G O A

Student Program

Geometry

LENGTH OF COURSE: 7 WEEKS

Course Overview

Geometry is one of the oldest fields of mathematics, tracing its roots back through ancient civilizations in Egypt and Mesopotamia centuries before the more well-known work done by Greek mathematicians. It is a field of math that everyone interacts with every day in a very tangible sense and has applications in industries as varied as architecture, astronomy, and communication. This intensive summer course is designed to provide an accelerated path through the traditional high school geometry curriculum. Focusing on Euclidean geometry, students will examine topics relating to parallel lines, similar and congruent triangles, quadrilaterals, polygons, circles, transformation, and other manipulations. Students can also expect to analyze lengths, areas, and volumes of two and three-dimensional figures, and introductory trigonometry with right triangles that include the trigonometric ratios. Emphasis will be placed on the development of a mature, logical thought process and effective communication skills through a formal introduction to arguments, deductions, theorems, and proofs.

Pre-requisites

A strong background in Algebra 1 that includes a study of quadratics and square roots, as well as their properties.

Competency-Based Learning

GOA courses use a competency-based learning approach in which students build both GOA core competencies and discipline-specific ones. Throughout the semester, we assess outcomes tied to each competency to track student progress with the goal of students leaving the course able to use and apply these competencies well beyond the final day of the semester.

Competencies

Curate and create content relevant to real-world issues.

- *Logical argument*: You use logic and deductive reasoning to make and evaluate conclusions.
- *Geometric properties*: You articulate properties of geometric objects and distinguish among them.
- *Calculation*: You accurately calculate unknown values in geometric objects.
- *Geometric Proof:* You apply axioms, postulates and theorems with accuracy and flexibility to create compelling and complete formal geometric proofs.

Organize your time and tasks to learn independently.

- *Resourcefulness*: When asking for help, you demonstrate that you have already attempted to identify the problem and leverage the resources available to you to solve it on your own.

Reflect on and take responsibility for your learning and that of others.

- *Specificity*: You communicate precisely using accurate terminology, notation, and calculations.
- *Respect*: Your feedback uses nonjudgmental and affirmative language that aims at understanding intent. When possible, you address your audience by name.

Course Outline

Week 0	Orientation and Fundamentals of Geometry and Reasoning
COMPETENCIES: • Curate and create content relevant to real-world issues. • Organize your time and tasks to learn independently. • Reflect on and take responsibility for your learning and that of others.	

TOPICS COVERED:

- The Building Blocks of Geometry
- Mathematical Statements
- Segments and Angles
- Mathematical Proofs
- Elementary Postulates and Theorems
- Postulates and Angles

DESCRIPTION:

Nearly all of what we know about geometric objects is the result of a relatively small number of axioms and postulates. The key content of this unit includes the building blocks of geometry: the basic objects, definitions, and figures. Starting with just a point, a line, and a plane, students will explore fundamental components of geometry including angles, distance and measurement from a geometric standpoint, the concept of congruence, and applying properties and theorems to diagrams in order to draw conclusions about geometric objects. Emphasis will also be placed on recognizing geometric properties in the world.

Students will also be formally introduced to elements of both deductive and inductive reasoning and the construction of geometric proofs. Terminology such as statement, conditional, converse, inverse, contrapositive, and biconditional will be introduced and utilized, and students will construct their first two-column and narrative proofs.

EXAMPLE ASSESSMENT:

In addition to problem sets and completing proofs, use GeoGebra to construct a diagram containing certain objects and properties and identify those properties in other students' work. Identify the building block geometric objects from the chapter in the world near where you live.

Week 1	Parallel Lines and Planes

COMPETENCIES:

- Curate and create content relevant to real-world issues.
- Organize your time and tasks to learn independently.
- Reflect on and take responsibility for your learning and that of others.

TOPICS COVERED:

- Parallel and Perpendicular Lines
- Parallel Lines and Angles

- Proving Lines Parallel
- Sum of Angles of a Triangle
- Sum of Angles of a Polygon

DESCRIPTION:

We take for granted the fact that parallel lines and planes do not intersect: railroad tracks, latitude lines on the globe, and airplanes flying at different altitudes all rely on the idea of parallel. In this unit, the key questions are “How do we know when two lines are parallel?”, “What are the properties of parallel lines?”, and “What do parallel lines tell us about polygons?”

