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Student Activity 3

Developing Optics Equations

Science Background

Light is a fascinating thing. Look around. The room is full of light. You need light to see, but you cannot actually see the light as it fills the room. Scientists use different behaviours of light to build models that help explain what is happening. In this activity, you will use these models of light to understand several useful optics equations.

Part 1: Magnification and Thin Lens Equations

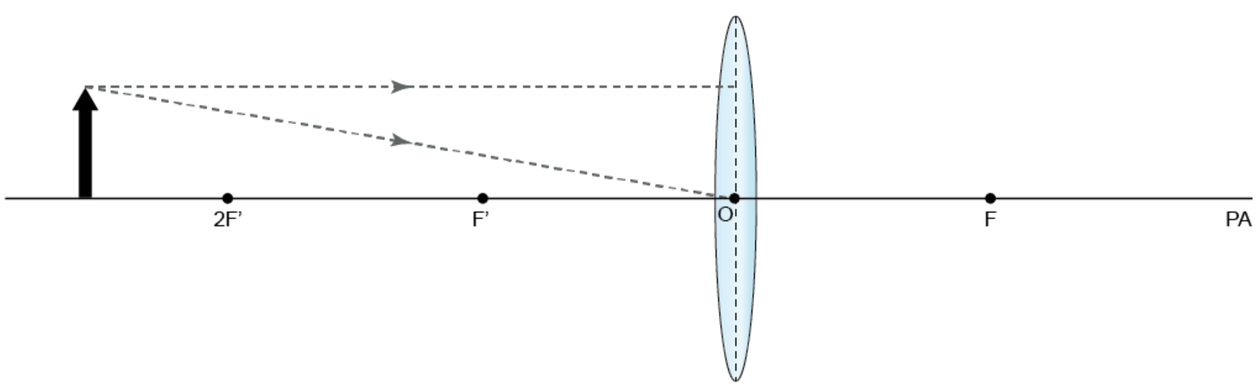
Light travels in straight lines. We can therefore represent it with straight lines or rays. We can use ray diagrams to locate the focused image formed by a lens, but they are not the only way to find the image. Equations, derived from ray diagrams, give us another way to approach lens problems. In this activity, you will examine ray diagrams to reveal connections between the object, image, and lens that are expressed in these two equations:

$$M = -\frac{h_i}{h_o} = \frac{d_i}{d_o} \qquad \frac{1}{d_i} + \frac{1}{d_0} = \frac{1}{f}$$

where

M is the magnification factor
h_o is the height of the object, measured from the principal axis (PA)
h_i is the height of the image, measured from the PA
d_o is the distance from the object to the centre of the lens along the PA
d_i is the distance from the image to the centre of the lens along the PA
f is the focal length of the lens, measured from the centre of the lens along the PA

1. The black arrow is the object. Extend the rays indicated by the dotted lines to locate the focused image of the arrow. Choose a different colour for each ray. Sketch the image of the arrow. Label h_o, h_i, d_o, d_i, and f. PA stands for principal axis.



2. Shade in the triangle formed by the object, the ray that goes through O, and the principal axis. Using the same colour, shade in the triangle formed by the image, the ray through O, and the principal axis. Consider the two angles in your triangles at point O. These are opposite angles. What property do opposite angles have?

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3. The object and the image are perpendicular to the principal axis, so the two triangles you shaded are *similar*. In similar triangles, the ratio of corresponding sides is a constant. Write an equation showing that the ratio of height over distance along the principal axis for the object is equal to the ratio of height over distance for the image.

4. Rearrange your equation to get h_i/h_o on the left side. What does this ratio of heights describe?

5. Is the image upright or inverted? What could you add to your equation to convey this information?

6. Two small right-angle triangles are formed to the right of the lens that both come to a point at F. Since the lens is thin, we can use the vertical dashed line to represent the lens. Shade in the triangle formed by the lens, the ray that goes through F, and the principal axis with a second colour. Use this second colour to also shade in the triangle formed by the image, the ray that goes through F, and the principal axis. These are similar triangles. What makes them similar?

7. Label the length of the sides along the principal axis in terms of d_i and f. Express the ratio of the height to the length of the sides for each triangle.

8. Equate the two ratios, and rearrange the terms to get h_i/h_o on the left side.

9. Equate this expression with the one you obtained in Step 4.

10. This combined expression is not very useful in its current form. Let's clean it up a little so that each term has only one variable in the numerator. Start by separating any fractions into individual terms, recalling that $\frac{x-y}{z} = \frac{x}{z} - \frac{y}{z}$.

