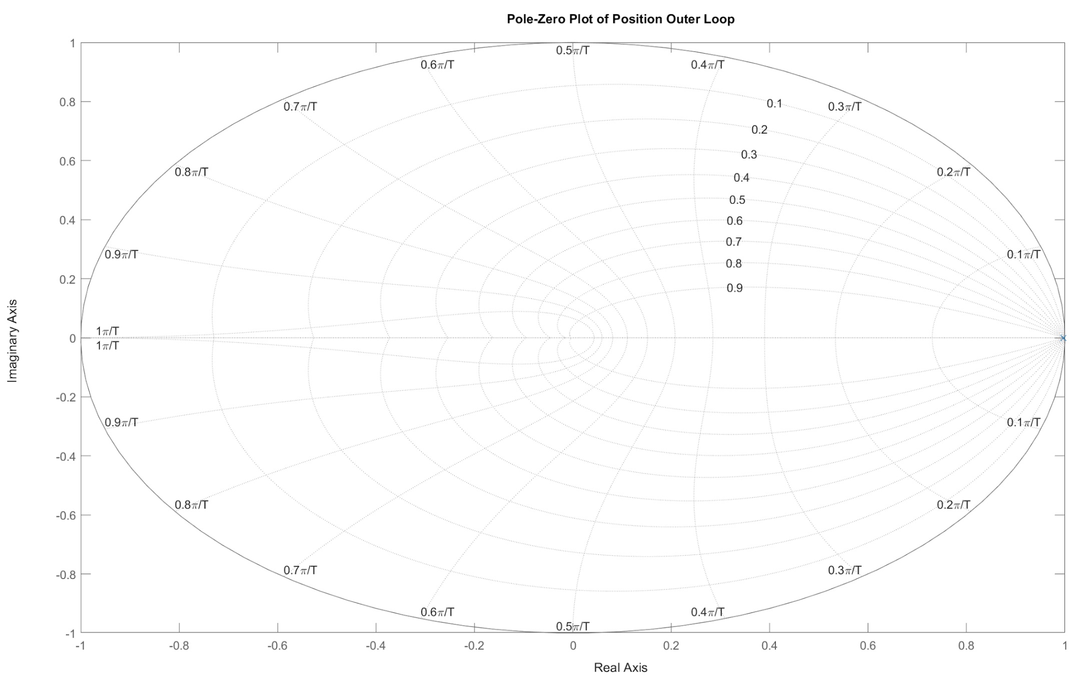
Gain around 29.71 selected for characteristics.

Setting **K_pp = 29.9**,



Zoomed:





No forced oscillations

Solve for Gain

K_pp = 29.9

Position Response

G_theta =

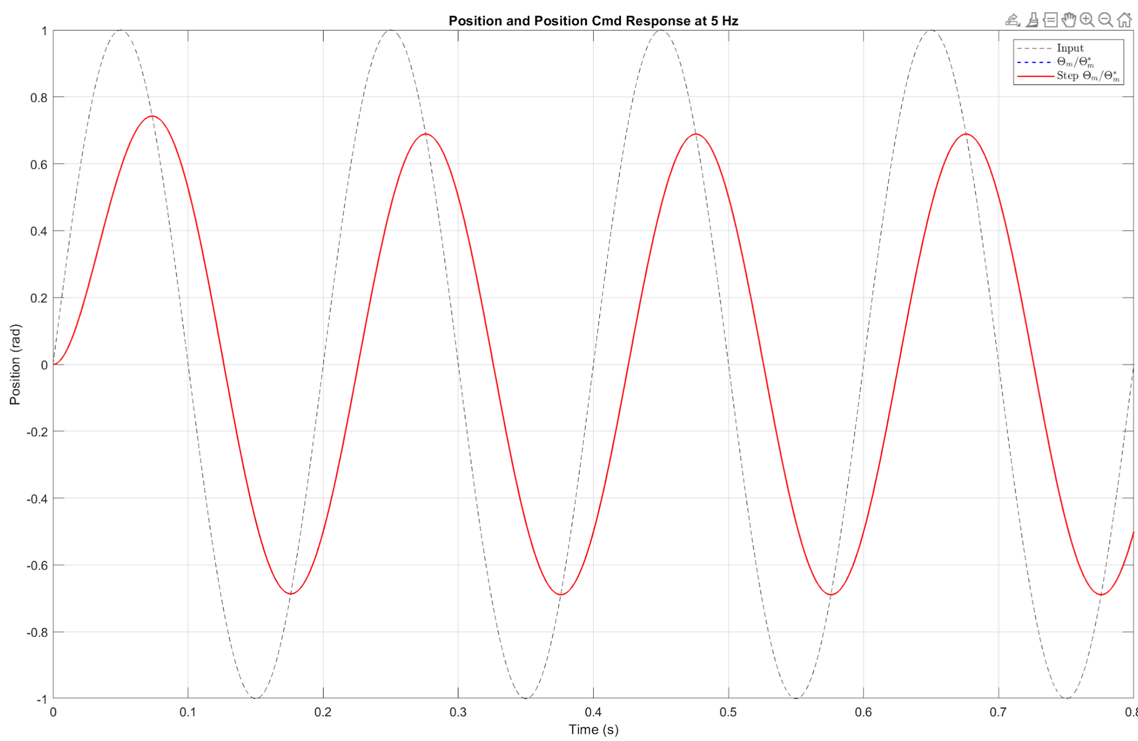
299

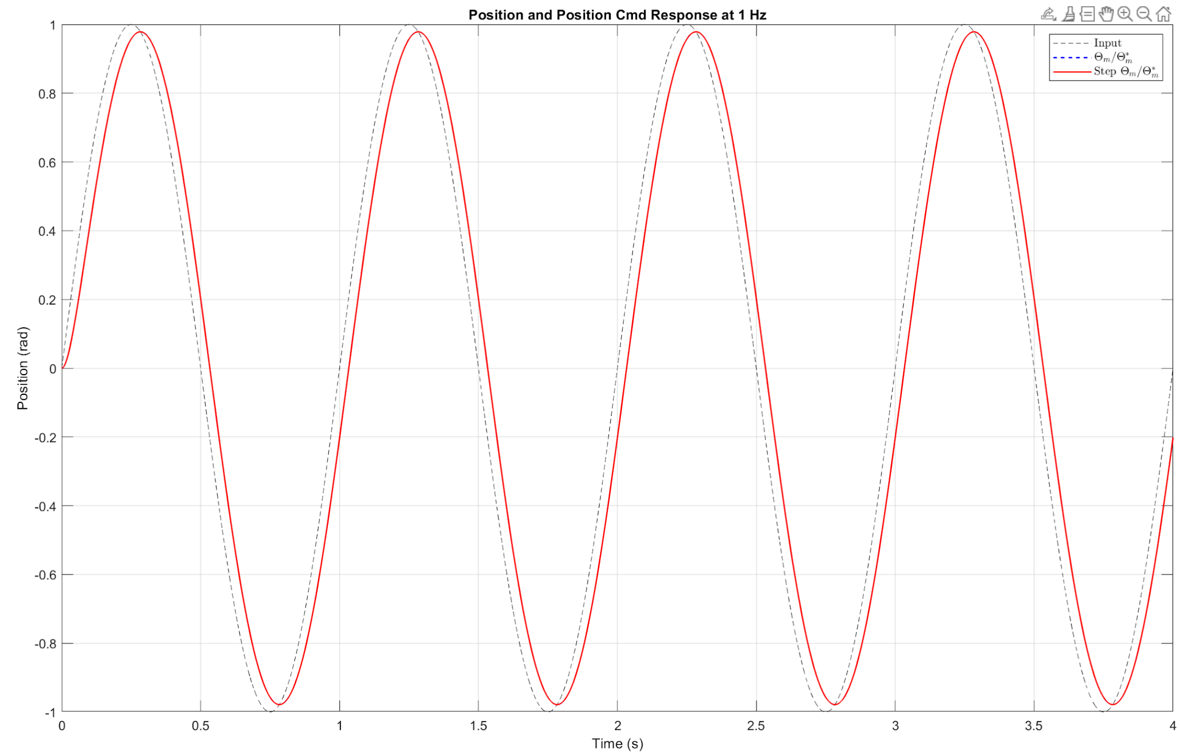
100299 z - 100000

Sample time: 0.0001 seconds

Discrete-time transfer function.

[Model Properties]





Accepting ~7deg phase lag

Full Motor Loop Response

Governing Equations

Without CFF:

Open Loop:

$$\frac{\Theta_{p,m}(z)}{\Theta_{p,\mathrm{err}}(z)} = \frac{(K_{\mathrm{pp}}\,K_{\mathrm{tot}}\,T_s)\,z^3 + (K_{\mathrm{pp}}\,K_{\mathrm{tot}}\,T_s - K_{\mathrm{pp}}\,K_{\mathrm{tot}}\,T_s\,\delta_c)\,z^2 + (-K_{\mathrm{pp}}\,K_{\mathrm{tot}}\,T_s\,\delta_c)\,z}{z^4 + (K_{\mathrm{tot}} - 3)\,z^3 + (3 - K_{\mathrm{tot}}\,\delta_c)\,z^2 + (-K_{\mathrm{tot}} - 1)\,z + K_{\mathrm{tot}}\,\delta_c}$$

Closed Loop:

$$\frac{\Theta_{p,m}(z)}{\Theta^*{}_{p,m}(z)} = \frac{(-K_{\mathrm{pp}}\, K_{\mathrm{tot}}\, T_s)\, z^3 + (K_{\mathrm{pp}}\, K_{\mathrm{tot}}\, T_s\, \delta_c - K_{\mathrm{pp}}\, K_{\mathrm{tot}}\, T_s)\, z^2 + (K_{\mathrm{pp}}\, K_{\mathrm{tot}}\, T_s\, \delta_c)\, z}{-z^4 + (3 - K_{\mathrm{pp}}\, K_{\mathrm{tot}}\, T_s - K_{\mathrm{tot}})\, z^3 + (K_{\mathrm{tot}}\, \delta_c - K_{\mathrm{pp}}\, K_{\mathrm{tot}}\, T_s + K_{\mathrm{pp}}\, K_{\mathrm{tot}}\, T_s\, \delta_c - 3)\, z^2 + (K_{\mathrm{tot}} + K_{\mathrm{pp}}\, K_{\mathrm{tot}}\, T_s\, \delta_c + 1)\, z - K_{\mathrm{tot}}\, \delta_c}$$

With CFF:

cff portion =

$$\frac{\Theta^*{}_{\mathrm{ps},m}(z)\ \left(K_{\mathrm{tot}}\, z^2 + (K_{\mathrm{tot}} - K_{\mathrm{tot}}\, \delta_c)\, z - K_{\mathrm{tot}}\, \delta_c\right)}{z^3 + (K_{\mathrm{tot}} - 2)\, z^2 + (K_{\mathrm{tot}} - K_{\mathrm{tot}}\, \delta_c + 1)\, z - K_{\mathrm{tot}}\, \delta_c}$$

feedback =

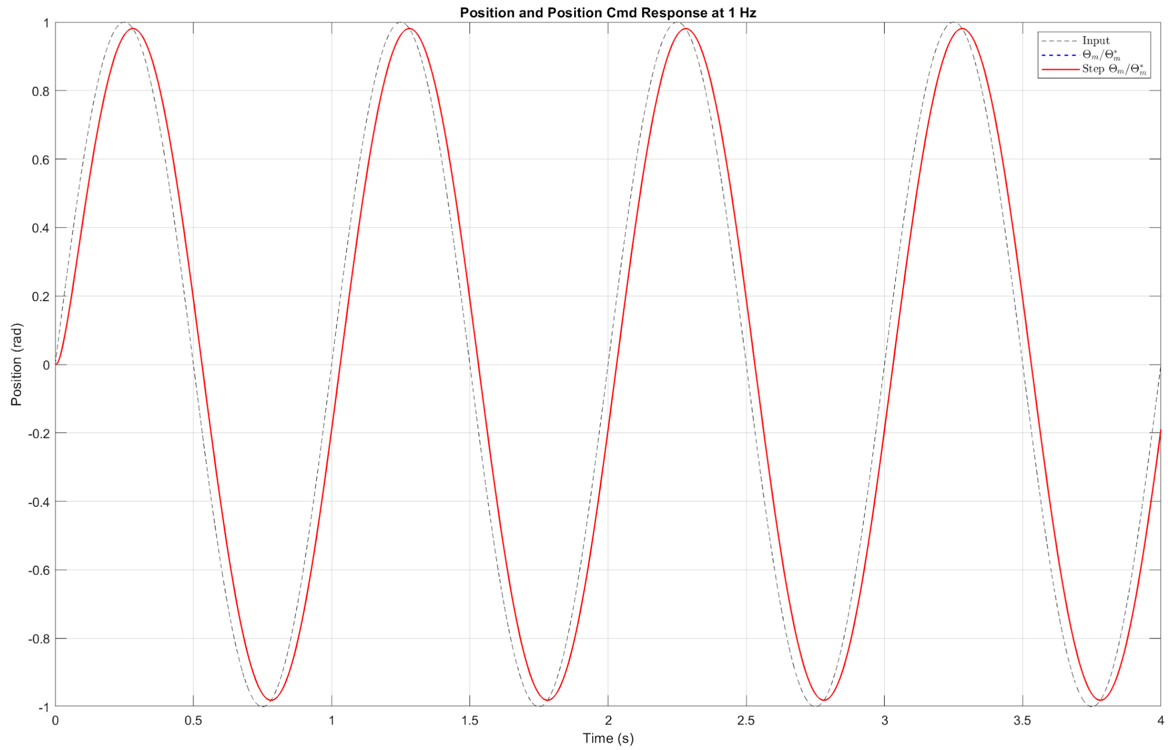
$$-\frac{\left(\Theta_{\mathrm{ps},m}(z) - \Theta^*{}_{\mathrm{ps},m}(z)\right)\ ((K_{\mathrm{pp}}\, T_s)\, z)}{z - 1}$$

