Linear momentum and impulse

- Definition of linear momentum
- Conservation of momentum
- Definition of Impulse – Graphs momentum-time
- Application to collisions – Elastic and inelastic collisions

1. Definition of linear momentum

1.1. Momentum - Characteristics

The linear momentum of an object of mass m is noted p and is a vector: p = m.v

Units: [p] = [m]x[v] = kg.m.s^{-1}

1.2. Momentum and Newton's second law

$$F_{net} = ma = m \cdot \frac{v_2 - v_1}{\Delta t} = \frac{mv_2 - mv_1}{\Delta t} = \frac{p_2 - p_1}{\Delta t} = \frac{\Delta p}{\Delta t}$$

We can write F = m.a only if the mass of the object is constant.

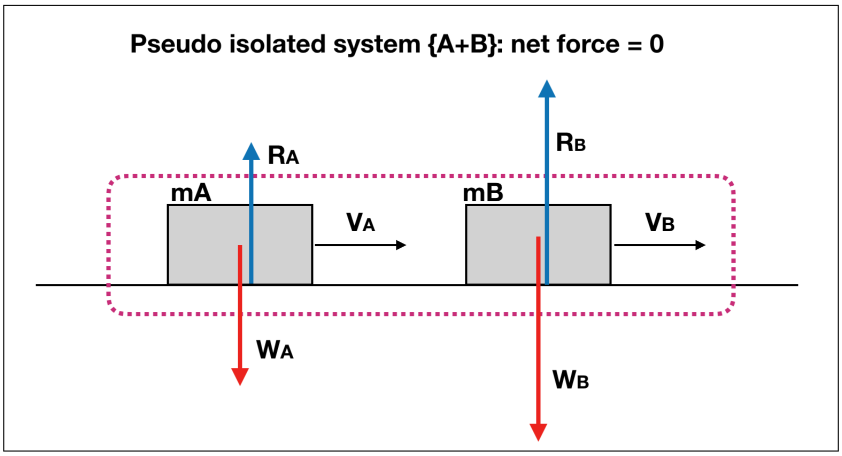
$$\text{Since } F_{net} = \frac{\Delta p}{\Delta t} \text{ we can write: } \Delta p = F_{net} \cdot \Delta t$$

Δp, the change in momentum, is called impulse. The units of impulse are N.s

So the other units of the momentum p is also N.s

2. Conservation of momentum

The momentum is a very important quantity in physics.
If a system (for example the system composed of the objects A and B) is isolated (no forces apply on it) or pseudo-isolated (the forces balanced each other) the total linear momentum is always conserved.

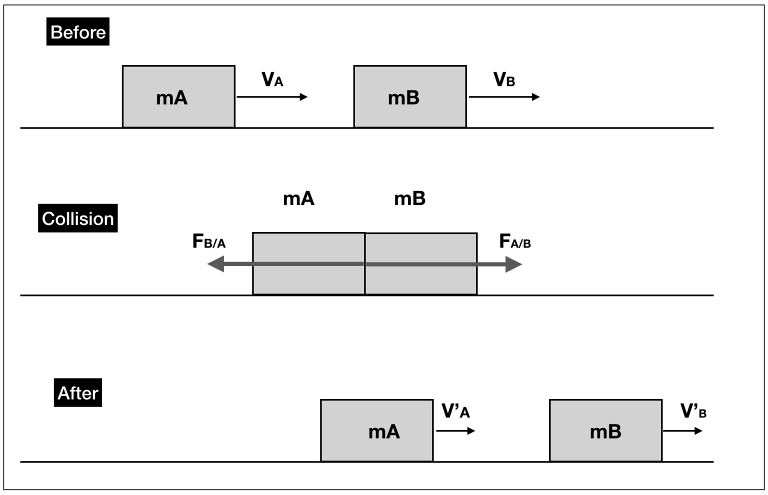


Demonstration:

Let's consider 2 objects A and B.
A and B will collide. The initial velocities are v_A and v_B before the collision and v'_A and v'_B after the collision.

Before the collision, the total linear momentum is:

$$p_{initial} = p_A + p_B = m_A v_A + m_B v_B$$



Let's consider the system during the collision.