11. Two of your terms should have the same numerator. Divide the equation by that numerator and simplify. You should now have an equation that relates the distances to the focal length of the lens.

12. Use your equations to verify the types, size, and location of an image formed when a 5.0 cm tall object is located 25.0 cm from a lens with a focal length of 15.0 cm. You should confirm that the image will be real, inverted, 7.5 cm tall, and 37.5 cm on the opposite side of the lens from the object.

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Part 2: Snell’s Law

Modelling light as a wave can help us understand why light is refracted at a boundary. Let’s use this model to take a closer look at where Snell’s law comes from. (Recall that Snell’s law describes the relationship between the angles of incidence and refraction for light travelling between media with different refractive indexes.) The index of refraction, *n*, expresses how much slower light travels in a medium, *v*, compared to in a vacuum, *c*. When light moves from one medium to another, the change in speed can cause the light to change direction. Snell’s law connects the angles with the indexes of refraction:

$$n_1 \sin sin\, \theta_1 = n_2 \sin sin\, \theta_2 , \quad n = \frac{c}{V}$$

where
n_1 is the index of refraction of the first medium
n_2 is the index of refraction of the second medium
θ_1 is the angle of incidence, measured from the normal
θ_2 is the angle of refraction, measured from the normal

Use the PhET Bending Light simulation with the settings selected as shown in ***Figure 1***. A beam of light is shown as a series of wave fronts travelling in the direction of the light ray. Notice that the wave fronts are perpendicular to the direction of the light ray and that successive waves are separated by an equal length of time because they are produced at regular intervals.

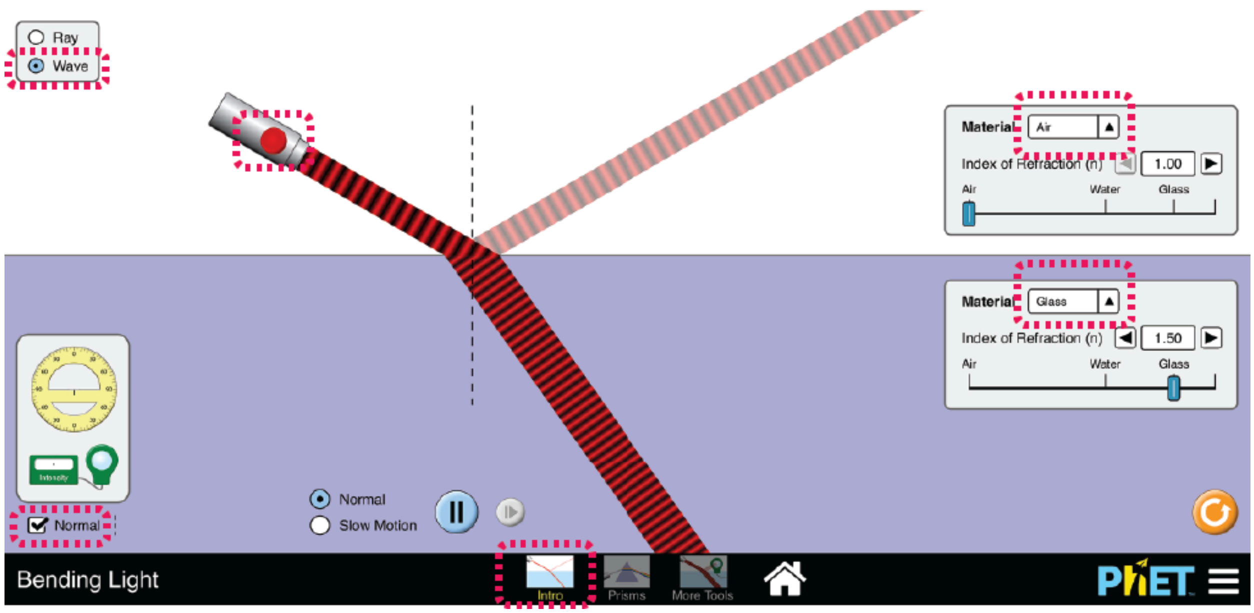

Figure 1 Screenshot of the PhET Bending Light simulation

1. Adjust the angle of incidence to be around 60°, using the protractor tool for guidance. Examine the incident and refracted beams.

(a) What happens to the distance between wave fronts in the glass?

(b) Would the change in medium affect the time intervals between successive wave fronts passing a

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point?

(c) What does that tell you about the speed of the wave in the glass?