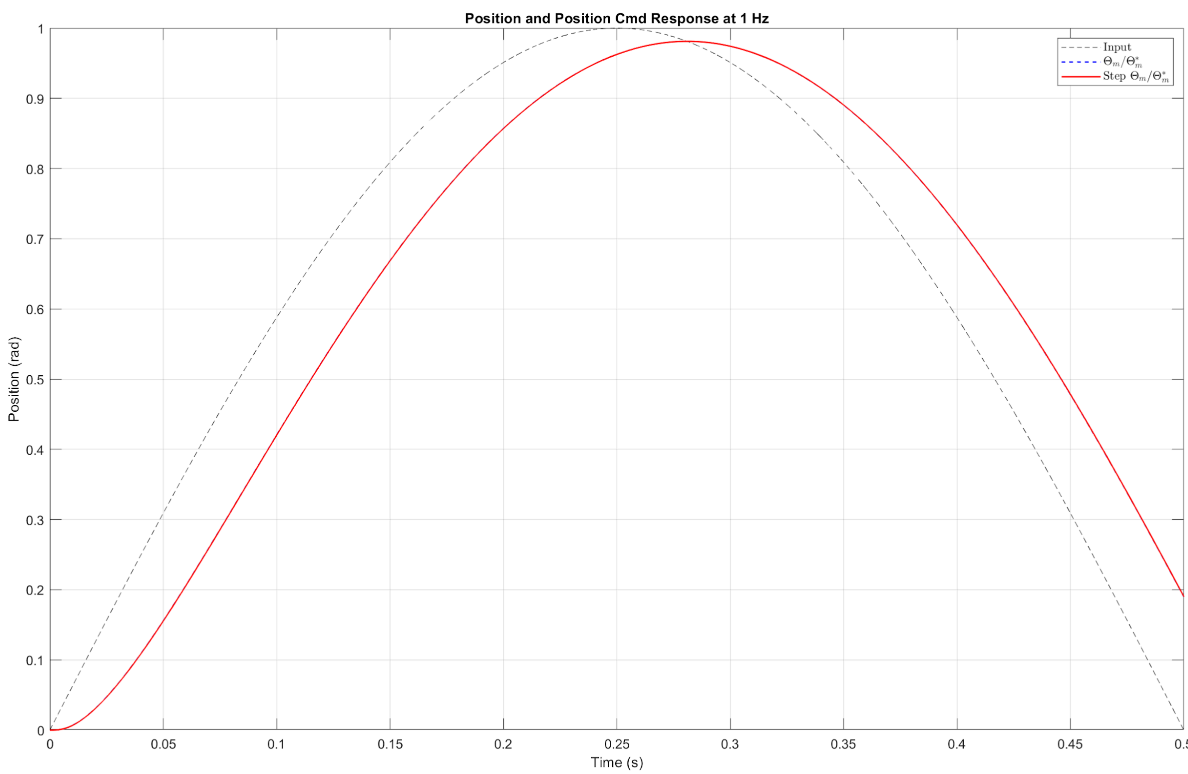
$$\Theta_{\mathrm{ps},m}(z) = \left(\frac{K_{\mathrm{tot}}\, z^2 + (K_{\mathrm{tot}} - K_{\mathrm{tot}}\, \delta_c)\, z - K_{\mathrm{tot}}\, \delta_c}{z^3 + (K_{\mathrm{tot}} - 2)\, z^2 + (K_{\mathrm{tot}} - K_{\mathrm{tot}}\, \delta_c + 1)\, z - K_{\mathrm{tot}}\, \delta_c} + \frac{z\ (K_{\mathrm{pp}}\, T_s)}{z - 1}\right) \Theta^*{}_{\mathrm{ps},m}(z) - \frac{z\, \Theta_{\mathrm{ps},m}(z)\ (K_{\mathrm{pp}}\, T_s)}{z - 1}$$

Closed Loop:

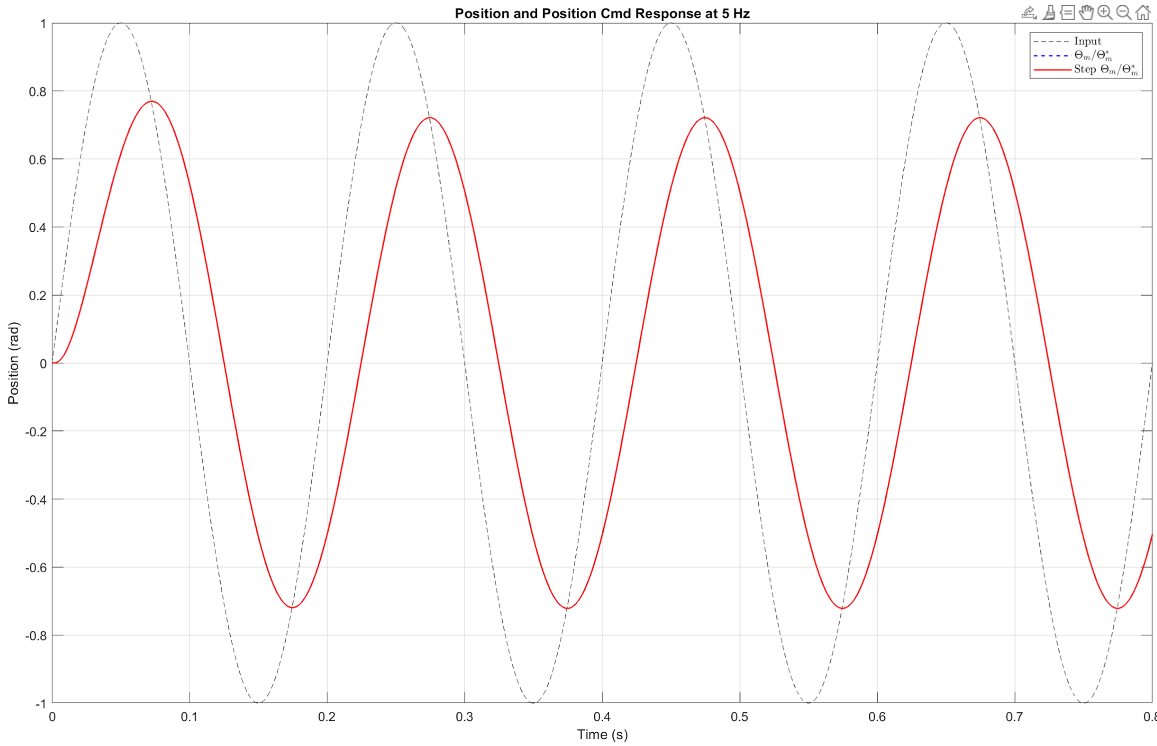
$$\frac{\Theta_{\mathrm{ps},m}(z)}{\Theta^*{}_{\mathrm{ps},m}(z)} = \frac{(-K_{\mathrm{pp}}\, T_s)\, z^4 + (2\, K_{\mathrm{pp}}\, T_s - K_{\mathrm{tot}} - K_{\mathrm{pp}}\, K_{\mathrm{tot}}\, T_s)\, z^3 + (K_{\mathrm{tot}}\, \delta_c - K_{\mathrm{pp}}\, T_s - K_{\mathrm{pp}}\, K_{\mathrm{tot}}\, T_s + K_{\mathrm{pp}}\, K_{\mathrm{tot}}\, T_s\, \delta_c)\, z^2 + (K_{\mathrm{tot}} + K_{\mathrm{pp}}\, K_{\mathrm{tot}}\, T_s\, \delta_c)\, z - K_{\mathrm{tot}}\, \delta_c}{(-K_{\mathrm{pp}}\, T_s - 1)\, z^4 + (2\, K_{\mathrm{pp}}\, T_s - K_{\mathrm{tot}} - K_{\mathrm{pp}}\, K_{\mathrm{tot}}\, T_s + 3)\, z^3 + (K_{\mathrm{tot}}\, \delta_c - K_{\mathrm{pp}}\, T_s - K_{\mathrm{pp}}\, K_{\mathrm{tot}}\, T_s + K_{\mathrm{pp}}\, K_{\mathrm{tot}}\, T_s\, \delta_c - 3)\, z^2 + (K_{\mathrm{tot}} + K_{\mathrm{pp}}\, K_{\mathrm{tot}}\, T_s\, \delta_c + 1)\, z - K_{\mathrm{tot}}\, \delta_c}$$

