Data-Driven Stoch Opt

Data-Driven Stochastic Optimization

(PhD School on Intersections of Algorithms and Machine Learning Theory)

Course Resources

- Links to Slides: Lec 1, Lec 2, Lec 3, Lec 4, Lec 5, Lec 6
- Video Recordings: Lec 1, Lec 2, Lec 3, Lec 4, Lec 5, Lec 6
- Link to Exercises
- Prereqs for Summer School: Many of the topics and relevant background is available at this course website. The video recordings for this GT course is available here. Some specific topics of interest are:
    - Basic concentration bounds: Lecture 2
    - Markov Decision Process: Lecture 14
    - Stochastic Multi-armed Bandits: Thomas' notes,
    - Pandora's Box: Lecture 15
    - Stochastic Probing: Lecture 17

Lec 1: What is Stochastic Optimization?

- Change inputs from being given to random variables from given distributions.
- We will often assume n independent random vars and maximize expected value.
- Examples: Revenue maximization, Optimal Stopping, Pandora's box
- Goal given distributions: Find a (near-)optimal algorithm:
    - Think of a decision tree/MDP.
    - Alternatively, think of it as a class of policies. Find a near-optimal soln.
    - Computation is the only bottleneck.
- Techniques:
    - Closed form solutions
    - Dynamic programming
    - Linear programming
    - Index-based policies
- But what if the input distributions are unknown and we only have historical data?
    - We want generic techniques that apply to many pbs (lots of pb specific work)
    - Next 3 lectures on general techniques for full and limited feedback.
    - Last 2 lectures on the important Online Resource Allocation problem.

Lec 2: What is Data-Driven Stoch Optimization?

- What if the input distributions are unknown and we only have historical data? We want to still obtain the best possible approximation.
- What does “historical data” mean?
  1. Offline Learning: Given a small number of full samples (full-feedback)
  2. Online Learning: We learn as we go with full/partial feedback.
- Simple bounds 1: Number of algorithms is small
  - Class of algos is actually small or we can consider a smaller interesting subclass
  - Apply Chernoff bounds.
  - Nice but often not a polytime algo. Did not exploit product dist.
- Simple bounds 2: Distributions have a small support
  - Basics of TV distance.
  - Learn each D_i in a small TV distance.
  - This implies we can learn D in a small TV distance.
  - Optimize for the learnt distribution.
- Challenges in extending to continuous distribution:
  - Can’t learn continuous distrib in TV distance
  - However, we can learn an “envelope”

Lec 3: Learning Strongly Monotone Stochastic Problems

- Previous algo either not polytime or assumed discrete distrib.
- Even though “fine discret” often works, it’s suboptimal to use above general bounds.
- Hope: Can we just run an algo for the product empirical distrib?
- Guo et al show: yes, for any strongly monotone problem
- Let’s again aim for T=n^2/\eps^2 since we only use TV distance. Algo is to again take the empirical distribution E and feed it to blackbox.
- **Proof**: First observe that for all t, |F_E(t) - F_D(t)| \leq 1/\sqrt{T}. Now we need to show D^+ \geq E \geq D^- where the TV distance between \delta(D^-,D)\leq 1/\sqrt{T}. This can be achieved by choosing the cdf of D^- to be 1/\sqrt{T}+F_D(t). The TV distance is small by considering the coupling X^- = X*1[X<Top(1-\eps)].
- Can we improve the bounds? Yes, we can always improve n^2 to n by working with Hellinger distance (see paper for details). In general this is tight, but for some problems we can remove n, like for optimal stopping.

Lec 4: Online Learning with Partial Feedback

- What is online learning for stochastic optimization?
- Bandit feedback: Simple bounds via Guo et al.
    - Use some rounds for exploring and some for exploiting
    - In general we cannot obtain \sqrt{T} with bandit feedback, but for some problems we can: e.g., optimal stopping and pandora's box in this paper.
- What is semi-bandit feedback feedback?
- General results for discrete distributions with semi-bandit feedback.

Lec 5: Online Resource Allocation

- What is Online Resource Allocation
- Why do we need some underlying distributions?
- How to solve this stochastic problem when distributions are known?
- What if the inputs are from an unknown identical distribution?
- An approach via Multiplicative Weights algorithm.

Lec 6: Online Resource Allocation for Non-Identical Distributions

- Recall Online Resource Allocation
- What if the distributions are not identical?
- Single-sample algorithms
- Exponential pricing algorithm (more details in this paper)
- Discuss exercises from the summer school.