Examine the region where the light moves from one medium to another (e.g., from air to glass). If we look at two successive wave fronts, shown in ***Figure 2*** as $\overline{AB}$ and $\overline{CD}$, we can see two right-angle triangles formed: one in air (ΔBAD) and the other in glass (ΔADC).

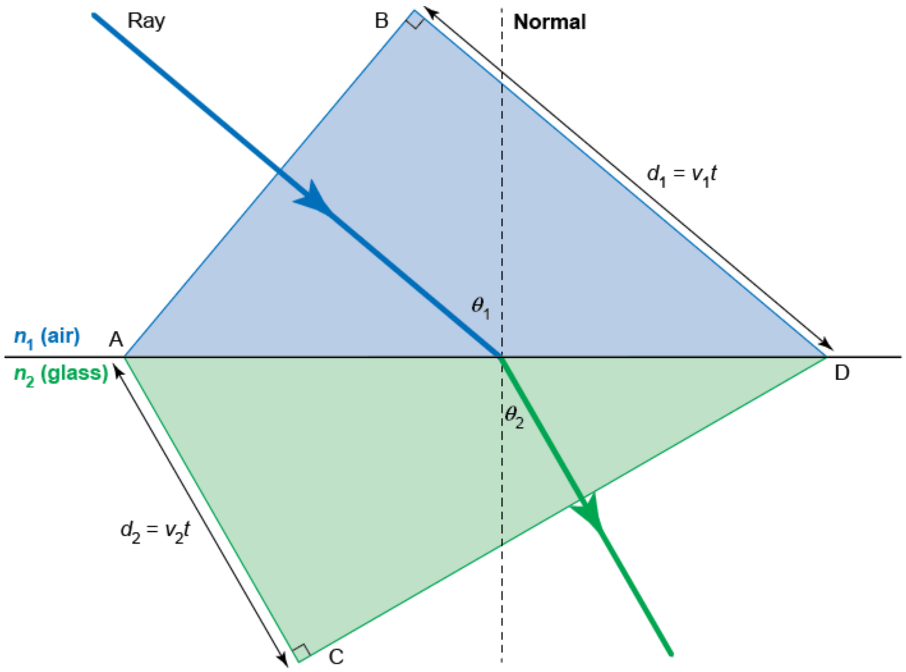

Figure 2 Light waves travelling from air into glass

2. Use geometry to show that $\angle BAD = \theta_1$ and $\angle ADC = \theta_2$.

3. ΔBAD and ΔADC are right-angle triangles. Using the definition $\sin x = \frac{opposite}{hypotenuse}$, write an expression for each triangle equating the sine of the angle (θ_1 or θ_2) with the sides.

4. The hypotenuse ($\overline{AD}$) is common to the two triangles. Rearrange your equations to solve for the hypotenuse and then combine your two equations into one. Simplify your equation.

5. The sides $\overline{AC}$ and $\overline{BD}$ show how far the light travels in the same amount of time. Express these distances using the index of refraction, *n*, and the speed of light in a vacuum, *c*.

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6. Substitute these expressions into your equation from Step 4 and simplify.

7. Use your equation to verify that when light in water (n_{water} = 1.33) is incident on glass (n_{glass} = 1.5) at an angle of 40°, the angle of refraction is 35°.

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Consolidate Your Learning

Answer the following questions to check your understanding of optics equations.

1. In Part 1, the real image is formed on the side of the lens that is opposite to the object. Where would a virtual image be formed? How could you indicate this when using the equations?

2. A biologist is examining a specimen using a magnifying glass (f = 12 cm). The lens is 10 cm above the 2.5 cm long specimen. Use your lens equation from Part 1, Step 11, to give a complete description (size, attitude, location, type) of the focused image produced by the lens.

3. Refer to the wave fronts in Part 2, ***Figure 2***.

(a) How would the diagram change if the light were moving from glass to air?

(b) Why does light bend away from the normal when it speeds up?

(c) What happens to the light when the angle of incidence is zero?

4. Total internal reflection (TIR) is a very important phenomenon in which all of the light that is incident on a transparent surface is reflected back into the original (more optically dense) medium. It is the basis of all fibre optics technology. TIR occurs when light that is in a higher-index material moves toward a lower-index material at a large angle.

   What is the largest angle of refraction that a light ray can reasonably have? Sketch this situation, and then use your equation from Part 2, Step 6, to develop an equation for the angle of incidence when this maximum angle of refraction happens. This angle of incidence is called the **critical angle**, θ_c.