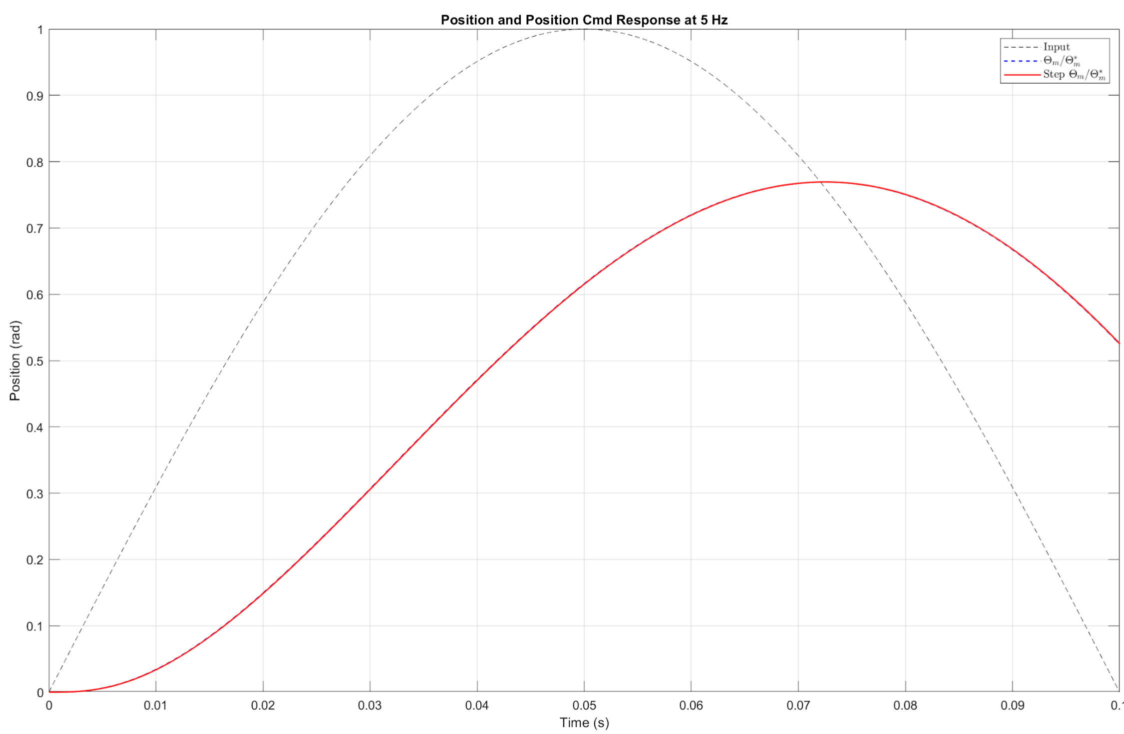
Without CFF


Position and Position Cmd Response at 1 Hz
Input
Θ_m/Θ_m^*
Step Θ_m/Θ_m^*
Position (rad)
Time (s)
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1
0
0.5
1
1.5
2
2.5
3
3.5
4