Students are introduced to the idea of transversals and the relationship between pairs of angles that are created by a transversal cutting two parallel lines. Those relationships are then applied to determine what information we need to know that two lines are parallel and to construct the relationship between the angles of a polygon and the number of sides it has.

At the end of the unit, we’ll also talk about the biggest assumption made in the history of geometry and why it took so long for anyone to determine whether or not it’s true, or if its truth even matters.

EXAMPLE ASSESSMENT:

In addition to problem sets and completing proofs, this unit will culminate with the first of three larger review problem sets that includes significant numbers of application and context-based problems.

Week 2	Triangles and Congruence

COMPETENCIES:

- Curate and create content relevant to real-world issues.
- Organize your time and tasks to learn independently.
- Reflect on and take responsibility for your learning and that of others.

TOPICS COVERED:

- Congruent Figures
- Congruent Triangles
- CPCTC
- Special Properties of Isosceles and Equilateral Triangles
- Special Segments in Triangles
- Transformations on a Coordinate Plane

DESCRIPTION:

Triangles are among the most widely used and foundational polygons in geometry. Throughout this substantial unit, students will explore the properties of triangles and introduce the idea of congruent figures.

Using several methods for determining whether triangles are congruent, students uncover important properties of triangles as well as other lines that interact with them, such as perpendicular bisectors, medians, angle bisectors, altitudes. Mathematical proof is a fundamental part of this unit, and this is where students will rely on proof methods most heavily.

Through those investigations, students will be introduced formally to the notion of a mathematical transformation, a systematic way of manipulating a given object or space, in the forms of translations, rotations, and reflections. Online platforms such as GeoGebra will be used to illustrate and explore both properties and transformations.

EXAMPLE ASSESSMENT:

In addition to problems sets and completing proofs, students will look at the process of tessellation - completely filling a plane by tiling congruent figures - and describe a particular tessellation of their choice using the language of congruent figures and transformations.

Week 3	Quadrilaterals and Similar Figures

COMPETENCIES:

- Curate and create content relevant to real-world issues.
- Organize your time and tasks to learn independently.
- Reflect on and take responsibility for your learning and that of others.

TOPICS COVERED:

- Quadrilaterals
- Properties of Parallelograms
- Midsegment Theorem
- Rectangles, Rhombuses and Squares
- Trapezoids and Kites
- Geometry and the Coordinate Plane
- Similar Figures
- Similar Triangles
- Similarity and Parallel Lines
- Three Parallel Lines Postulate

- Angle Bisectors
- Dilations

DESCRIPTION:

The study of quadrilaterals focuses on parallelograms and trapezoids, with emphasis given to how to determine if a quadrilateral is a parallelogram, properties of parallelograms, and properties of trapezoids, as well as the use of mathematical proof to determine the relationships described above

The relationships between polygons extends beyond whether they are congruent and their properties. This unit extends the idea of congruence to similarity.

Key aspects of this topic include the properties of proportions, the definition of similarity, properties of similar figures, and ways to prove that triangles are similar. Emphasis will also be placed on the application of similar figures, particularly triangles, in a variety of ways in the world. Students will also be introduced to the final major category of transformation: dilation.

EXAMPLE ASSESSMENT:

In addition to problem sets and completing proofs, students will investigate a common use of similar figures in the world of their choice, paying particular attention to the mathematics and scale factor involved. As a part of that investigation, students will also provide a specific example or that transformation and receive peer feedback on their work. This unit will also contain the second of three larger review problem sets.

Week 4	Right Triangles and Trigonometry

COMPETENCIES:

- Curate and create content relevant to real-world issues.
- Organize your time and tasks to learn independently.
- Reflect on and take responsibility for your learning and that of others.

