Javier Garcia Mat 2680HD42 Project

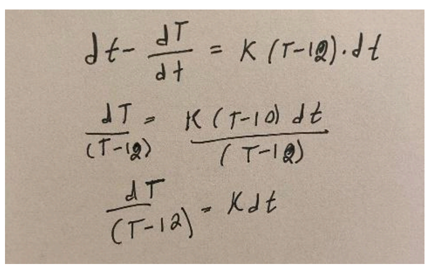
The question you are looking at is from Section 4.2 question one on Cooling and mixing using or demonstrating Newtons law of cooling. 1. A thermometer is moved from a room where the temperature is 70F to a freezer where the temperature is 12F. After 30 seconds the thermometer reads 40F. What does it read after 2 minutes?

$$\frac{dT}{dt} = K(T - Tm)$$

Today’s differential equation is basically telling us how fast an object is cooling is going to be a proportional to the difference in temperature between the object and the ambient temperature. The greater the difference between an object and the surrounding, the faster the object will cool. And the lower the difference between the object and the surrounding, the slower that object will cool. We know that the thermometer begins at a temperature of 70 degrees. So, we can say that T of time zero is equal to 70 And it's moved inside where the ambient temperature is 12 degrees. So, t sub m will equal 12 and then after one half of a minute, the thermometer's temperature is 40 degrees. So, we could say that T of one half is equal to 40. The first objective is to figure out what the thermometer's temperature is after 2 minutes.

$T(0) = 70$ $\quad$ $Tm = 12$ $\quad$ $T(1/2) = 40$ $\quad$ $T(1) = ?$

We need T of two and in order for us to do that, we have to solve that differential equation. Notice we have plugged the 12 in for the ambient temperature, t sub M. Now, to solve this we are going to separate variables. From there, we would be multiplying both sides by DT. That would cancel it out on the left side. We will divide both sides of the equation by this term T minus 12, which will cancel it out on the right hand side. We have separated the variables. Notice that the temperate variable is gathered on the left side and the time variable is gathered on the right side.



Next, we have to integrate both sides of the equation. You would have to do a U substitution. You would let U equal the term in the denominator. The capital T minus 12. You would differentiate both sides with respect to temperature. So, du, DT, the derivative of one t is just one, and the constant 12 goes to zero when you differentiate it. Multiply both sides of that by DT, and you can see that du is equal to DT. You have the integral of du over U. From calculus, we know that this equal to the natural log of the absolute value of. And if we replace the U back with the T minus 12, we have the natural log of T minus 12.

$$\int \frac{dT}{(T-12)} = \int k\,dt$$

$$u = T-12$$

$$\frac{du}{dT} = 1$$

$$du = dT$$

$$\int \frac{du}{u} = \ln|u| = \ln|T-12|$$

Here we have the natural log of the absolute values of T minus 12 and then this was equal to the integral of k DT. You can factor out the constant of proportionality and then you're just integrating DT. Integrating DT which is just T. The right hand side becomes K multiplied by t plus arbitrary constant of integration. We would have to raise R to both sides of this equation. What that does is cancels out the natural log. Remember that the natural log and the base e exponential function are inverses so they cancel each other out. This leaves us with t minus 12 is equal to E to the power of KT plus c. We want to rewrite the right hand side and to do that, one of the laws of exponents allows us to take an exponential that is raised to the power of a sum and rewrite that as E to the power of A times E to the power of B. We would have E to the power of KT times E to the power of C. It is looking really good now and we are almost there!

$$\Rightarrow \ln|T-12| = \int k\,dt$$

$$= k\int dt$$

$$= kt$$

$$\cancel{\ln}|T-12| = kt + c$$

$$e \qquad\qquad e$$

$$T-12 = e^{kt+c} \qquad e^{a+b} = e$$

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$$T = e^{kt}\cdot e^{c}$$

We plug zero in for the little T and 70 in for the capital T and the left side becomes 70 minus 12 which is just 28 R to the power of zero times E to the power of C. We should recall that E to the power of zero is just one. T minus 12 equals 58 times E to the KT and input T half a minute equals 40 which is the last part of the question. You are going to have e to the one half times K and divide from both sides by 58. Divide and you get 14/29 equals e to the 1/2k power. By solving for K we have to take the natural log of both sides. Natural log and E cancel each other out. You have Ln of 14/29 to one half Multiple both sides of this equation by two and have two Ln of 14/29.

$T - 12 = e^{kt} \cdot e^{c}$ $T(0) = 70$

$70 \uparrow$ t T

$58 = e^{0} \cdot e^{c}$

$T - 12 = e^{kt} \cdot 58$

$T - 12 = 58e^{kt}$ $T(\frac{1}{2}) = 40$

$40 \uparrow$

$\frac{28}{58} = \frac{58e^{kt}}{58}$

$\frac{14}{29} = e^{\frac{1}{2}k}$

$\ln \frac{14}{29} = \cancel{\ln} e^{\frac{1}{2}k}$

After you have used a calculator now its time to find after two minutes which is T equals Tc + Ce power of kt equals 12 plus 58e to the power negative 1.45647 multiply by 2. Capital T euqals 15.15F and the last thing you have to do is T(2) equals 15.15 and there is your answer.

$\ln\left(\frac{14}{29}\right) = \frac{1}{2}k$

$2\ln\left(\frac{14}{29}\right) = \cancel{2} \cdot \cancel{\frac{1}{2}}k$

$-0.7282385 = 0.5k$

$-1.456477 = k$

After 2 min

$T = T_c + Ce^{kt} = 12 + 58e^{-1.45647 \times 2}$

$T = 15.15f$

$T(2) = 15.15$