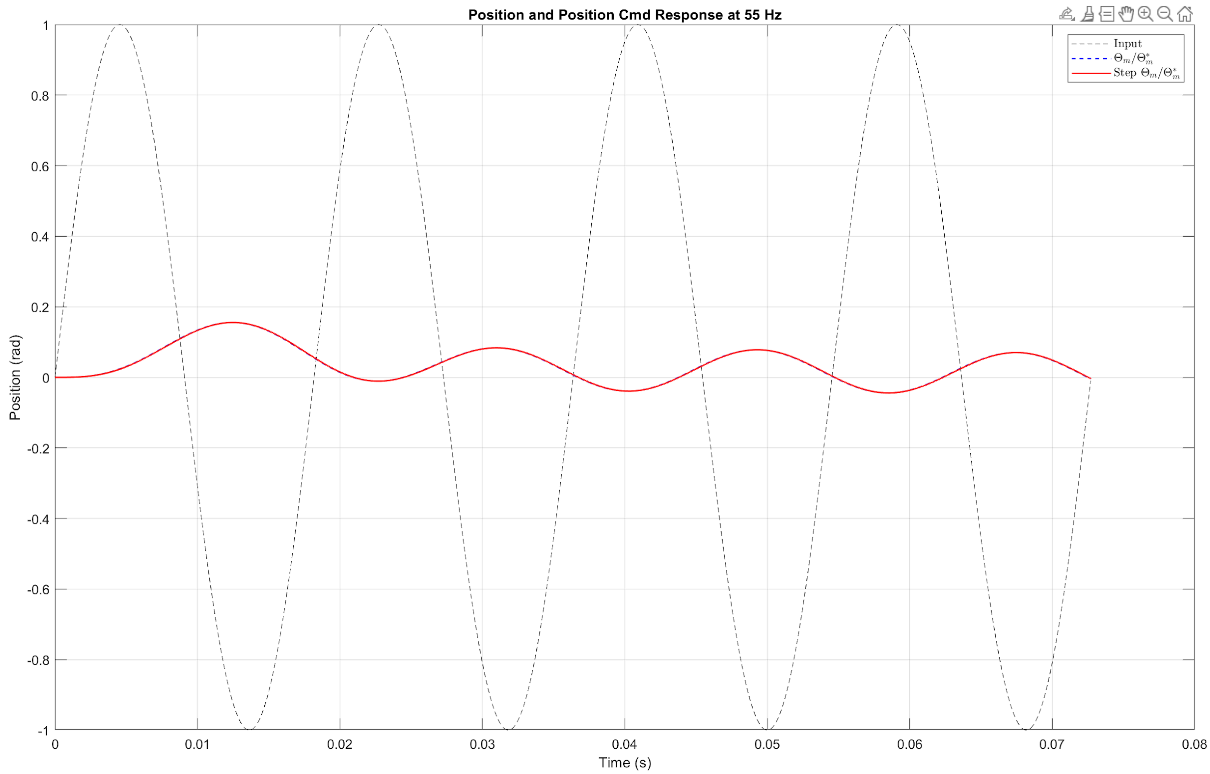

Position and Position Cmd Response at 1 Hz
Input
Θ_m/Θ_m^*
Step Θ_m/Θ_m^*
Position (rad)
Time (s)
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
0.05
0.1
0.15
0.2
0.25
0.3
0.35
0.4
0.45
0.5

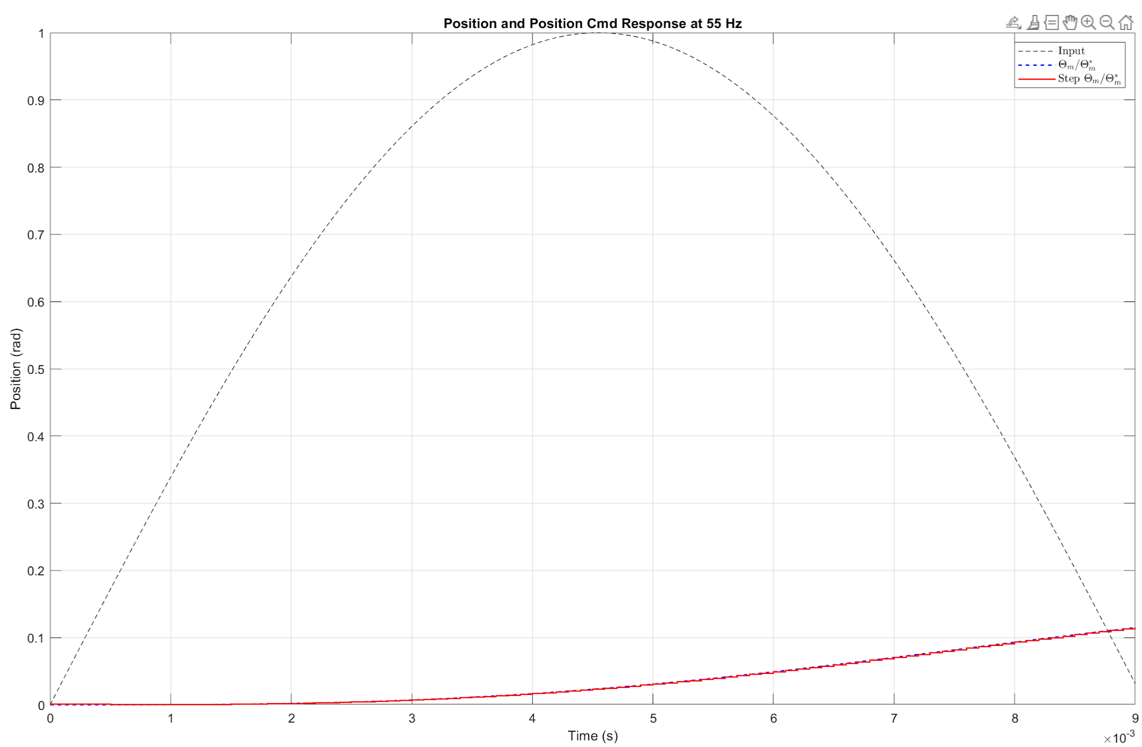
Controller drops off to 3dB at 5Hz:



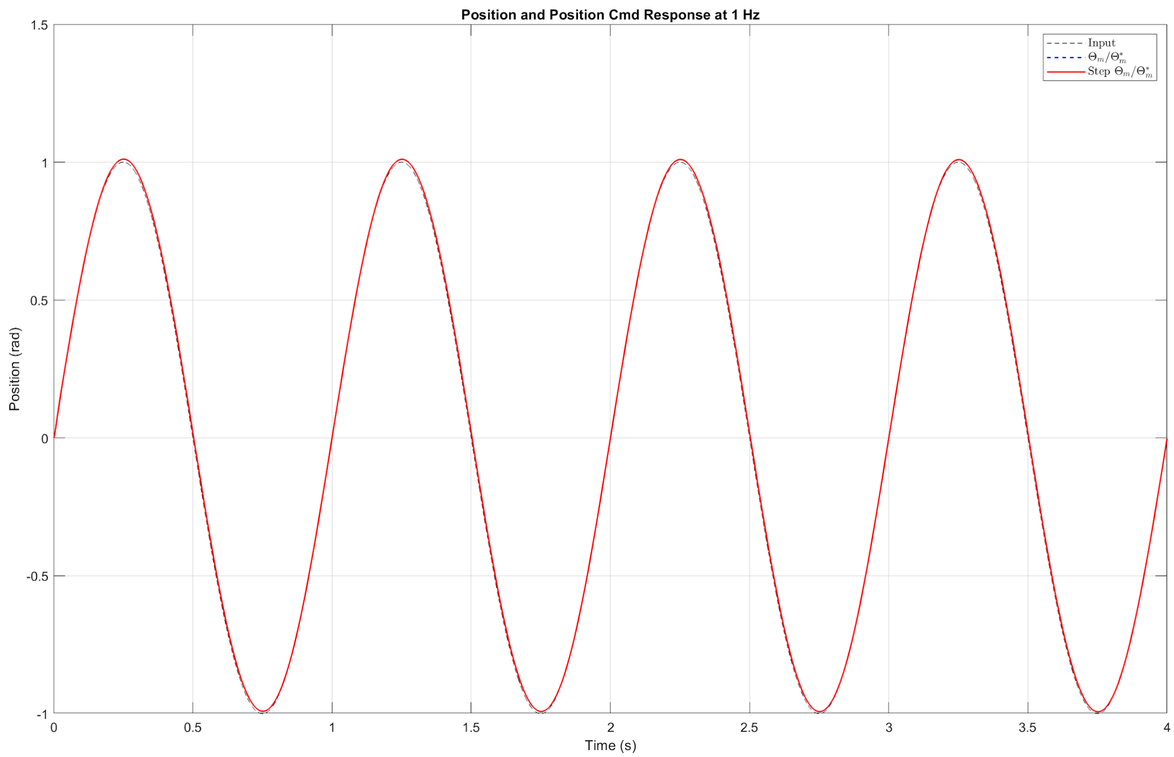


Controller significantly attenuated and ~180 deg lag at +50Hz

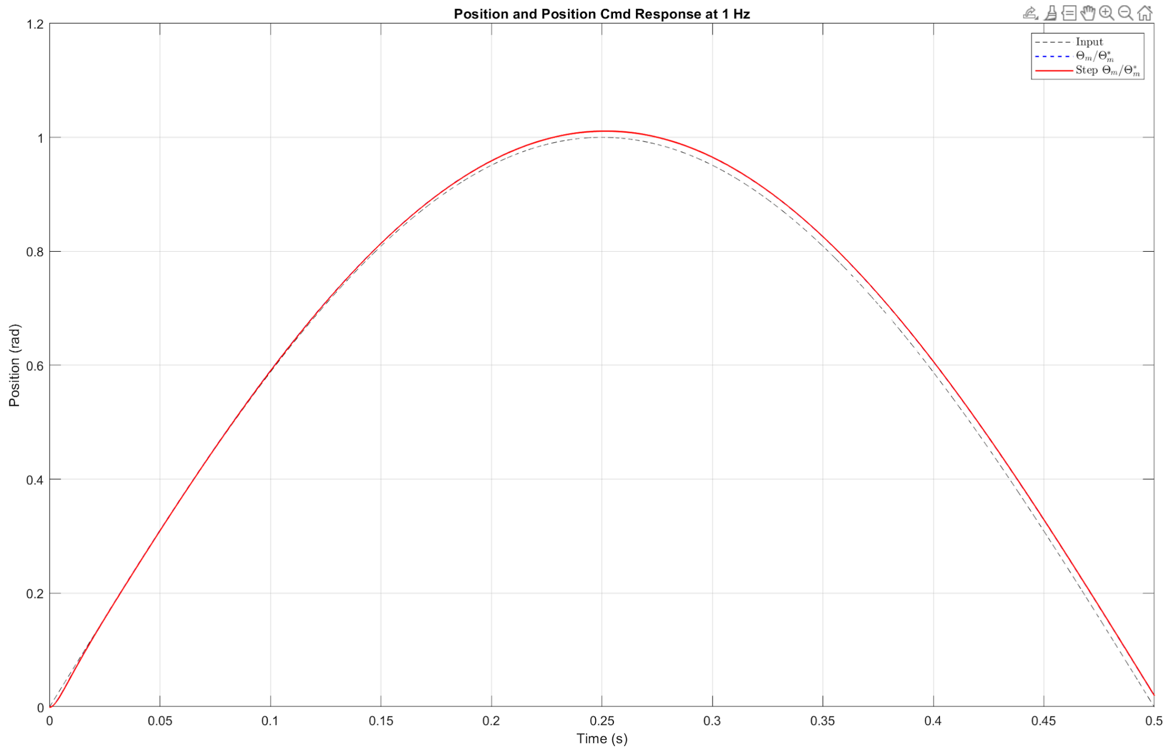




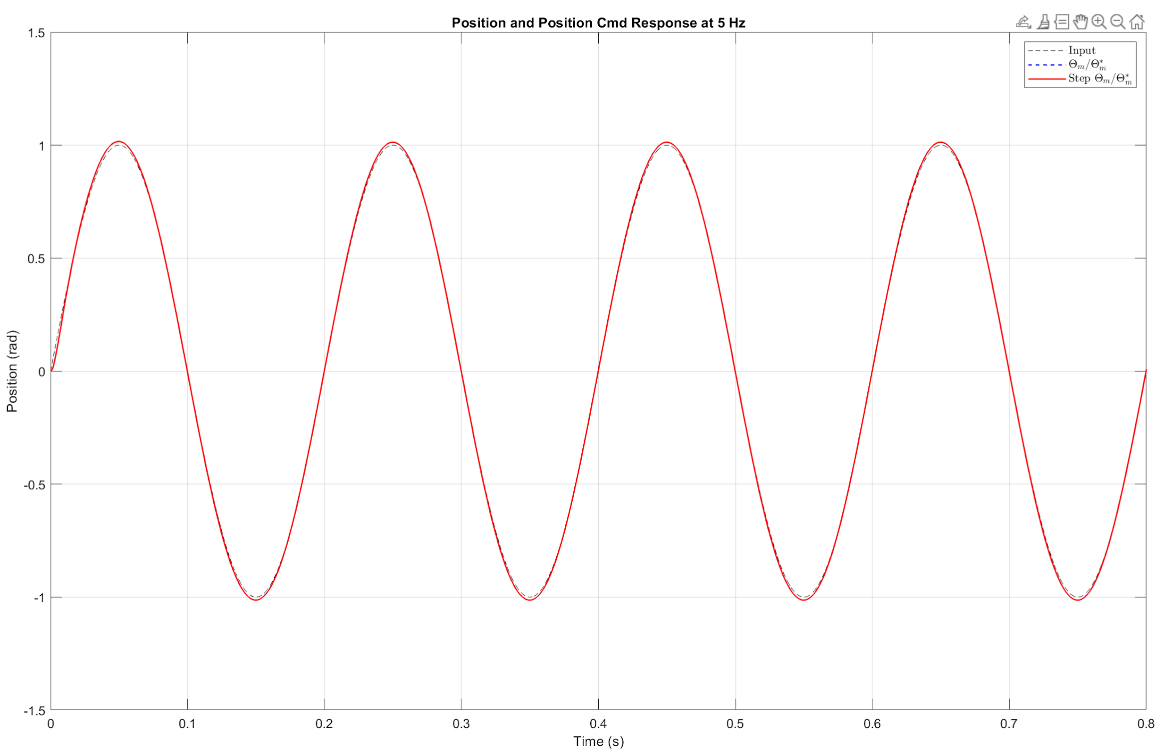
With CFF

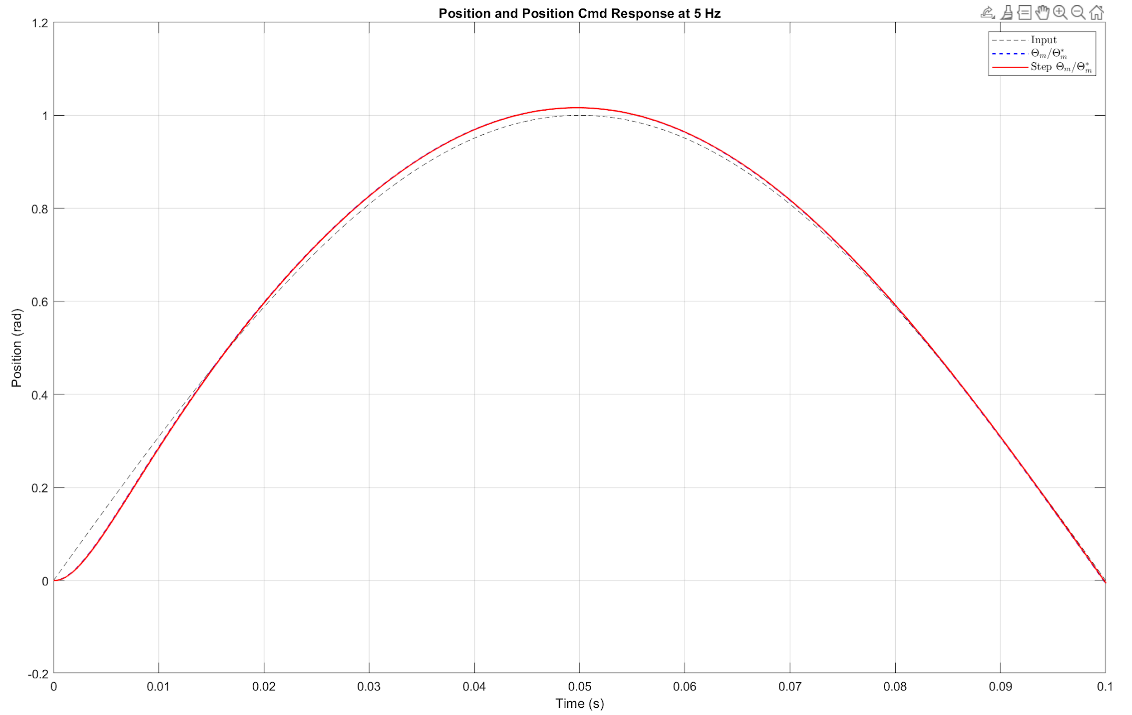