- Based on the third Newton's law:

$$F_{A/B} = -F_{B/A}$$

- Using the second Newton's law we can write:

$$F_{B/A} = \frac{m_A(v'_A - v_A)}{\Delta t} = \frac{m_A v'_A - m_A v_A}{\Delta t}$$

$$F_{A/B} = \frac{m_B(v'_B - v_B)}{\Delta t} = \frac{m_B v'_B - m_B v_B}{\Delta t}$$

$$\frac{m_B v'_B - m_B v_B}{\Delta t} = -\left(\frac{m_A v'_A - m_A v_A}{\Delta t}\right) = \frac{m_A v_A - m_A v'_A}{\Delta t}$$

- We finally obtain the following equation:

$$m_B v'_B - m_B v_B = m_A v_A - m_A v'_A$$

$$m_B v'_B + m_A v'_A = m_A v_A + m_B v_B$$

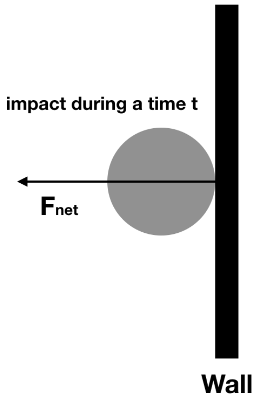
$$p'_{final} = p_{initial}$$

3. Definition of impulse

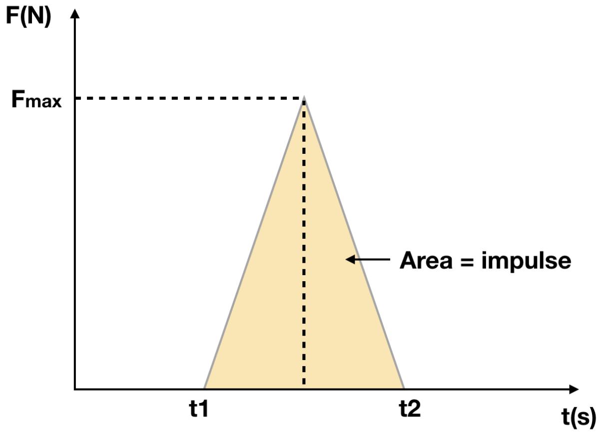
Impulse is the change in time of the momentum:

$$\Delta p = F_{net} \cdot \Delta t$$

Example: a ball of mass m hit a wall during a time interval t:



If we plot the graph F versus time, the area under the line is the impulse:



4. Application to collisions – Elastic and inelastic collisions

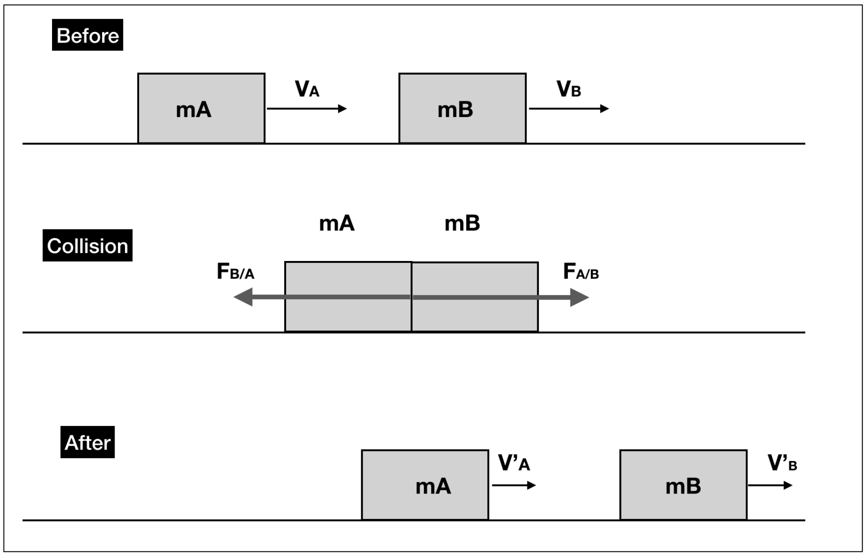
During a collision, the total linear momentum is always conserved.

- If the total kinetic energy KE is conserved the collision is considered as an elastic collision
- If the total kinetic energy KE is not conserved the collision is considered as an inelastic collision

Note - the kinetic energy is given by the following relation:

$$KE = \frac{1}{2}mv^2$$

With KE in Joule, m in kg and v in ms^{-1}



- Before the collision:

$$KE_{tot-initial} = \frac{1}{2} m_A v_A{}^2 + \frac{1}{2} m_B v_B{}^2$$

- After the collision:

$$KE_{tot-final} = \frac{1}{2} m_A v_A{}^{'2} + \frac{1}{2} m_B v_B{}^{'2}$$

If $KE_{final} = KE_{initial}$ the collision is elastic