A tutorial on pH and Logarithms

Logarithms:
The pH scale for measuring the relative strength of acids and bases is built upon the base 10 logarithm, sometimes called common logarithm. In general, the logarithm of any number is the power to which the base must be raised in order to equal this number. So, for common logarithms, the value of the logarithm of a number is the power to which 10 must be raised in order to equal the number.

Examples:

$10 = 10^1$	log (10) = 1
$1 = 10^0$	log (1) = 0
$100 = 10^2$	log (100) = 2
$1000 = 10^3$	log (1000) = 3
$0.01 = 10^{-2}$	log (0.01) = -2

pH
The pH scale was developed by Sören Sörenson in 1909. The name "pH" comes from the German *Potenz Hydrogen*, which simply means "power of hydrogen," sometimes referred to as the hydrogen ion exponent. The pH is found by taking the negative of the common logarithm on the molar concentration of hydrogen ions in solution.

$$pH = -\log([H^+])$$

where $[H^+]$ is the molarity of hydrogen ions in solution.

Common properties of logarithms
Logarithms have a number of properties that can be useful in calculations. The most frequency used properties are:

1. $\log(a^n) = n \log(a)$
2. $\log(a \cdot b) = \log(a) + \log(b)$
3. $\log(a/b) = \log(a) - \log(b)$

Approximating logarithms
Often when using our calculators it is convenient to know if the answer is reasonable. We can use estimation to do this. For example, 5×10^{-6} is half-way between 1×10^{-5} and 1×10^{-6}. We can express this as:

$$1 \times 10^{-5} < 5 \times 10^{-6} < 1 \times 10^{-6}$$

Since pH is lower for higher concentrations, and we know that the pH for a $[H^+]$ concentration of 1×10^{-5} and 1×10^{-6} are 5 and 6, respectively, the pH for $[H^+] = 5 \times 10^{-6}$ must bet between 5 and 6. We can use calculators to determine that $-\log(5 \times 10^{-6}) = 5.3$, which is in the range that we had already found.

Note: Although 5×10^{-6} is half-way between 1×10^{-5} and 1×10^{-6}, the value of the logarithm is not. Values of the logarithm function are not linear. Some samples are included below:

number	log (number)
0	0
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	1

Inverse logarithms

When we are given the pH, we can find the concentration.

$$[H^+] = 10^{-pH}$$

Examples:

pH = 2.0 $[H^+] = 1.0 \times 10^{-2}$
pH = 7.0 $[H^+] = 1.0 \times 10^{-7}$
pH = 10. $[H^+] = 1.0 \times 10^{-10}$
pH = 1.7 $[H^+] = 2.0 \times 10^{-2}$
pH = 6.4 $[H^+] = 4.0 \times 10^{-7}$
pH = 9.1 $[H^+] = 8.0 \times 10^{-10}$

Autoionization of water

Hydroxide ion concentration

It is also possible to measure the hydroxide ion concentration.