Position and Position Cmd Response at 1 Hz
Input
Θ_m/Θ_m^*
Step Θ_m/Θ_m^*
Position (rad)
Time (s)
1.5
1
0.5
0
-0.5
-1
0
0.5
1
1.5
2
2.5
3
3.5
4

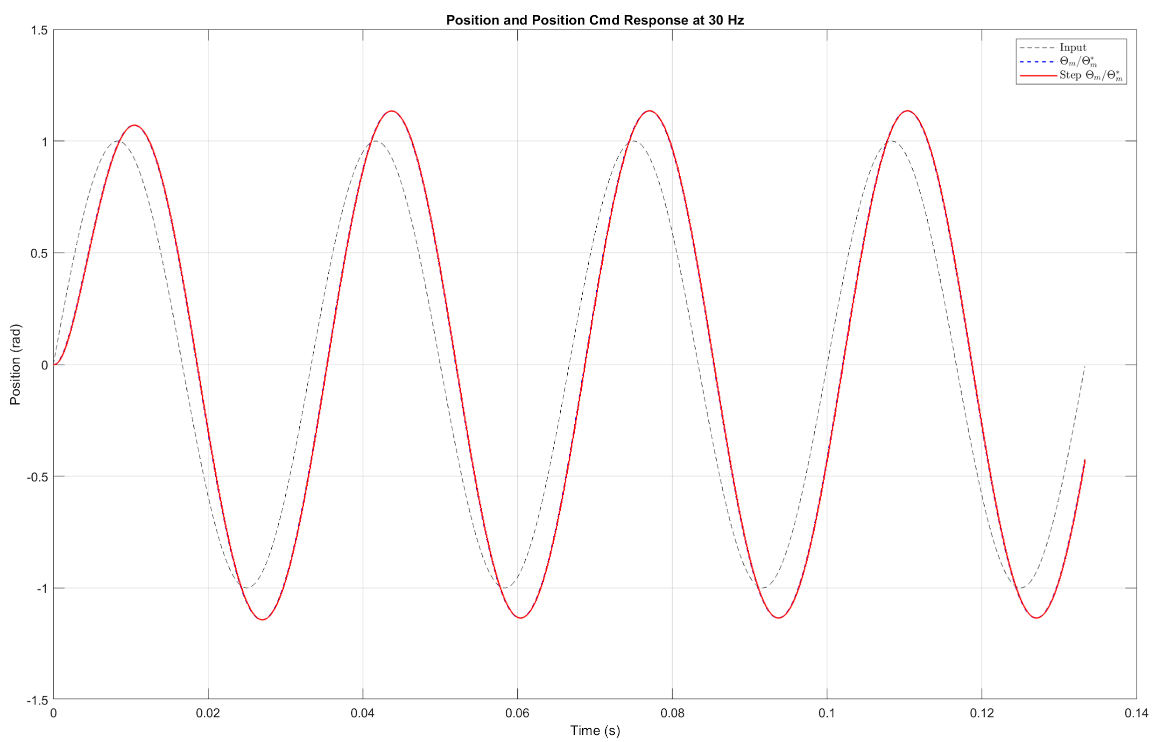


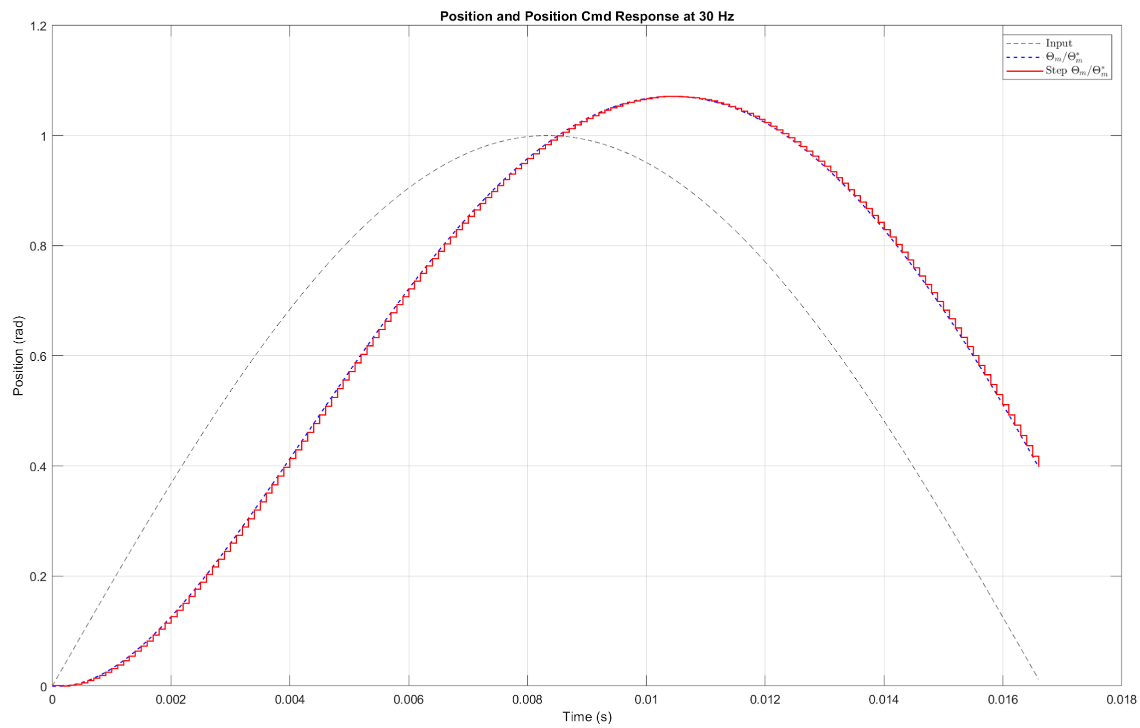
Controller response is acceptable at 5Hz:



