financialalgebra

6.6 Intro to Logarithms

Student Activity Packet

UNIT: INVESTING STRATEGIES & EXPONENTIAL FUNCTIONS

Name:

IN THIS LESSON, YOU WILL:

- Explain what the purpose of a logarithm is
- Convert between exponential and logarithmic functions
- Use the change of base formula to evaluate logarithms
- Solve exponential equations using logarithms
- Use the Rule of 72
- Determine the time required to meet investing goals

INTRO

EXPLORE: How Long Does Your Investment Take to Mature?
Delilah makes an investment of $10,000 at 7% annual interest and wants to wait until it reaches $15,500 before withdrawing her investment.

1. **Write an exponential equation that represents this situation where x represents years.**

2. **What problems do you run into trying to solve an equation like this?**

3. **For now, estimate the amount of time it will take to reach $15,500. (We'll revisit this later!)**

LEARN IT

VIDEO: Introduction to Logarithms
As you saw in the intro, we sometimes need to solve for an exponent but we haven't learned how to isolate a variable in the exponent part of an equation. That's where logarithms come in! Watch this introduction to logarithms. Then, answer the questions.

1. **Briefly summarize what logarithms are used for.**

PRACTICE IT

PRACTICE: Converting Logarithms
Complete each problem by converting to the specified form.

1. **Convert each exponential equation into logarithmic form.**
   a. $8^2 = 64$
   b. $10^0 = 1$
   c. $4^x = 128$

2. **Convert each logarithmic equation into exponential form.**
   a. $\log_2 32 = 5$
   b. $\log_7 49 = 2$
   c. $\log_5 x = 3$

LEARN IT

EXAMPLE
Before we can solve for an exponential equation, we need to convert it into logarithmic form.

1. **Review the completed example below.**

1	**Isolate:** If there are any numbers on the same side of the equation as the exponent and base, move them to the other side	**$5 \cdot 2^3 + 6 = 46$** **$5 \cdot 2^3 = 40$** **$2^3 = 8$**
2	**Recall:** Recall the connection between exponential equations and logarithmic equations and their meaning	**Exponential: $b^x = c$** What number do you get when you raise b to the x power? The answer is c. **Logarithm: $\log_b c = x$** What number did you raise b to in order to get an answer of c? You raised b to the x power.

3 **Rewrite:**
Rewrite your example using the conversion above. Then explain what it means.

Exponential: $2^3 = 8$
What number do you get when you raise 2 to the 3 power? You get 8.

Logarithm: $\log_2 8 = 3$
What number did you raise 2 to in order to get an answer of 8? You raised 2 to the 3 power.

PRACTICE IT

PRACTICE: Converting to and Evaluating Logarithms
Complete each problem by converting to logarithmic form and, if necessary, solving for the variable.

1. **Isolate each base and exponent, then convert to logarithmic form.**
   a. $5^3 + 1 = 126$
   b. $3 \cdot 7^2 - 4 = 143$
   c. $4 \cdot 2^x + 1 = 33$

2. **Evaluate each logarithm**
   a. $\log_2 128$
   b. $\log_4 64$
   c. $\log_3 81$

3. **Convert each of the following to logarithmic form, then solve for the variable**
   a. $6^x = 36$
   b. $3^x - 7 = 74$
   c. $2 \cdot 4^x + 1 = 33$

LEARN IT

The Change of Base Formula
Some logarithms are easy to evaluate in your head, but there are some that aren't. Let's take a look at a new formula that will help you solve all logarithms.

Let's consider the following two logarithms to see why we need the change of base formula:

What is $\log_5 25$?
This one is easy to do in your head because this is asking the question: "What power did you raise 5

to to get an answer of 25?" Another way of thinking about this is the exponential form: $5^x = 25$. **We can easily get an answer of 2.**

What is $\log_5 30$?

This logarithm is a little bit harder. The question here is "What power did you raise 5 to to get an answer of 30?" This is not obvious because it's a decimal. **We need the change of base formula for this example.**

The change of base formula lets us use the log button on our calculator to get an exact value for a logarithm.

$$\log_5 30 = \frac{\log 30}{\log 5}$$

This means that $\log_5 30 = 2.11328$. We can verify this by checking $5^{2.11238}$ to see if we get 30.

PRACTICE IT

PRACTICE: Evaluating Logarithms using the Change of Base Formula

1. **Evaluate each of the following using the change of base formula. Round to 3 decimal places.**
   a. $\log_4 35$

b. $\log_8 50$

c. $\log_{10} 1200$

Now that we know how to use logarithms to solve for an exponent, let's revisit Delilah's situation from the Intro. Recall that Delilah is investing $10,000 at 7% annual interest until her account reaches $15,500.

2. Use the equation that you set up in the intro and solve it to calculate how long her money must be invested before it reaches its target amount.

3. Why might it be important for Delilah to know the exact amount of time it takes instead of an estimate?

APPLY IT

Follow your teacher's directions to complete the Application Problems.

Teachers, you can find the Application problems linked in the Lesson Guide.