TOPICS COVERED:

- Right Triangles
- Triangle Inequality
- Similarity in Right Triangles
- Geometric Mean
- Calculating Line Segment Lengths in Right Triangles
- The Pythagorean Theorem

- Sine, Cosine, and Tangent Ratios

DESCRIPTION:

In this unit, students are introduced to two of the most powerful, and well-known, concepts in geometry: the Pythagorean Theorem and right triangle trigonometry.

When working with right triangles, the Pythagorean Theorem simplifies many of the concepts from previously in the course, including congruence and determining the length of sides in triangles. A study of the theorem itself, its converse, and the properties of altitudes drawn from the right angle in a triangle.

The second half of the unit uses the properties of right triangles to introduce students to right triangle trigonometry. The sine, cosine, and tangent ratios are introduced in the context of right triangles, and emphasis is placed on the application of right triangle trigonometry to the world.

EXAMPLE ASSESSMENT:

In addition to problem sets and completing proofs, students will research a common historical or modern application of right triangle trigonometry in the world. The research will focus on the significance of the application at the time it was used and will include an example of how the mathematics was applied.

Week 5	Circles

COMPETENCIES:

- Curate and create content relevant to real-world issues.
- Organize your time and tasks to learn independently.
- Reflect on and take responsibility for your learning and that of others.

TOPICS COVERED:

- Circles
- Arcs, Central Angles, and Inscribed Angles
- Arcs and Chords in Circles
- Congruent Arcs and Chords
- Tangent Lines and Circles
- Other Angles in Circles
- Other Segment Lengths in Circles
- Intersecting Segments in Circles

DESCRIPTION:

So many aspects of our modern world rely on circular and spherical objects. Sports, transportation, and astronomy are just a few of the areas that would be entirely different without utilizing the properties of circles and spheres.

Fittingly, the study of circles and spheres is the culmination of all of the previous units of study as students rely on congruence, similarity, properties of parallel lines, and right triangles to explore the properties of circle, including arcs, chords, interior and exterior angles, and tangent lines. Significant emphasis is placed on the application of properties of circles to the world.

EXAMPLE ASSESSMENT:

In addition to completing the normal problem sets and completing proofs, this unit contains the final review problem set of the course, which continues to focus heavily on the application of acquired geometric skills to world contexts.

Week 6 & 7	Area and Volume

COMPETENCIES:

- Curate and create content relevant to real-world issues.
- Organize your time and tasks to learn independently.
- Reflect on and take responsibility for your learning and that of others.

TOPICS COVERED:

- Area, Surface Area and Volume
- Area of Parallelograms, Triangles, Trapezoids, Circles and Regular Polygons
- Compound Area and Circle Sectors
- Surface Area and Volume of Prisms, Cylinders, Pyramids and Cones

DESCRIPTION:

The most visually accessible part of geometry, everyone interacts with area, surface area, and volume on a daily basis. How much turf will it take to cover a field? How much paint will it take to completely paint the walls on the inside of a house? The geometry of surfaces and filling space is all around us.

The final unit in the geometry course is almost entirely application-based, as students use the area, surface area, and volume formulas to calculate values and make qualitative decisions for context-driven questions based on area and volume.

EXAMPLE ASSESSMENT:

In addition to problem sets, students will research and write a short biography of a mathematician who was influential in the development of geometry (particularly if they were from the same area of the world as the students), with particular attention paid to the contributions of that person to mathematics. As a part of that research, students will describe the impact of that contribution and its application to the world.

Course Notes

- **NOTE:** At the start of the term, students will need access to a printer in order to print out GOA's Geometry notebook that is required in this course
- **Optional** Textbook for additional practice and reference: *Geometry*, Jurgensen et al. - McDougal Littell - 2000
  ISBN - 9780395977279

Required Materials

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Material / Access / Cost:

- Print Geometry notebook / Printer / Varies
- At the start of the term, students will need access to a printer in order to print the notebook that is required in this course. Printers may be accessed at school, home, or local business.
- Optional, for reference: Geometry (Jurgensen et al., McDougal Littell- 2000) (ISBN - 9780395977279) / Print version available online or in-person bookstores / $33+ USD