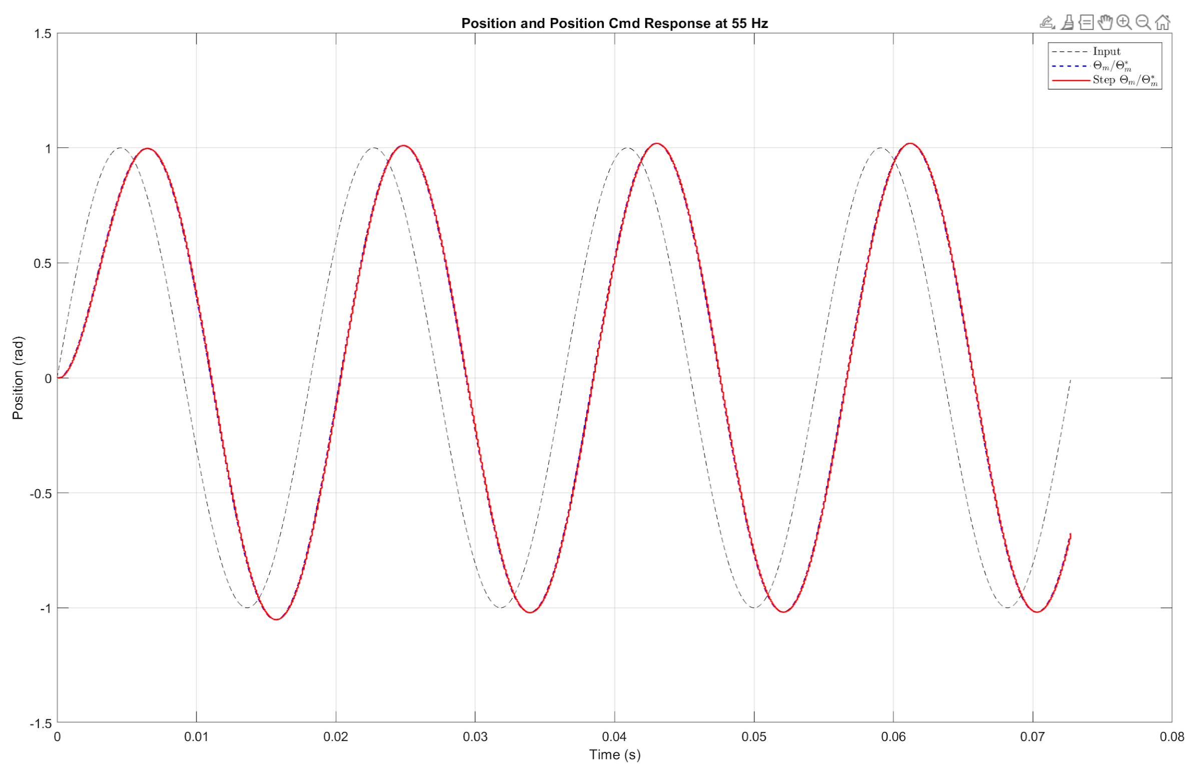


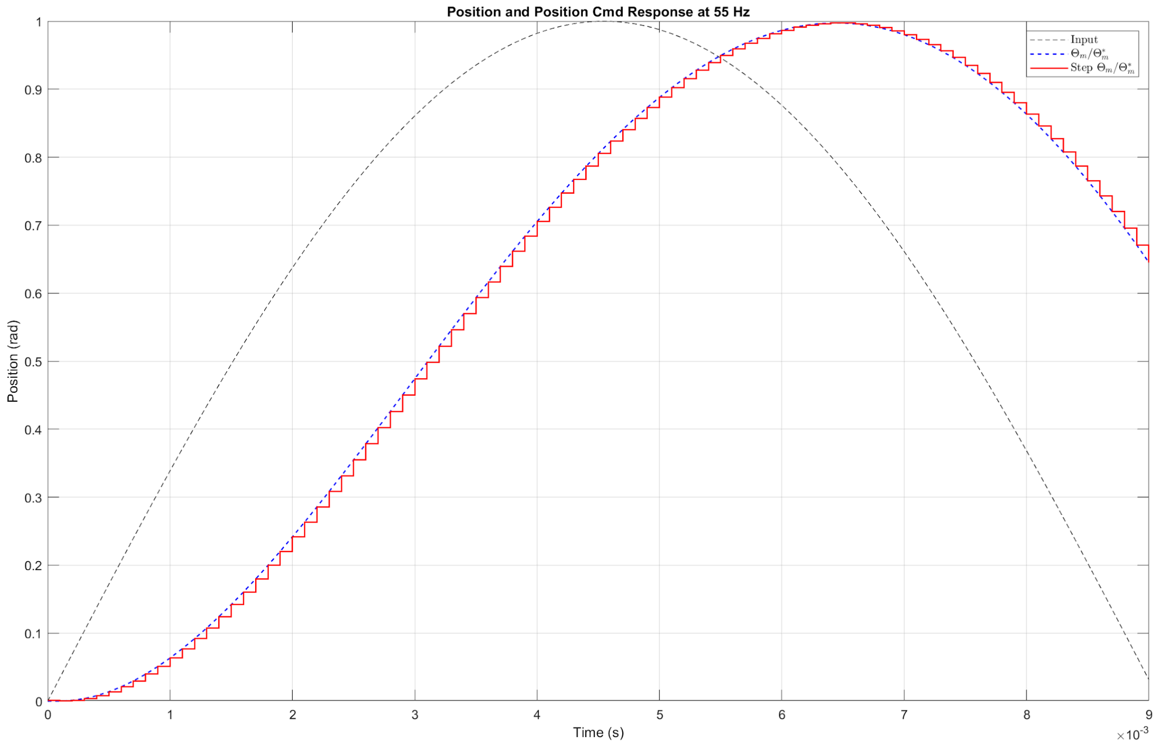
Velocity gain evident in this region of response (30Hz):





Controller avg velocity compensates for attenuation at 50Hz, only lag evident:





Appendix (Values):

Variable	Value	Solved?
K_pp	29.9	Iteratively
K_pv	0.0079	From [K_tot, delta_c]
K_iv	1.04	From [K_tot, delta_c]
K_tot	0.01	Iteratively
delta_c	0.987	Iteratively